Assignment 8: Applications of Integration, Pappus Theorem

1. **(D)** Find the area of the region in the first quadrant bounded on the left by the Y-axis, below by the curve $x = 2\sqrt{y}$, above left by the curve $x = (y-1)^2$, and above right by the line $x = 3 - y$.

2. **(D)** Sketch the graph of $r = 1 + \sin\theta$. Find the area of the region that is inside the circle $r = 3\sin\theta$ and also inside $r = 1 + \sin\theta$.

3. **(T)** Sketch the graphs $r = \cos(2\theta)$ and $r = \sin(2\theta)$. Also, find their points of intersection.

4. **(T)** A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a $45°$ angle at the center of the cylinder. Find the volume of the wedge.

5. **(D)** The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the X-axis to generate a solid. Find the volume of the solid.

6. **(D)** The region in the first quadrant bounded by the parabola $y = x^2$, the Y-axis and the line $y = 1$ is revolved about the line $x = 2$ to generate a solid. Find the volume of the solid.

7. **(T)** Let f be a continuous function on $\mathbb{R}$. A solid is generated by rotating about the X-axis, the region bounded by the curve $y = f(x)$, the X-axis and the lines $x = a$ and $x = b$. For fixed a, the volume of this solid between a and b is $b^3 + b^2 - ab - a^3$ for each $b > a$. Find $f(x)$.

8. **(D)** Find the area of the surface generated by revolving the curve $y = x^3$, $0 \leq x \leq 1/2$, about the X-axis.

9. **(T)** A square is rotated about an axis lying in the plane of the square, which intersects the square only at one of its vertices. For what position of the axis, is the volume of the resulting solid of revolution the largest?

10. **(D)** A regular hexagon is inscribed in the circle $x^2 + (y-2)^2 = 1$ and is rotated about the X-axis. Find the volume and the surface area of the solid so formed.

11. **(T)** Find the centroid of the semicircular arc $(x-r)^2 + y^2 = r^2$, $r > 0$ described in the first quadrant. If this arc is rotated about the line $y + mx = 0$, $m > 0$, determine the generated surface area A and show that A is maximum when $m = \pi/2$.