Practice Problems, No. 3

1. Show that $f(x) = x^{\frac{1}{3}}$, $x \in R$ is not differentiable at $x = 0$.

2. Let $f : R \to R$ be defined by $f(x) = x^2$ for x rational, $f(x) = 0$ for x irrational. Show that f is differentiable at $x = 0$ find $f'(0)$.

3. Suppose $f(R) \to R$ is differentiable at c and that $f(c) = 0$. Show that $g(x) = |f(x)|$ is differentiable at c if and only if $f'(c) = 0$.

4. Assume that f is differentiable at a. Find

$$\lim_{x \to a} \frac{x\, f(a) - a\, f(x)}{x - a}.$$

5. Assume that f is continuous on $[a, b]$, $a > 0$, and differentiable on an open interval (a, b). Show that if $\frac{f(a)}{a} = \frac{f(b)}{b}$, then there is $x_0 \in (a, b)$ such that $x_0 f'(x_0) = f(x_0)$.

Assume that $a_0\ a_1, \ldots\ldots\ldots, a_n$ are real numbers such

that $$\frac{a_0}{n+1} + \frac{a_1}{n} + \ldots\ldots\ldots\ldots + \frac{a_{n-1}}{2} + a_n = 0.$$

Prove that the polynomial $p(x) = a_0 x^n + a_1 x^{n-1} + \ldots\ldots\ldots + a_n$ has at least one root in (0,1).