Practice Problems , No. 1

1. Use the sandwich Theorem to find the limit of the following sequence (x_n).

(i) $x_n = \left(\sqrt{2} - 2^{\frac{1}{3}}\right)\left(\sqrt{2} - 2^{\frac{1}{5}}\right)\cdots\cdots\cdots\left(\sqrt{2} - 2^{\frac{1}{2n+1}}\right)$

(ii) $x_n = \frac{1}{1+n^2} + \frac{2}{2+n^2} + \cdots\cdots + \frac{n}{n+n^2}$

(iii) $x_n = (a^n + b^n)^{\frac{1}{n}}$ where $0 < a < b$.

2. Use problem 3 of the previous assignment to find the limit of the sequence (x_n), where (x_n) is

(i) $\frac{n^2}{2^n}$ (ii) $\frac{n!}{2^{n^2}}$ (iii) $\frac{2^{3n}}{3^{2n}}$ (iv) $\frac{b^n}{2^n}$, $b > 1$

3. Determine the convergence and divergence of the sequence (x_n), where (x_n) is

(i) $x_n = \frac{n!}{(2n+1)!!}$ (ii) $\frac{(2n)!!}{(2n+1)!!}$

4. Show that the following sequence (x_n) is bounded and monotone. Find the limit.

(i) $x_1 = 8$ and $x_{n+1} = \frac{1}{2}x_n + 2$ (ii) $x_1 = 1$ and $x_{n+1} = 2 - \frac{1}{x_n}$.

5. Use Problem 6 of the previous assignment to show that the following sequence (x_n) statisfies Cauchy criterion and find the limit.

(i) $x_1 = 1$ and $x_{n+1} = \frac{1}{2+x_n}$ (ii) $x_1 = 2$ and $x_{n+1} = 2 + \frac{1}{x_n}$.