Practice Problems, No. 2

1. Show that the absolute value function $f(x) = |x|$ is continuous at every point $c \in R$.

2. Let $K > 0$ and let $f : R \to R$ satisfy the condition $|f(x) - f(y)| \le K|x - y|$ for all $x, y \in R$. Show that f is continuous at every point $c \in R$.

3. Define $g : R \to R$ by

   $g(x) = 2x$ if x rational.
   $= x + 3$ if x irrational.

   Show that g is continuous only at $x = 3$.

4. Let $f : R \to R$ be continuous at c and $f(c) > 0$. Show that there exists a $\delta > 0$ such that $f(x) > 0$ for all $x \in (c - \delta, c + \delta)$.

5. Show that the polynomial $p(x) = x^4 + 7x^3 - 9$ has at least two real roots.

6. Show that the equation $x = \cos x$ has a solution in the interval $[0, \pi/2]$.

7. If $g : [a,b] \to [a,b]$ is continuous show that there is some $c \in [a,b]$ such that $g(c) = c$.

8. Let f be continuous on the interval $[0,1]$ to R and such that $f(0) = f(1)$. Prove that there exists a point c in $\left[0, \frac{1}{2}\right]$ such that $f(c) = f(c + \frac{1}{2})$.