Assignment 9: Vectors, Curves, Surfaces, Vector Functions

1. **(T)** Consider the planes $x-y+z=1,\ x+ay-2z+10=0$ and $2x-3y+z+b=0$, where a and b are parameters. Determine the values of a and b such that the three planes

   (a) intersect at a single point,

   (b) intersect in a line,

   (c) intersect (taken two at a time) in three distinct parallel lines.

2. **(D)** Determine the equation of the cylinder generated by a line through the curve $(x-2)^2+y^2=4,\ z=0$ moving parallel to the vector $\vec{i}+\vec{j}+\vec{k}$.

3. **(T)** Determine the equation of a cone with vertex $(0,-a,0)$ generated by a line passing through the curve $x^2=2y,\ z=h$.

4. **(T)** The velocity of a particle moving in space is $\frac{d}{dt}c(t)=(\cos t)\vec{i}-(\sin t)\vec{j}+\vec{k}$. Find the particle's position as a function of t if $c(0)=2\vec{i}+\vec{k}$. Also find the angle between its position vector and the velocity vector.

5. **(T)** Show that $c(t)=\sin t^2\vec{i}+\cos t^2\vec{j}+5\vec{k}$ has constant length and is orthogonal to its derivative. Is the velocity vector of constant magnitude?

6. **(T)** Find the point on the curve $c(t)=(5\sin t)\vec{i}+(5\cos t)\vec{j}+12t\vec{k}$ at a distance 26π units along the curve from the origin in the direction of increasing arc length.

7. **(T)** Reparametrize the curves

   (a) $c(t)=\frac{t^2}{2}\vec{i}+\frac{t^3}{3}\vec{k},\quad 0\le t\le 2,$

   (b) $c(t)=2\cos t\vec{i}+2\sin t\vec{j},\quad 0\le t\le 2\pi$

   in terms of arc length.

8. **(D)** If a plane curve has the Cartesian equation $y=f(x)$ where f is a twice differentiable function, then show that the curvature at the point $(x,f(x))$ is $\dfrac{|f''(x)|}{[1+f'(x)^2]^{3/2}}$.

9. **(D)** For the curve $c(t)=t\vec{i}+t^2\vec{j}+\frac{2}{3}t^3\vec{k}$ find the equations of the tangent, principal normal and binormal. Also calculate the curvature of the curve.

10. **(T)** Show that the parabola $y=ax^2,\ a\neq 0$ has its largest curvature at its vertex and has no minimum curvature.