Assignment 7: Improper Integrals

1. **(T)** Test the convergence/divergence of the following improper integrals:

   $(a)\ \int_0^1 \frac{dx}{\log(1+\sqrt{x})}$ $\quad (b)\ \int_0^1 \frac{dx}{x-\log(1+x)}$ $\quad (c)\ \int_0^1 \frac{\log x}{\sqrt{x}}$ $\quad (d)\ \int_0^1 \sin(1/x)dx.$

   $(e)\ \int_1^\infty \frac{\sin(1/x)}{x}dx$ $\quad (f)\ \int_0^\infty e^{-x^2}dx$ $\quad (g)\ \int_0^\infty \sin x^2 dx,$ $\quad (h)\ \int_0^{\pi/2} \cot x dx.$

2. **(D)** In each case, determine the values of p for which the following improper integrals converge

   $(a)\ \int_0^\infty \frac{1-e^{-x}}{x^p}dx$ $\quad (b)\ \int_0^\infty \frac{t^{p-1}}{1+t}dt.$

3. **(T)** Show that the integrals $\int_0^\infty \frac{\sin^2 x}{x^2}dx$ and $\int_0^\infty \frac{\sin x}{x}dx$ converge. Further, prove that $\int_0^\infty \frac{\sin^2 x}{x^2}dx = \int_0^\infty \frac{\sin x}{x}dx.$

4. **(T)** Show that $\int_0^\infty \frac{x\log x}{(1+x^2)^2}dx = 0.$

5. **(D)** Prove that improper integral $\int_1^\infty \frac{\sin x}{x^p}dx$ converges conditionally for $0 < p \leq 1$ and absolutely for $p > 1$.

6. **(T)** Show that $\int_0^s \frac{1+x}{1+x^2}dx$ and $\int_{-s}^0 \frac{1+x}{1+x^2}dx$ do not approach a limit as $s \longrightarrow \infty$. However $\lim_{s\longrightarrow\infty} \int_{-s}^s \frac{1+x}{1+x^2}dx$ exists.