

### Assignment 13 : Triple Integrals, Surface Integrals, Line integrals

1. (D) Integrate  $ze^{x^2+y^2} dx dy dz$  over the cylinder  $x^2 + y^2 \leq 4$ ,  $2 \leq z \leq 3$ .
2. (T) Evaluate the integral  $\iiint_W \frac{dz dy dx}{\sqrt{1+x^2+y^2+z^2}}$ ; where  $W$  is the ball  $x^2 + y^2 + z^2 \leq 1$ .
3. (D) Find the area of the surface of the portion of the sphere  $x^2 + y^2 + z^2 = 4a^2$  that lies inside the cylinder  $x^2 + y^2 = 2ax$ .
4. (T) What is the integral of the function  $x^2z$  taken over the entire surface of a right circular cylinder of height  $h$  which stands on the circle  $x^2 + y^2 = a^2$ . What is the integral of the given function taken throughout the volume of the cylinder.
5. (D) Compute  $\iint_S xy d\sigma$ , where  $S$  is the surface of the cone  $x = r \cos t$ ,  $y = r \sin t$ ,  $z = r$  for  $0 \leq r \leq 1$  and  $0 \leq t \leq 2\pi$ .
6. (T) Find the line integral of the vector field  $F(x, y, z) = y\vec{i} - x\vec{j} + \vec{k}$  along the path  $\mathbf{c}(t) = (\cos t, \sin t, \frac{t}{2\pi})$   $0 \leq t \leq 2\pi$  joining  $(1, 0, 0)$  to  $(1, 0, 1)$ .
7. (D) Evaluate  $\int_C \frac{-y dx + x dy}{x^2 + y^2}$ , where  $C := \{(x, y) : x^2 + y^2 = 1\}$ .
8. (T) Evaluate  $\int_C T \cdot dR$ , where  $C$  is the circle  $x^2 + y^2 = 1$  and  $T$  is the unit tangent vector.
9. (T) Show that the integral  $\int_C yz dx + (xz + 1) dy + xy dz$  is independent of the path  $C$  joining  $(1, 0, 0)$  and  $(2, 1, 4)$ .