Assignment 14 : Green's /Stoke's /Gauss's Theorems

1. **(T)** Use Green's Theorem to compute $\int_C (2x^2 - y^2)\, dx + (x^2 + y^2)\, dy$ where C is the boundary of the region $\{(x, y) : x, y \geq 0 \;\; \& \;\; x^2 + y^2 \leq 1\}$.

2. **(D)** Show that the value of the line integral $\int xy^2 dx + (x^2 y + 2x) dy$ around any square depends only on the size of the square and not on its location in the plane.

3. **(D)** Evaluate $\int_C \frac{xdy - ydx}{x^2 + y^2}$ along any simple closed curve in the xy plane not passing through the origin. Distinguish the cases where the region R enclosed by C:
   (a) includes the origin (b) does not include the origin.

4. **(T)** Use Stoke's Theorem to evaluate the line integral $\int_C -y^3\, dx + x^3\, dy - z^3\, dz$, where C is the intersection of the cylinder $x^2+y^2 = 1$ and the plane $x+y+z = 1$ and the orientation of C corresponds to counterclockwise motion in the xy-plane.

5. **(D)** Verify the Stoke's Theorem where $\overrightarrow{F} = (y, z, x)$ and S is the part of the cylinder $x^2 + y^2 = 1$ cut off by the planes $z = 0$ and $z = x + 2$, oriented with $\overrightarrow{n}$ pointing outward.

6. **(T)** Let $\overrightarrow{F} = \frac{\overrightarrow{r}}{|\overrightarrow{r}|^3}$ where $\overrightarrow{r} = x\,\overrightarrow{i} + y\,\overrightarrow{j} + z\,\overrightarrow{k}$ and let S be any surface that surrounds the origin. Prove that $\iint_S \overrightarrow{F}.n\; d\sigma = 4\pi$.

7. **(T)** Let D be the domain inside the cylinder $x^2 + y^2 = 1$ cut off by the planes $z = 0$ and $z = x + 2$. If $\vec{F} = (x^2 + ye^z, y^2 + ze^x, z + xe^y)$, use the divergence theorem to evaluate $\iint_{\partial D} F \cdot \mathbf{n}\; d\sigma$.