Beamforming Techniques in Wireless Communications

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Outline

- Beamforming: Introduction & Overview
- Transmit & Receive Beamforming :- Examples
- Common Beamforming Techniques
- Beamforming
  - In MIMO Systems
  - In Massive MIMO Systems
  - In MIMO-OFDM Systems
  - For Interference Mitigation in Multi-antenna, Multi-carrier Systems
  - In Wireless Sensor Networks
- Future Work

Beamforming

- Directional transmission/reception to optimize a design criterion [1]
- **Objective:** Design transmit & receive beamformers to nullify the interference and enhance system performance
- Can be used at both transmitting and receiving ends
- Application in different areas of wireless communications:
    - Multiple-Input and Multiple-output(**MIMO**)
    - Massive MIMO(**m-MIMO**)
    - Interference mitigation
    - Wireless sensor Networks(**WSN**)

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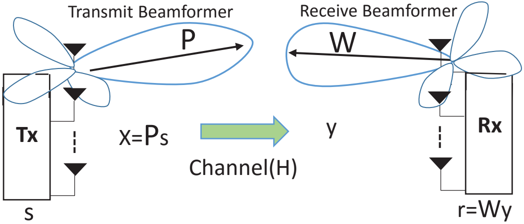
[1] Mietzner, J.; et al., *"Multiple-antenna techniques for wireless communications - a comprehensive literature survey,"* in Communications Surveys & Tutorials, 2009 "

System Model: An Overview



- The general model for a MIMO wireless communication is given as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}; \quad \mathbf{H} \in \mathbb{C}^{N_R \times N_T} \tag{1}$$

$$= \mathbf{HPs} + \mathbf{n} \tag{2}$$

- $\mathbf{x} = \mathbf{Ps}$ is the precoded and $\mathbf{r} = \mathbf{Wy}$ is the filtered output

Transmit & Receive Beamforming:- Examples

Transmit and Receive Beamforming - Example

- Consider singular value decomposition(**SVD**) of **H**

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{3}$$

- where $\mathbf{\Sigma}$ is a diagonal matrix with entries σ_i (singular values)
- Choosing $\mathbf{P} = \mathbf{V}$ and $\mathbf{W} = \mathbf{U}^H$ in eq. (2) gives

$$\begin{aligned} \mathbf{r} &= \mathbf{\Sigma}\,\mathbf{s} + \mathbf{U}^H\mathbf{n} && (4) \\ &= \mathbf{\Sigma}\,\mathbf{s} + \tilde{\mathbf{n}} && (5) \end{aligned}$$

- Then, one data stream per singular value can be transmitted as

$$\mathbf{r}_i = \sigma_i \mathbf{s}_i + \tilde{\mathbf{n}}_i; \;\; 1 \leq i \leq \min\{N_R, N_T\} \tag{6}$$

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[2] E. Telatar, *"Capacity of multiantenna Gaussian channels, European Transactions on Telecommunications"*, 1999

Precoder: Transmit Beamforming - Example

- Multiple Input Single Output(**MISO**) system[3]

$$y = \mathbf{h}^H \mathbf{f} s + n \tag{7}$$

- Here, $\mathbf{P} = \mathbf{f}$ and $\mathbf{W} = \mathbf{I}$
- Antenna Selection: Send data on the antenna that maximizes the receive SNR$= |\mathbf{h}^H \mathbf{f}|^2$

$$m_{opt} = \arg \max_{1 \leq m \leq N_T} |\mathbf{h}(m)|^2 \tag{8}$$

- Transmit beamforming vector is restricted to rank one covariance matrix
- Optimal selected antenna can be fedback to the transmitter using $\lceil \log_2(N_T) \rceil$ bits

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[3] D. Love, R. Heath, et al., ”*An overview of limited feedback in wireless communication systems*”, IEEE Journal on Selected Areas in Communications, 2008.

Common Beamforming Techniques

MRC: Maximal Ratio Combining

- All received signals are coherently combined at the receiver [5]
- Here, $\mathbf{W} = \mathbf{w}$ (vector with N_R elements) and $\mathbf{P} = \mathbf{I}$

$$r = \mathbf{w}^H \mathbf{y} = \mathbf{w}^H \mathbf{h} x + \mathbf{w}^H \mathbf{n} \tag{9}$$

- Maximizes the output SNR for the intended user

$$\gamma = \frac{|\mathbf{w}^H \mathbf{h}|^2}{\mathbb{E}|\mathbf{w}^H \mathbf{n}|^2} = \frac{|\mathbf{w}^H \mathbf{h}|^2}{\sigma^2 \mathbb{E}|\mathbf{w}^H \mathbf{w}|} \tag{10}$$

- By Cauchy-Schwartz inequality it is found that the SNR is maximized when, $\mathbf{w} \propto \mathbf{h}$

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[4] Tse, David, and Pramod Viswanath, *Fundamentals of wireless communication*, Cambridge university press, 2005

ZF: Zero Forcing

Precoding:

- MISO system with (K)-user is considered in [6]
- Received signal at k^{th} user user is given as

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k; \quad k = 1, 2, \cdots, K \tag{11}$$

- where, $\mathbf{x} = \Sigma_{i=1}^{K} s_i \mathbf{w}_i$ is the $N_T \times 1$ transmitted symbols
- $\mathbf{w}_i$ is the linear precoder, orthogonal to all other user channel vectors
- Hence,

$$y_k = \mathbf{h}_k^H \Sigma_{i=1}^{K} \mathbf{w}_i s_i + n_k = \mathbf{h}_k^H \mathbf{w}_k s_k + n_k; \quad k = 1, 2, \cdots, K \tag{12}$$

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[5] Peel, et. al., "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part I: channel inversion and regularization," IEEE Tran. on Comm., 2005

ZF: Zero Forcing

Equalization:

- Considering the MIMO signal model given in eq. (1)
- To decouple the detection of each symbol at the receiver $\mathbf{W} = \mathbf{H}^{\dagger}$
- so,

$$\mathbf{r} = \mathbf{H}^{\dagger}\mathbf{y} = \mathbf{x} + \mathbf{H}^{\dagger}\mathbf{n} \tag{13}$$

- where, $\mathbf{H}^{\dagger}$ is the pseudo inverse given as $(\mathbf{H}^{H}\mathbf{H})^{-1}\mathbf{H}^{H}$

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[6] Tse, David, and Pramod Viswanath, *Fundamentals of wireless communication*, Cambridge university press, 2005

State of the Art

- Multi-cell MIMO Networks [a],[b]
- Cognitive Radio Networks [c],
- Massive-MIMO Networks [a]
- Dirty Paper Coding (DPC) based algorithms[d]

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[a] Lakshminarayana, S.; Assaad, M.; Debbah, M., "*Coordinated Multicell Beamforming for Massive MIMO: A Random Matrix Approach*", IEEE Tran. on Inform. Th., 2015

[b]Venkategowda, N.K.D.; et al.,"*MVDR-Based Multicell Cooperative Beamforming Techniques for Unicast/Multicast MIMO Networks With Perfect/Imperfect CSI*", Tran. on Veh. Tech., 2015

[c]Afana, A.; et al., "*Distributed Beamforming for Two-Way DF Relay Cognitive Networks Under PrimarySecondary Mutual Interference*", Tran. on Veh. Tech., 2015

[d] N. Jindal and A. Goldsmith, *Dirty-paper coding versus TDMA for MIMO broadcast channels,* IEEE Trans. Inf. Theory, 2010

Beamforming in MIMO

Beamforming in MIMO

- In [7], transmission of data symbols subject to (possibly different) **QoS** constraints is considered
- System model is same as in eq. (1)
- Optimum Receiver: for a given transmit matrix $\mathbf{P}$

$$\min_{\mathbf{W}} \mathrm{Tr}[(\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^H] \tag{14}$$

- where $\hat{\mathbf{x}} = \mathbf{W}\mathbf{x}$ and the optimal $\mathbf{W}$ is the linear minimum MSE (LMMSE) filter[8]

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[7] D. P. Palomar, et al., *Optimum linear joint transmit-receive processing for MIMO channels with QoS constraints,* IEEE Trans. Signal Process., 2004

[8] Palomar, D.P. and Jiang, Y.,"*MIMO transceiver design via majorization theory. Foundations and trends in communications and information theory*", 2006

Beamforming in MIMO [7] contd.

- Optimum Precoder:

$$\min_{\mathbf{P}} \mathrm{Tr}(\mathbf{P}\mathbf{P}^H) \tag{15}$$

$$\text{s.t} \;\; [(\mathbf{I} + \mathbf{P}^H \mathbf{R}_H \mathbf{P})^{-1}]_{ii} \leq \rho_i, \;\; 1 \leq i \leq L \tag{16}$$

- where, L is the number of established links and $\mathbf{R}_H = \mathbf{H}^H \mathbf{R}_n^{-1} \mathbf{H}$
- Above problem is nonconvex in $\mathbf{P}$
- Majorization theory is used to reformulate it as a simple convex optimization problem
- Contribution:
    - A practical and efficient multilevel water-filling algorithm is proposed
    - A simple robust design under channel estimation errors is also proposed

Beamforming in MIMO: Robust Framework

- In [9], the design of linear transceivers with robustness for imperfect CSI is considered
- The MIMO channel matrix distribution is modeled as

$$\mathbf{H} = \overline{\mathbf{H}} + (\mathbf{R}_{\mathbf{H}}^{Rx})^{1/2}\mathbf{G}(\mathbf{R}_{\mathbf{H}}^{Tx})^{(1/2)^{H}} \tag{17}$$

- where $\mathbf{G}$ is the unknown part in the fading estimate
- $\overline{\mathbf{H}}$ is the channel mean & covariance matrix

$$\mathbf{R}_{\mathbf{H}} = (\mathbf{R}_{\mathbf{H}}^{Rx}) \otimes (\mathbf{R}_{\mathbf{H}}^{Tx}) \tag{18}$$

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[9] Xi Zhang; et al., *"Statistically Robust Design of Linear MIMO Transceivers,"* in TSP, 2008

Beamforming in MIMO [9] contd.

- General Problem:

$$\min_{\mathbf{W},\mathbf{P}} \ \mathcal{F}_o\{\mathrm{Tr}[\mathbb{E}\{(\hat{\mathbf{x}}-\mathbf{x})(\hat{\mathbf{x}}-\mathbf{x})^H\}]\} \tag{19}$$

$$\text{s.t } \mathrm{Tr}[\mathbf{P}\mathbf{P}^*] \leq p_{max} \tag{20}$$

- where, $\mathcal{F}_o$ is an arbitrary increasing function of the average MSE
- Case1: Imperfect CSIR & CSIT
  - A closed-form expression for the average MSE matrix is

$$\mathbb{E}\{\mathbf{E}(\mathbf{W},\mathbf{P})\} = [(\mathbf{W}^H\overline{\mathbf{H}}\mathbf{P} - \mathbf{I}_l)(\mathbf{P}^H\overline{\mathbf{H}}^H\mathbf{W} - \mathbf{I}_l) + \mathbf{W}^H\mathbf{R}_n^{'}\mathbf{W}] \tag{21}$$

$$\text{where}, \mathbf{R}_n^{'} = \mathbf{R}_n + \mathrm{Tr}[\mathbf{P}\mathbf{P}^H(\mathbf{R}_{\mathbf{H}}^{Tx})^T]\mathbf{R}_{\mathbf{H}}^{Rx}$$

  - The optimal receiver **W** is similar to the Wiener filter
  - The optimal design of the transmitter **P** is derived for $\mathbf{R}_{\mathbf{H}}^{Tx} = \mathbf{I}_{N_T}$

Beamforming in MIMO [9] contd.

- Case2: Perfect CSIR & imperfect CSIT
    - The optimal receiver **W** is exactly the same as for perfect CSI case
    - A robust transmitter design is proposed based on a tight lower bound of $\mathbb{E}\{\mathbf{E}\}$
    - Cost function of the design problem (19) is also lower-bounded

Beamforming in Massive MIMO

Beamforming in Massive MIMO

- Massive MIMO employs hundreds of antennas to enable a large beamforming gain
- In [10], quantized versions of the channel estimates are used for conjugate beamforming
- Desirable to use A/D and D/A converters with resolutions as coarse as possible
- M-antenna base station, K autonomous single antenna terminals with Time Divison Duplexing(**TDD**) operation is considered
- The minimum mean-square error (MMSE) estimate for $\mathbf{H}$ is $\hat{\mathbf{H}} = \frac{\sqrt{\tau_u \rho_u}}{1+\tau_u \rho_u}\mathbf{y}$
- where τ_u is coherence slot for up-link pilots
- ρ_u is a measure of the expected SNR of the up-link

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[10] Hong Yang; Marzetta, T.L., "*Quantized beamforming in Massive MIMO*", Annual Conference on Information Sciences and Systems (CISS), 2015

Beamforming in Massive MIMO[10] contd..

- Conjugate beamforming :

$$\mathbf{x} = \frac{\hat{\mathbf{H}}^H \mathbf{q}}{\sqrt{MK \frac{\tau_u \rho_u}{1+\tau_u \rho_u}}} \tag{22}$$

- The down-link data channel is given by

$$\mathbf{y} = \sqrt{\rho_d}\mathbf{H}^H \mathbf{x} + \mathbf{n} \tag{23}$$

- ρ_d is a measure of the expected SNR of the down-link channel
- Effects of quantization : $\mathbf{x} = \frac{\hat{\mathbf{H}}_{\Theta}^H \mathbf{q}}{\sqrt{c}}$
- where, the constant $c = \mathbb{E}[\mathrm{Tr}(\hat{\mathbf{H}}_{\Theta} \hat{\mathbf{H}}_{\Theta}^H)]$
- Θ is the quantization scheme on the channel response estimate $\hat{H}$
- Open Problems:
  - Consideration of multi-cell interference and pilot contamination
  - Analysis and impact of multi-antenna receiver

Beamforming in Multi-Antenna & Multi-Carrier System

Beamforming in MIMO-OFDM System

- In [11], robust transceiver designs with statistical channel uncertainties are considered
- The channel matrices on each subcarrier have the following relationship

$$\mathbf{H}_k = \hat{\mathbf{H}}_k + \Delta\mathbf{H}_k \tag{24}$$

- where, $\Delta\mathbf{H}_k = \sigma_k^2\mathbf{H}_w$
- σ_k^2 is the estimation error variance of each channel element
- $\mathbf{H}_w$ is the random matrix whose elements are i.i.d. with zero mean and unit variance

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[11] Chengwen Xing; et al., *"Robust Transceiver Design for MIMO-OFDM Systems Based on Cluster Water-Filling,"* in IEEE Communications Letters, 2013

Beamforming in MIMO-OFDM Systems[11] contd.

- The weighted MSE optimization problem of transceiver design is formulated as

$$\min_{\mathbf{P}_k} \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k[\mathbf{P}_k^H \hat{\mathbf{H}}_k^H \mathrm{K}_{\mathbf{P}_k}^{-1} \hat{\mathbf{H}}_k \mathbf{P}_k + \mathbf{I}]^{-1}\} \tag{25}$$

$$\text{s.t} \quad \mathrm{K}_{\mathbf{P}_k} = [\sigma_{e,k}^2 \mathrm{Tr}(\mathbf{P}_k \mathbf{P}_k^H) + \sigma_n^2]\mathbf{I} \tag{26}$$

$$\sum_{k=1}^{K} \mathrm{Tr}(\mathbf{P}_k \mathbf{P}_k^H) \leq p_{max} \tag{27}$$

- where, $\mathbf{W}_k$ is the weighting matrix in the k^{th} subcarrier
- A series of auxiliary variables P_k are introduced
- Thus enabling optimization problem to be decoupled into a series of subproblems
- Cluster water-filling is proposed to solve the robust transceiver design

Beamforming for Interference Mitigation

Interfernce Alignment

- Interference Alignment (IA) concepts have been well explored in [12],[13]
- Interference may arise due to multiuser, multicell, Inter-Block Interference(IBI), Inter-Symbol Interference (ISI) etc.
- Align signal space and interference space in disjoint subspaces
- Key idea is to maximize the space for the desired signal
- Minimize the interference space at the receiver which can be suppressed using proper filter (e.g; ZF)

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[12] S. A. Jafar and S. Shamai, *Degree of freedom region for the MIMO X channel,* IEEE Trans. Inf. Theory, 2008

[13] V. R. Cadambe and S. A. Jafar, *Interference alignment and the degrees of freedom for the K user interference channel,*IEEE Trans. Inf. Theory, Aug. 2008.

Beamforming for Interference Mitigation in MIMO-OFDM

- In [14], an interference nulling based precoding is examined for MIMO-OFDM insufficient Cyclic Prefix (**CP**)
- Where, L and ν are the number of channel taps and CP length
- Insufficiency of $(L - \nu)$, incurs IBI at the receiver
- Interference can be aligned to a subspace of dimensions no more than $(L - \nu)/2$
- Thus the other half can be used for sending more information symbols by the insufficient CP

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[14] Yuansheng Jin; et al., *"An Interference Nulling Based Channel Independent Precoding for MIMO-OFDM Systems with Insufficient Cyclic Prefix,"* Tran. on Comm., 2013

Beamforming for Interference Mitigation in MIMO-OFDM[14] contd.

- The time domain received block after CP is removed

$$\tilde{\mathbf{y}}_k = (\mathbf{H} - \mathbf{A})\overline{\mathbf{W}}_N \tilde{\mathbf{x}}_k + \mathbf{B}\overline{\mathbf{W}}_N \tilde{\mathbf{x}}_{k-1} + \mathbf{n}_k; \ \forall \ k \geq 2 \tag{28}$$

- where, $\overline{\mathbf{W}}_N = \mathbf{W}_N \otimes \mathbf{I}_{N_T}$ and $\mathbf{W}_N$ is the N-pt. DFT matrix
- $\mathbf{A}$ & $\mathbf{B}$ are the Inter-carrier Interference(ICI) and IBI component respectively
- Structure of precoding matrix ($\mathbf{Q} = \overline{\mathbf{W}}_N \mathbf{P}$) is as $\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22}. \end{bmatrix}$

Beamforming for Interference Mitigation in MIMO-OFDM[14] contd.

- The two submatrices $\mathbf{Q}_{21}$ & $\mathbf{Q}_{22}$ are designed so as to suppress the IBI
- Submatrices $\mathbf{Q}_{11}$ & $\mathbf{Q}_{12}$ are designed to achieve more transmission rate
- Total number of independent information symbols for $N_R \leq N_T$:

$$\begin{cases} N_T(N-L+\nu)+\lfloor\frac{N_RN-N_T(N-L+\nu)}{2}\rfloor & \text{if} \quad N_T(N-L+\nu) < N_RN \\ N_RN & \text{if} \quad N_T(N-L+\nu) \geq N_RN \end{cases}$$

- No precoding, or CP or ZP is needed to eliminate the IBI when $N_R \geq N_T$

Beamforming for Interference Mitigation in OFDM

- In [15], an IA based precoder design for OFDM with insufficient CP is proposed
- Received signal is similar to eq. (28)
- Structure of the precoder $\mathbf{Q} = \mathbf{W}_N\mathbf{P}$ is given as $\mathbf{Q} = [\mathbf{Q}_u \ \mathbf{Q}_z]$
- $\mathbf{Q}_z$ is chosen to be linearly dependent with the columns of $\mathbf{Q}_u$
- $\mathbf{Q}_u$ has the structure $\begin{bmatrix} [\mathbf{Q}_k]_{n_\beta \times n_\beta} \\ 0_{(N-n_\beta)\times n_\beta}. \end{bmatrix}$
- n_β is the number of the independent information symbols that can be recovered

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[15] Yuansheng Jin; et al., *"An interference alignment based precoder design using channel statistics for OFDM systems with insufficient cyclic prefix,"* in GLOBECOM, 2012 IEEE

Beamforming for Interference Mitigation in OFDM[15] contd.

- IBI ($:= \mathbf{B}\mathbf{Q}_u\mathbf{x}_{k-1}$) is restricted in an $n_1 = (L-\nu)/2$ dimensional space
- To eliminate the IBI completely the ZF matrix chosen

$$\mathbf{F}_{zf} = diag(\underbrace{0,\ldots,0}_{n_1},1,\ldots,1) \tag{29}$$

- Then the optimal precoder design problem is given as

$$\min_{\mathbf{Q}_u} \ \mathrm{Tr}\{[\frac{1}{\sigma_s^2}\mathbf{I} + \mathbf{Q}_u{}^H\mathbf{R}_c\mathbf{Q}_u]^{-1}\} \tag{30}$$

$$\text{s.t} \ \mathrm{Tr}(\mathbf{Q}_u{}^H\mathbf{Q}_u) \leq n_\beta \tag{31}$$

- Statistical CSIT is assumed to be known

Beamforming for Interference Mitigation in MIMO OFDM

- In [16], robust transceiver design problem for a MIMO system with imperfect CSIT for sufficient CP is considered
- Imperfect CSIT is in the form channel mean and covariance
- Develops a set of optimization criteria based on the tight lower bound of average MSE matrix
- In [17], robust transceiver design problem for MIMO-OFDM with sufficient CP is considered
- Channel estimation errors at both transmitter and receiver is assumed for transceiver design

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[16] X. Zhang, D. P. Palomar, and B. Ottersten, Statistically robust design of linear MIMO transceivers, IEEE Trans. Signal Process.,2008

[17] C. Xing, et al., Robust transceiver design for MIMO-OFDM systems based on cluster water-filling, IEEE Commun. Lett.,2013

Beamforming for Interference Mitigation in MIMO-OFDM

- A precoder design using statistical CSIT in the form of covariance matrix is proposed for insufficient CP [18]
- The system model and structure of the precoder is similar as in [15]
- Optimization problem is to minimize the maximum stream MSE of all data streams

$$\min_{\mathbf{Q}_u} \quad \max_{1\leq i\leq n_\beta} \{[\frac{1}{\sigma_s^2}\mathbf{I} + \mathbf{Q}_u{}^H \mathbf{R}_c \mathbf{Q}_u]^{-1}\}_{i,i} \tag{32}$$

$$\text{s.t} \quad \mathrm{tr}(\mathbf{Q}_u{}^H \mathbf{Q}_u) \leq n_\beta \tag{33}$$

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[18] Yuansheng Jin; et al., *"A Robust Precoder Design Based on Channel Statistics for MIMO-OFDM Systems with Insufficient Cyclic Prefix,"* Tran. on Comm., 2014

Beamforming for Interference Mitigation in OFDM

- In [19], precoder design for OFDM systems with insufficient CP has been considered
- IA is utilized to formulate the precoder design problem as a BER minimization problem
- $\mathbf{Q}_k$ has the following structure for all $k \geq 2$

$$\mathbf{Q}_k = \begin{bmatrix} \tilde{\mathbf{Q}}_k & 0_{n_\beta \times \kappa} \\ 0_{\kappa \times n_\beta} & 0_{\kappa \times \kappa}. \end{bmatrix} \tag{34}$$

- where, $\kappa := \lceil \frac{L-\nu}{2} \rceil$ is the dimension of the interference space

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[19] Bedi, A.S.; Akhtar, J.; Rajawat, K.; Jagannatham, A.K., *"BER-Optimized Precoders for OFDM systems with Insufficient Cyclic Prefix,"* in Communications Letters, IEEE

Beamforming for Interference Mitigation in OFDM[18] contd.

- The BER-optimized precoder design problem can be formulated as:

$$\min_{\tilde{\mathbf{P}}_k} \frac{1}{\eta} \sum_{m=1}^{\eta} Q\left(\sqrt{[\mathbf{V}_{mse}]_{mm}^{-1} - 1}\right) \tag{35}$$

$$\text{s.t } \operatorname{Tr}(\tilde{\mathbf{P}}_k \tilde{\mathbf{P}}_k^H) \leq \eta_\beta \tag{36}$$

- where $\mathbf{V}_{mse} = \left[\frac{1}{N_0}\left(\tilde{\mathbf{P}}_k^H \tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_k \tilde{\mathbf{P}}_k\right) + \mathbf{I}\right]^{-1}$
- The resulting precoder is also shown to be MSE-optimal

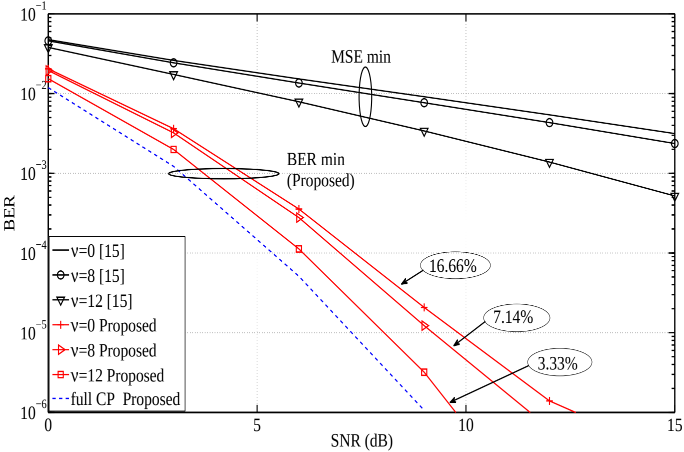

Figure: BER performance for channels with constant power delay profile

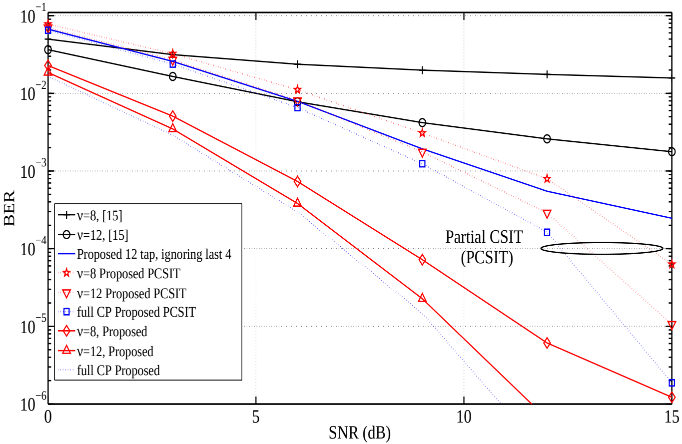

Figure: BER performance for channels with exponential PDP

Beamforming in WSN’s

WSN's : Introduction

- A spatially distributed system with multiple sensors for estimation of unknown source parameter
- Used for monitoring and remote surveillance applications in inaccessible terrains

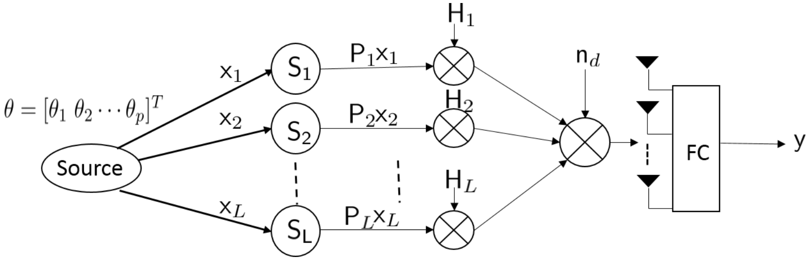

Figure: Wireless Sensor Networks System ($\mathbf{x}_i = \mathbf{A}_i\theta;\ \mathbf{A}_i \in \mathbb{C}^{l_i \times p};\ \mathbf{P}_i \in \mathbb{C}^{q_i \times l_i}$)

Beamforming in WSN's

- Centralized vs de-centralized system
- Compression of data at each sensor (power & bandwidth constraints)
- Two common ways to model the finite bandwidth constraint
  - Limit the number of bits per sensor [20],[21]
  - Limit the number of real-valued messages per sensor [22]-[23]

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[20] J.-J. Xiao, et al., *Distributed compression-estimation using wireless sensor networks*, IEEE Signal Process. Mag.,2006

[21] P. K. Varshney,*Distributed Detection and Data Fusion*. New York:Springer-Verlag, 1997

[22] T. J. Goblick,*Theoretical limitations on the transmission of data from analog sources*, IEEE Trans. Inf. Theory, 1965

[23] M. Gastpar, et al., *To code, or not to code: Lossy source-channel communication revisited*, IEEE Trans. Inf. Theory, 2003

Beamforming in WSN's

- Orthogonal channel usage between sensors and the Fusion Centre (FC), has been studied in [24]
- Design of optimal linear decentralized estimation is shown to be NP-hard for orthogonal channel in [25]
- In [26], MSE based criteria is used to design the transceiver for a linear model
- Channels between sensors and the FC are assumed to be non-orthogonal and Multiple Access Channels (MAC) to be coherent

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[24] K. Zhang, et al., *Optimal linear estimation fusion Part VI: Sensor data compression,* in Proc. Int. Conf. Inf. Fusion, 2003

[25] Z.-Q. Luo, et al, *Optimal linear decentralized estimation in a bandwidth constrained sensor network,* in Proc. IEEE Int. Symp. Inf. Theory,2005

[26] Jin-Jun Xiao; et al., *"Linear Coherent Decentralized Estimation,"* IEEE Tran. on S/g Process., 2008

Beamforming in WSN's [26]

- Joint estimation of the unknown source is obtained for noisy and noiseless FC scenarios
- **Noiseless FC case:** Design Problem (Bandwidth constraint)

$$\min_{\mathbf{B}_i; 1 \leq i \leq L} \text{Tr}(\mathbf{D}) \tag{37}$$

$$\text{s.t} \;\; \mathbf{D}^{-1} = \mathbf{I}_p + \mathbf{A}^T \mathbf{B}^T (\mathbf{B}\mathbf{B}^T)^{\dagger} \mathbf{B}\mathbf{A} \tag{38}$$

- where, $\mathbf{B} \in \mathbb{R}^{q \times l}$ and $\mathbf{B}_i = \mathbf{H}_i \mathbf{P}_i$
- $q = \sum_{i=1}^{L} q_i$ and $l = \sum_{i=1}^{L} l_i$
- MSE obtained matches the centralized WSN (benchmark) when q reaches p

Beamforming in WSN's contd.[26]

Noisy FC Case: Design Problem (Power & BW Constraint)

$$\min_{\mathbf{P}_i,\mathbf{B}_i;1\leq i\leq L} \mathrm{Tr}(\mathbf{D}) \tag{39}$$

$$\text{s.t}\ \ \mathbf{D}^{-1} = \mathbf{I}_p + \mathbf{A}^T\mathbf{B}^T(\mathbf{I}_q + \mathbf{B}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{A} \tag{40}$$

$$\mathrm{Tr}[\mathbf{P}_i(\mathbf{A}_i\mathbf{A}_i^T + \mathbf{I}_{l_i})\mathbf{P}_i^T] \leq p_i \tag{41}$$

- Above problem is not convex over $\{\mathbf{P}_i, \mathbf{B}_i : 1 \leq i \leq L\}$
- With certain relaxation the problem is transformed into an SDP(Semi-Definite programming)
- MSE is matched with the centralized benchmark for $q \geq p$ by increasing the power to infinite

Beamforming in WSN's

- In[27], the same system model as in [26] is considered
- The FC is equipped with multiple (N_{FC}) antennas
- **Noiseless FC case:**

$$\hat{\mathbf{P}} = \arg\min_{\mathbf{P}} \{\mathbf{E}\} \tag{42}$$

  - where, MSE matrix $(\mathbf{E}) = \mathbb{E}||\mathbf{HPA}\theta + \mathbf{HPn}_r - \theta||^2$
  - Number of antennas at the FC is assumed to be $N_{FC} \geq p$
  - For solution to exist, the number of transmit message $N_i = N \leq p$
  - Performance of the system approaches the centralized benchmark when $N = p$

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[27] Behbahani, A.S.; et al., *"Linear Decentralized Estimation of Correlated Data for Power-Constrained Wireless Sensor Networks,"* in Signal Processing, IEEE Transactions on , 2012

Beamforming in WSN's contd.[27]

Noisy FC case:

- The received signal at the FC is given by:

$$\mathbf{y} = \mathbf{HPA}\theta + \mathbf{HPn}_r + \mathbf{n}_d \tag{43}$$

- The optimization problem is given as

$$\min_{\{\mathbf{W}\},\{\mathbf{P}_i\}_{i=1,\cdots L}} \zeta(\mathbf{W}, \{\mathbf{P}_i\}_{i=1}^{L}) \tag{44}$$

$$\text{s.t} \ \ \mathrm{Tr}[\mathbf{P}_i(\mathbf{A}_i\mathbf{R}_\theta\mathbf{A}_i^H + \mathbf{R}_{n_{r_i}})\mathbf{P}_i^H] \leq p_i, \quad i = 1, 2\cdots, L \tag{45}$$

- where,
  $\zeta(\mathbf{W}, \{\mathbf{P}_i\}_{i=1}^{L}) = \mathbb{E}||\mathbf{Wy} - \theta||^2 = \mathbb{E}||\mathbf{W}(\mathbf{HPA}\theta + \mathbf{HPn}_r + \mathbf{n}_d) - \theta||^2$

Beamforming in WSN's contd.[27]

Noisy FC case:

- An iterative algorithm is used that decrease the MSE monotonically to find the set of $\{\mathbf{P}_i\}_{i=1}^{L}$ and $\mathbf{W}$
- The proposed algorithm is also shown to converge
- The average MSE performance is shown to approach the benchmark as the number of sensor increases

Beamforming in WSN's

- In [28], an online algorithm for tracking of time varying source is proposed
- The random source varies according to a state space model

$$\theta(t+1) = \mathbf{B}\theta(t) + \mathbf{Q}\mathbf{n}_s(t) \tag{46}$$

- where, **B** is a known state transition matrix
- The system model is similar to the one considered in [27]

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[28] Singh, Rahul R.; Rajawat, Ketan, "*Online precoder design for parameter tracking in wireless sensor networks,*" in PIMRC, 2015

Beamforming in WSN's contd.[28]

- The designed optimization problem is given by:

$$\min_{\{\mathbf{W}\},\{\mathbf{P}_i\}_{i=1,\cdots L}} \zeta(\mathbf{W},\{\mathbf{P}_i\}_{i=1}^{L}) \tag{47}$$

$$\text{s.t} \;\; \mathrm{Tr}[\mathbf{P}_i(\mathbf{A}_i\mathbf{R}_\theta\mathbf{A}_i^H+\mathbf{R}_{n_{r_i}})\mathbf{P}_i^H] \le p_i, \quad i=1,2\cdots,L \tag{48}$$

- where, p_i is the maximum transmit power for sensor i
- where, $\zeta(\mathbf{W},\{\mathbf{P}_i\}_{i=1}^{L}) = \mathrm{Tr}(\mathbb{E}(\tilde{\theta}(t|t)\tilde{\theta}^H(t|t)))$
- Also,

$$\begin{aligned}\tilde{\theta}(t|t) =& \mathbf{B}\tilde{\theta}(t-1|t-1)+\mathbf{W}(t)[-\mathbf{HP}(t)\mathbf{AB}\tilde{\theta}(t-1|t-1)\\ &+\mathbf{HP}(t)\mathbf{AQn}_s(t-1)+\mathbf{HP}(t)\mathbf{n}_r(t)+\mathbf{n}_f(t)]-\mathbf{Qn}_s(t-1)\end{aligned} \tag{49}$$

Beamforming in WSN's contd.[28]

- A **BCD**(Block Coordinate Descent) approach is applied to solve the above non-convex problem
- Convergence of BCD algorithm is guaranteed as the problem is strictly convex in $\mathbf{W}$ & $\mathbf{P}_i$ separately
- The online algorithm is also proved to be convergent for some special cases:
  - when $\mathbf{Q} = 0$ and $\mathbf{B}$ is symmetric with $0 < |\lambda(\mathbf{B})| \leq 1$
  - when $\mathbf{B}$=0

Future Proposed work

- Robust Transceiver design for distributed WSN
- Optimal transceiver design for Interference mitigation in MIMO-OFDM system(Insufficient CP)
- Online Resource Allocation

Thank You!