Practice Problems, No.-4

1. Use mean value theorem to show that $\sin x \le x$ for all $x \ge 0$.

2. Let $f:[a,b] \to R$ be differentiable. If $f'(x)=0$ for all $x \in [a,b]$, show that f is constant on $[a,b]$.

3. Let $f:[0,\infty) \to R$ be differentiable on $(0,\infty)$ and assume that $f'(x) \to b$ as $x \to \infty$

   (a) Show that for any , $h>0$, we have $\lim\limits_{x\to\infty} \dfrac{f(x+h)-f(x)}{h} = b.$

   (b) Show that if $f(x) \to a$ as $x \to \infty$, then $b = 0$.

   (c) Show that if f is bounded then $\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = b.$

4. Prove that if f is differentiable on $[a,b]$ and if the derivative f' is bounded on then $\exists\, M > 0$ such that $|f(x) - f(y)| \le M|x-y|$ for all $x, y \in [a,b]$.

5. Let f, g be differentiable on R and suppose that $f(0) = g(0)$ and $f'(x) \le g'(x)$ for all $x \ge 0$. Show that $f(x) \le g(x)$ for all $x \ge 0$.

1. Let $f:[a,b] \to R$ be continuous on $[a, b]$ and differentiable on (a,b). If $f(a) < f(b)$, then show that $f'(c) > 0$ for some $c \in (a,b)$.

2. Sketch the graph of the function $f(x) = x^3 - 6x^2 + 9x + 1$.