Assignment 11: Directional derivatives, Maxima, Minima, Lagrange Multipliers

1. **(D)** Let $f(x,y) = \frac{y}{|y|}\sqrt{x^2+y^2}$ if $y \neq 0$ and $f(x,y) = 0$ if $y = 0$. Show that f is continuous at $(0,0)$, it has all directional derivatives at $(0,0)$ but it is not differentiable at $(0,0)$.

2. **(T)** Let $f(x,y) = \frac{1}{2}\Big(\big|\, |x| - |y| \,\big| - |x| - |y| \Big)$. Is f continuous at $(0,0)$? Which directional derivatives of f exist at $(0,0)$? Is f differentiable at $(0,0)$?

3. **(T)** Find the equation of the surface generated by the normals to the surface $x + 2yz + xyz^2 = 0$ at all points on the z-axis.

4. **(T)** Examine the following functions for local maxima, local minima and saddle points:

   $i)\ 4xy - x^4 - y^4 \qquad ii)\ x^3 - 3xy$

5. **(D)** Let $f(x,y) = 3x^4 - 4x^2y + y^2$. Show that f has a local minimum at $(0,0)$ along every line through $(0,0)$. Does f have a minimum at $(0,0)$? Is $(0,0)$ a saddle point for f?

6. **(T)** Find the absolute maxima of $f(x,y) = xy$ on the unit disc $\{(x,y) : x^2 + y^2 \leq 1\}$.

7. **(D)** Assume that among all rectangular boxes with fixed surface area of 20 square meters, there is a box of largest possible volume. Find its dimensions.

8. **(D)** L&T produces steel boxes at three different plants in amounts x, y and z, respectively, producing an annual revenue of $R(x,y,z) = 8xyz^2 - 200(x+y+z)$. The company is to produce 100 units annually. How should production be distributed to maximize revenue?

9. **(T)** Minimize the quantity $x^2+y^2+z^2$ subject to the constraints $x+2y+3z = 6$ and $x + 3y + 9z = 9$.