Assignment 12 : Double Integrals

1. **(D)** Evaluate the integral $\iint\limits_R (x+y)^2 dxdy$ over the triangle R with vertices $(0,0),\ (2,2)$ and $(0,1)$.

2. **(T)** Evaluate the following integrals:

   $i)\ \int\limits_0^1 \int\limits_0^{\sqrt{1-x^2}} \sqrt{1-y^2}dydx \qquad ii)\ \int\limits_0^{\pi}\int\limits_x^{\pi} \frac{\sin y}{y} dydx \qquad iii)\ \int\limits_0^1\int\limits_y^1 x^2 \exp^{xy} dxdy.$

3. **(D)** Evaluate the integral $\iint\limits_R (x^2 - y^2)dxdy$ over the square R with vertices $(0,0),\ (1,-1),\ (2,0)$ and $(1,1)$.

4. **(T)** Evaluate $\iint\limits_R xdxdy$ where R is the region $1 \leq x(1-y) \leq 2$ and $1 \leq xy \leq 2$.

5. **(D)** The cylinder $x^2 + z^2 = 1$ is cut by the planes $y = 0, z = 0$ and $x = y$. Find the volume of the region in the first octant.

6. **(T)** Compute $\lim\limits_{a \longrightarrow \infty} \int\int\limits_{D(a)} \exp^{-(x^2+y^2)} dxdy$, where

   $i)$ $D(a) = \{(x,y) : x^2 + y^2 \leq a^2\}$ and $ii)$ $D(a) = \{(x,y) : 0 \leq x \leq a,\ 0 \leq y \leq a\}$.

   Hence prove that $\int\limits_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

7. **(D)** Change the order of integration to prove that

   $i)\ \int\limits_0^x \int\limits_0^u e^{m(x-t)} f(t)dtdu = \int\limits_0^x (x-t)e^{m(x-t)} f(t)dt,$

   $ii)\ \int\limits_0^x \int\limits_0^v \int\limits_0^u e^{m(x-t)} f(t)dtdudv = \int\limits_0^x \frac{(x-t)^2}{2} e^{m(x-t)} f(t)dt.$