1. INTRODUCTION

Spectrum is a scarce resource. There is significant competition for available spectrum resources for new military, commercial and civil applications. Futuristic military communication devices will require more spectral resources. Cognitive radio provides a new platform for military communication system by opportunistic exploitation of available spectrum holes and thus allowing guaranteed voice, data or video communication services to military personnel in hostile environment by maintaining connectivity.

Cognitive radio (CR) is an intelligent radio which adapts its transmission parameters according to surrounding wireless environment to use the spectrum efficiently. In a military scenario, CR can detect bad quality of radio channel, congestion or unwanted interference and switch and adapt to the frequency bands unaffected by the above problems enhancing the quality of the military communications.

Cognitive radio comprises of two types of users. First is primary user (PU) who has the license to use the given frequency band. Second is secondary user (SU) who is not a licensed user of the given frequency band, but can use band whenever it is vacant. As soon as the PU returns to the frequency band, SU has to vacate it and find another vacant frequency band. To detect whether a frequency band is vacant or not, SU needs to perform spectrum sensing. There are various detection techniques available for spectrum sensing like energy detection, cyclostationary detection and matched filter detection. Energy detection is a suitable detection method having much low complexity compared to other detection methods and it is optimum when SUs do not have any information about PU signals.

In conventional energy detection, received signal samples by the SU are squared, summed and compared with the predefined threshold to determine whether a PU is present or absent. An improved energy detector has been proposed in which squaring operation is replaced by an arbitrary power operation $p$ such that $p > 0$ and it has been shown that with optimum $p$, improved energy detector performs better than that of the conventional energy detector. However, the sensing performance by a single SU may be degraded due to fading of the radio channels and shadowing effect. To overcome degraded detection performance, cooperative spectrum sensing (CSS) has been proposed. Cooperative spectrum sensing takes advantage of spatial diversity and is performed in two steps: sensing and reporting of local decisions over reporting channel, unlike single step in single user sensing. The local decisions are combined at a central fusion center to make final decision on presence or absence of PU. Optimization of CSS for single threshold improved energy detector has been proposed for perfect reporting channel and for imperfect reporting channel.

It has been shown that bandwidth of the reporting control...
channel is limited. To reduce the bandwidth needed, a censoring method based on double threshold has been proposed for OR fusion rule in CSS\cite{ref10} in which if the received energy lies between upper and lower threshold, no decision is communicated to the fusion center. In modified double threshold energy detection, SUs receiving energies between upper threshold and lower threshold report actual energy values to the fusion center\cite{ref11}. This method gives rise to slight better detection performance at the cost of communication bandwidth of the reporting channel. A fusion rule \( n\)-ratio\cite{ref12} for CSS is proposed with double threshold energy detection showing significant improvement detection performance over single threshold energy detection for CSS. Also performance of CSS is optimized against optimum \( n\).

In this paper, we combine improved energy detection with double threshold for CSS in CR. We find optimum \( p\) that maximizes the probability of detection of PU and minimizes total error rate, i.e., sum of probability of miss detection and probability of false alarm. We also show the effect of probability of unreliable local decision in double threshold on the detection and error performance of the CSS and try to find optimum difference in upper and lower threshold such that probability of detection is maximized and total error is minimized. In cooperative sensing, we use \( k\)-out-of-\( M\) fusion rule\cite{ref13} to combine local decisions from SUs and find optimum pair \((p,k)\) such that minimized total error rate is the lowest. Further, the effect of imperfect reporting channel is considered on the performance of CSS.

2. IMPROVED ENERGY DETECTION

Authors consider CSS scenario where there is a single primary user (PU), \( M\) secondary users (SU) and one fusion center (FC) as shown in Fig. 1. The channel between PU and each SU is modeled by an additive white Gaussian noise (AWGN) channel. Improved energy detection is used at SUs to sense the PU. Then the local binary decisions of each SU are conveyed to the FC over the reporting channel which is modeled as a binary symmetric channel. Fusion center then takes the final binary decision to determine whether PU is present or absent by combining the hard local decisions according to a fusion rule.

![Figure 1. Cooperative spectrum sensing.](image)

The sensing of PU is a binary detection problem. In this case, the binary hypothesis testing problem can be given as

\[
 r(m) = \begin{cases} 
 n(m), & H_0 \text{ for } m = 1, \ldots, N \\
 s(m), & H_1 \end{cases}
\]

where \( r(m) \) is the \( m\)-th sample of real additive white Gaussian noise (AWGN) with mean zero and variance \( \sigma^2 \), that is, \( n(m) \sim N(0, \sigma^2) \). \( H_0 \) and \( H_1 \) are the hypotheses corresponding to the absence and presence of PU. \( N \) is the total number of samples. The signal \( s(m) \) is independent and identically distributed real Gaussian random variable with mean zero and variance \( \sigma^2 \). We assume that \( s(m) \) and \( n(m) \) to be independent.

We define \( X(m) = \frac{r(m)}{\sigma_n} \). Then the test statistic \( E_i \) for improved energy detection is given by

\[
 E_i = \frac{1}{N} \sum_{m=1}^{N} |X(m)|^p
\]

where \( p \) is an arbitrary positive constant. Thus improved energy detector is same as the conventional energy detector when \( p = 2 \). Local decision \( L_i \) for improved energy detector of \( i\)-th SU about presence or absence of PU is taken as follows:

\[
 L_i = \begin{cases} 
 0, & E_i \leq T \\
 1, & E_i > T 
\end{cases}
\]

where \( T \) is the threshold to decide between \( H_0 \) and \( H_1 \). ‘0’ and ‘1’ correspond to absence and presence of PU respectively.

We assume that the samples of the received signal are independent in time. For any \( p \)-random variables \( \{X(m)|^p\} \) are independent and identically distributed. We can write mean \( \mu_0 \) and variance \( \sigma^2_0 \) of \( \{X(m)|^p\} \) under \( H_0 \) as

\[
 \mu_0 = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right)
\]

\[
 \sigma^2_0 = \frac{2^p}{\sqrt{\pi}} \left[ \Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right]
\]

and under hypothesis \( H_1 \), the mean \( \mu_1 \) and variance \( \sigma^2_1 \) of \( \{X|^p\} \) can be given as

\[
 \mu_1 = \frac{2^{p/2}(1+\gamma)p^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right)
\]

\[
 \sigma^2_1 = \frac{2^p(1+\gamma)p^{p/2}}{\sqrt{\pi}} \left[ \Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right]
\]

where \( \Gamma(\cdot) \) is complete Gamma function and \( \gamma = \frac{\sigma^2}{\sigma^2_n} \) is received signal-to-noise ratio (SNR). Since \( \{X|^p\} \) are Gaussian random variables, the sum of such \( N \) random variables is also Gaussian distributed. Thus the test statistic \( E_i \) is Gaussian distributed. Now if number of samples \( N \) are large enough, we can invoke Central limit theorem. Then \( E_i \) is Gaussian distributed with means \( \mathbb{E}(E_i) \)

\[
 \mathbb{E}(E_i) = \begin{cases} 
 \mu_0, & H_0 \\
 \mu_1, & H_1 
\end{cases}
\]

and with variances \( \text{var}(E_i) \)

\[
 \text{var}(E_i) = \begin{cases} 
 \sigma^2_0/N, & H_0 \\
 \sigma^2_1/N, & H_1 
\end{cases}
\]
Using Eqns (8) and (9), we can write probability of detection \( P_d \) as

\[
P_d = Q\left(\frac{T - \mu_1}{\sigma_1 / \sqrt{N}}\right)
\]

and probability of false alarm is

\[
P_f = Q\left(\frac{T - \mu_0}{\sigma_0 / \sqrt{N}}\right)
\]

where \( Q(x) \) is a Q-function defined as

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy,
\]

where \( T \) is threshold, \( \mu_1, \sigma_1^2, \mu_0 \) and \( \sigma_0 \) are given by Eqns (4), (5), (6), and (7) respectively.

3. COOPERATIVE DOUBLE THRESHOLD ENERGY DETECTION

Double threshold energy detection method uses two thresholds to make local decision instead of a single threshold. Double threshold energy detection offers an advantage over conventional energy detection in terms of bandwidth needed for reporting channel to report local sensing results to the fusion center. From Fig. 2, it can be seen that there is a region of uncertainty between upper threshold \( T_2 \) and lower threshold \( T_1 \). Whenever the received energy falls in the uncertainty region, no local decision is taken and no reports are sent to the fusion center. This brings down the bandwidth needed for reporting control channel since secondary users receiving energy in uncertainty region do not send any local decision over the reporting channel.

For double threshold energy detection,

\[
L_i = \begin{cases} 
\text{No Decision}, & E_i \leq T_1 \\
1, & T_1 < E_i < T_2 \\
0, & E_i \geq T_2
\end{cases}
\]

We can define \( P_d \) and \( P_f \) for single threshold improved energy detector as follows

\[
P_d = \text{Pr}\{E_i \geq T | H_1\}
\]

and

\[
P_f = \text{Pr}\{E_i \geq T | H_0\}
\]

Figure 3 shows the probabilities involved in double threshold detection.

Mathematically, probability of detection is given by

\[
P_{d1} = \text{Pr}\{E_i \geq T_2 | H_1\}
\]

Probability of no decision under \( H_1 \) is

\[
\Delta_1 = \text{Pr}\{T_1 < E_i < T_2 \ | H_1\}
\]

Probability of miss detection is

\[
P_m = \text{Pr}\{E_i \leq T_2 | H_1\} = 1 - P_{d1} - \Delta_1
\]

Probability of false alarm can be given by

\[
P_f = \text{Pr}\{E_i \geq T_2 | H_0\}
\]

Probability of no decision under \( H_0 \) is

\[
P_{d0} = \text{Pr}\{E_i \leq T_1 | H_0\} = 1 - P_f - \Delta_0
\]

It can be seen that \( \Delta_1 \) and \( \Delta_0 \) are dependent on threshold difference \( T_2 - T_1 \).

Cooperative spectrum sensing setting is given in Fig. 1. There are various fusion rules used in literature like AND, OR, majority and \( k \)-out-of-\( M \) for hard decision combining of local decisions at the fusion center. In this paper, we use \( k \)-out-of-\( M \) fusion rule. We define \( k \) as an integer such that \( 0 < k \leq M \). In double threshold energy detection, we assume that \( K_1 \) is the number of SUs favouring \( H_1 \) i.e. absence of PU while \( K_2 \) secondary users favour the hypothesis \( H_2 \) i.e. presence of PU. Then we have \( K_1 + K_2 \leq M \). Inequality is due to the fact that in double threshold energy detection, there might be some SUs whose received energies fall in the uncertainty region and so they do not report any local decision to the fusion center. The fusion center in this case takes the final decision as follows:

\[
H_1 \text{ when } K_2 \geq k \\
H_0 \text{ when } K_2 < k
\]

That is when number of SUs favouring \( H_1 \) is greater than or equal to \( k \), fusion center takes final decision saying PU is present, otherwise PU is assumed to be absent. All other fusion rules can be derived from \( k \)-out-of-\( M \) fusion rule easily by choosing suitable \( k \) as shown in Table 1.

Cooperative probability of detection \( Q_d \) for \( k \)-out-of-\( M \) fusion rule at the fusion center in double threshold energy detection can be calculated easily with some modifications for \( n \)-ratio logic and is given as follows:

\[
Q_d = \sum_{K_2=k} M \sum_{K_1=0}^{M-K_2} \binom{M}{K_2} \binom{M-K_1}{K_1} p_{d1}^{K_1} \Delta_1^{M-K_1-K_2} p_{m1}^{K_2} p_{m2}^{K_2}
\]
and cooperative probability of false alarm $Q_f$ at fusion center can be given by

$$Q_f = \sum_{K_d=1}^{M-K_1} \left( \sum_{K_1=0}^{M-K_2} \binom{M-K_1}{K_1} p_f^{K_1} q_f^{K_d-K_1-K_2} q_d^{K_d+K_1-K_2} \right)$$  \hspace{1cm} (22)$$

and cooperative probability of miss detection $Q_m$ at fusion center is

$$Q_m = 1 - Q_d$$  \hspace{1cm} (23)$$

where $P_d, \Delta_1, P_m, P_f, \Delta_0$, and $P_d$ are defined in Eqs (15) to (20) respectively.

Table 1. Various fusion rules as a special case of $k$-out-of-$M$ fusion rule

<table>
<thead>
<tr>
<th>Value of $k$ in $k$-out-of-$M$ fusion rule</th>
<th>Specific fusion rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>OR fusion rule</td>
</tr>
<tr>
<td>$k = [M/2]$</td>
<td>Majority fusion rule</td>
</tr>
<tr>
<td>$k = M$</td>
<td>AND fusion rule</td>
</tr>
</tbody>
</table>

4. IMPERFECT REPORTING CHANNEL

In realistic scenarios, the reporting channel between SUs and fusion center is subjected to the errors. We assume that the reporting channel to be a binary symmetric channel with probability of error $P_e$ as shown in Fig. 4.

Figure 4. Binary symmetric imperfect reporting channel.

Then the erroneous probability of detection $P_{d,e}$ by fusion center is given by

$$P_{d,e} = P_{d} (1-P_e) + P_e P_{m}$$  \hspace{1cm} (24)$$

and the erroneous probability of false alarm $P_{f,e}$ by fusion center is given by

$$P_{f,e} = P_{f} (1-P_e) + P_e P_{d_0}$$  \hspace{1cm} (25)$$

and the erroneous probability of miss detection $P_{m,e}$ by fusion center is given by

$$P_{m,e} = P_m (1-P_e) + P_e P_{d_1}$$  \hspace{1cm} (26)$$

and the erroneous probability of detecting 0 under $H_0$ by fusion center is given by

$$P_{d_0,e} = P_{d_0} (1-P_e) + P_e P_f$$  \hspace{1cm} (27)$$

Erroneous $\Delta_{d,e}$ and $\Delta_{f,e}$ are calculated by replacing $P_{d_1}, P_{m}, P_{f}$ and $P_{d_0}$ by $P_{d,e}, P_{m,e}, P_{f,e}$ and $P_{d_0,e}$ in Eqs (17) and (20) respectively. After putting $P_{d,e}$ of Eqn (24), $P_{f,e}$ of Eqn (25), $P_{m,e}$ of Eqn (26), $P_{d_0,e}$ of Eqn (27), $\Delta_{d,e}$ and $\Delta_{f,e}$ instead of $P_{d_1}, P_{m}, P_{f}, P_{d_0}, \Delta_1$ and $\Delta_0$ respectively in Eqs (21), (22), and (23), we get the erroneous cooperative probability of detection, erroneous cooperative probability of false alarm and erroneous cooperative probability of miss detection at the fusion center for the imperfect reporting channel.

5. SIMULATION RESULTS

In this section, we present simulation results for optimizing the performance of CSS with double threshold improved energy detector. Here, $M$ is total number of SUs participating in cooperation and $N$ is number of samples.

5.1 Optimization of Cooperative Probability of Detection $Q_d$

We perform two step optimization to enhance cooperative probability of detection $Q_d$.

Step 1: Maximizing $Q_d$ against threshold difference $\Delta T$

In Fig. 5, cooperative probability of detection $Q_d$ is plotted against threshold difference $\Delta T = T_2 - T_1$ for different values of $p$ when cooperative probability of false alarm $Q_f$ is fixed to 0.0001. It can be seen that for different values of $p$, $Q_d$ is maximum for different value of $\Delta T$. Thus choosing an appropriate $\Delta T$ in double threshold will optimize the detection performance.

Step 2: Finding optimum power constant $p$ such that maximized $Q_d$ is the highest.

For each value of $p$, maximized value of $Q_d$ is different. This is shown in Fig. 6 where maximized $Q_d$ is plotted against corresponding power operation value $p$ for different received SNR. Maximized $Q_d$ is the highest for $p = 2.8$ for different SNR with $Q_f = 0.0001$, $M = 20$, $N = 50$. This shows that conventional energy detector i.e. $p = 2$ is not an optimum energy detector. Also as the received SNR increases, the value of maximized $Q_d$ also increases, but optimum $p$ remains the same. Similarly in Fig. 7, maximized $Q_d$ is plotted against $p$ for different number of cooperating SUs $M$. As the number of cooperating SUs increases, detection performance improves. Optimum $p$ in this case is also 2.8. Thus we observe from Figs. 6, 7, and 8, maximized $Q_d$ is the highest for $p = 2.8$ irrespective of changes in SNR, and $M$. In this simulation study, we have used majority fusion rule i.e. $k = [M/2]$ at the fusion center to decide on presence or absence of PU.

5.2 Minimization of Total Error Rate $Q_m + Q_f$

The expressions for $Q_f$ and $Q_m$ are given by Eqns (22) and (23) respectively. Total error rate $Q_f + Q_m$ is also optimized by two step optimization. Figure 9 shows the total error rate versus threshold difference $\Delta T$ for different values of $p$. Figure 10 shows minimized total error rate (TER) $Q_m + Q_f$ versus $p$ for $k$-out-of-$M$ fusion rule with $M = 10$. In this case, first TER is minimized for a particular by finding optimum $\Delta T$ as shown in Fig. 9. Then minimized TER is plotted against $p$ for different values of $k$ i.e. number of SUs supporting hypothesis $H_1$ as shown in Fig. 10. Thus we are able to find optimum $(p, k)$ pair for which minimized TER is the lowest. In this case, optimum pair is $(2.3, 4)$. That is, to have lowest minimized TER, one should choose $p = 2.3$ and 4-out-of-10 fusion rule.
Figure 5. Cooperative probability of detection $Q_d$ vs $\Delta T$ for different $p$, $Q_f = 0.0001$, SNR = -5 dB, $M = 20$, $N = 50$.

Figure 6. Maximized cooperative probability of detection $Q_d$ vs $p$ for different SNR, $Q_f = 0.0001$, $M = 20$, $N = 50$.

Figure 7. Maximized cooperative probability of detection $Q_d$ vs $p$ for different $Q_f$, SNR = -5 dB, $M = 20$, $N = 50$.

Figure 8. Maximized cooperative probability of detection $Q_d$ vs $p$ for different $M$, SNR = -5 dB, $Q_f = 0.0001$, $N = 50$.

Figure 9. Total error rate vs $\Delta T$ for different $p$, SNR = -5 dB, $Q_f = 0.0001$, $M = 20$, $N = 50$.

Figure 10. Minimized total error rate $Q_m + Q_f$ vs $p$ for different $k$, SNR = -5 dB, $M = 10$, $N = 50$. 
5.3 Effect of Imperfect Reporting Channel

Figure 11 shows the effect of imperfect reporting channel on the detection performance. We have considered binary symmetric channel with error probability $P_e = 10^{-3}$. Cooperative probability of detection $Q_d$ is plotted against $\Delta T$ for different $P_e$. It can be seen that detection performance degrades with imperfect reporting channel and imperfection is higher as the threshold difference $\Delta T$ increases. Thus $\Delta T$ if chosen properly, the effect of imperfect reporting channel on the detection performance can be minimized. In Fig. 12, the effect of reporting channel for different probability of error $P_e$ is shown on $Q_d$ for specific $p = 2.8$. It can be seen that detection performance deteriorates with increase in error probability $P_e$ of the reporting channel.

6. CONCLUSION

In this work, cooperative spectrum sensing with double threshold improved energy detection is studied. Two step optimization is performed to enhance the system performance. It has been shown that optimum power operation value is $p = 2.8$ that corresponds to the highest value of maximized cooperative probability of detection which is different from conventional energy detection i.e. $p = 2$. Similarly the lowest value of minimized total error rate corresponds to $p = 2.3$ and $k = 4$ for $k$-out-of-$M$ fusion rule with $M = 10$. Thus optimum value of $p$ changes according to the parameter chosen to optimize. Also the effect of binary symmetric imperfect reporting channel is studied on the performance of cooperative spectrum sensing. It is shown that the performance degrades with increase in error probability of reporting channel and effect of imperfect reporting channel is more profound for higher threshold difference. Thus if the threshold difference $\Delta T$ chosen properly, the effect of imperfect reporting channel can be minimized.

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