Assignment 9: Vectors, Curves, Surfaces, Vector Functions

2. **(D)** Determine the equation of the cylinder generated by a line through the curve $(x-2)^2 + y^2 = 4,\ z = 0$ moving parallel to the vector $\vec{i} + \vec{j} + \vec{k}$.

8. **(D)** If a plane curve has the Cartesian equation $y = f(x)$ where f is a twice differentiable function, then show that the curvature at the point $(x, f(x))$ is $\dfrac{|f''(x)|}{[1 + f'(x)^2]^{3/2}}$.

9. **(D)** For the curve $c(t) = t\vec{i} + t^2\vec{j} + \frac{2}{3}t^3\vec{k}$ find the equations of the tangent, principal normal and binormal. Also calculate the curvature of the curve.

Assignment 9 - Solutions

2. Any point on the curve is of the form $(x_0, y_0, 0)$. The equation of a line passing through $(x_0, y_0, 0)$ and parallel to $(1, 1, 1)$ is
$\frac{x-x_0}{1} = \frac{y-y_0}{1} = \frac{z-z_0}{1}$. We get $x_0 = x - z$ and $y_0 = y - z$.
Since $(x_0, y_0, 0)$ lies on the curve, we get the equation of the cylinder to be $(x - z - 2)^2 + (y - z)^2 = 4$.

8. The equation of the plane curve is $R(t) = xi + f(x)j$. Use the formula $\kappa = \frac{\|v \times a\|}{\|v\|^3}$.
(The detailed solution is given in the notes)

9. Note that $c'(t) = i + 2tj + 2t^2$. Use the formulae $T(t) = \frac{c'(t)}{\|c'(t)\|}, N(t) = \frac{T'(t)}{\|T'(t)\|}$ and $B(t) = T(t) \times N(t)$.

   For the equations of tangent, principle normal and binormal at a point on the curve, we have to fix a point on the curve.

   The solution is given in the notes.