Burr-XII Distribution Parametric Estimation and Estimation of Reliability of Multicomponent Stress-Strength

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Abstract: *In this paper, we estimate the multicomponent stress-strength reliability by assuming the Burr-XII distribution. The research methodology adopted here is to estimate the parameters by using maximum likelihood estimation. The reliability is estimated using the maximum likelihood method of estimation and results are compared using the Monte-Carlo simulation for small samples. By using real data sets we well illustrate the procedure.*

Key Words*: Burr-XII distribution, reliability estimation, stress-strength, ML estimation, confidence intervals.*

1. Introduction

The Burr-XII distribution, which was originally derived by Burr (1942) and received more attention by the researchers due to its broad applications in different fields including the area of reliability, failure time modeling and acceptance sampling plan. Reader can find the applications in various fields from Ali and Jaheen (2002) and Burr (1942).

The two parameters Burr-XII distribution has the following density function

$$f(x;\alpha,\beta)=\alpha\beta x^{\beta-1}\left(1+x^{\beta}\right)^{-(\alpha+1)}; \quad \text{for } x>0 \tag{1}$$

and the distribution function

$$F(x;\alpha,\beta)=1-\left(1+x^{\beta}\right)^{-\alpha}; \quad \text{for } x>0. \tag{2}$$

Here $\alpha>0$ and $\beta>0$ are the shape parameters respectively. It is important to note that when α =1, the Burr-XII reduces to the log-logistic distribution. Wu and Yu (2005) show that the shape parameter β plays a vital role for the Burr-XII. Now onwards two parameters Burr-XII distribution with shape parameters α and β will be denoted by BD(α,β).

Let the random samples $Y, X_1, X_2, \ldots X_k$ be independent, G(y) be the continuous distribution function of Y and F(x) be the common continuous distribution function of $X_1, X_2, \ldots X_k$. The reliability in a multicomponent stress-strength model developed by Bhattacharyya and Johnson (1974) is given by

$$R_{s,k} = \mathrm{P}\left[\text{at least } s \text{ of the } (X_1, X_2, \ldots X_k) \text{ exceed } Y\right]$$
$$= \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} [1 - F(y)]^i [F(y)]^{k-i} \, dG(y), \tag{3}$$

where $X_1, X_2, \ldots X_k$ are identically independently distributed (iid) with common distribution function $F(x)$ and subjected to the common random stress *Y*. The probability in (3) is called reliability in a multicomponent stress-strength model [Bhattacharyya and Johnson (1974)]. The survival probabilities of a single component stress-strength version have been considered by several authors for diffent distributions. Enis and Geisser (1971), Downtown (1973), Awad and Gharraf (1986), McCool (1991), Nandi and Aich (1994), Surles and Padgett (1998), Raqab and Kundu (2005), Kundu and Gupta (2005& 2006), Raqab *et al.* (2008), Kundu and Raqab (2009). The reliability in a multicomponent stress-strength was developed by Bhattacharyya and Johnson (1974), Pandey and Borhan (1985). Pandey and Uddin (1991) assumed the parameters not involved in reliability as known using the Bayesian estimation. Recently Rao and Kantam (2010) studied estimation of reliability in multicomponent stress-strength for the log-logistic distribution and Rao (2012) developed an estimation of reliability in multicomponent stress-strength based on generalized exponential distribution.

The aim of this paper is to study the reliability in a multicomponent stress-strength based on X, Y being two independent random variables, where $X \sim BD(\alpha_1, \beta)$ and $Y \sim BD(\alpha_2, \beta)$. We will use the parametric estimation and estimation reliability. Suppose a system, with *k* identical components, functions if at least $s\,(1 \le s \le k)$ components simultaneously operate. In its operating environment, the system is subjected to a stress *Y* which is a random variable with distribution function $G(.)$. The strengths of the components, that is the minimum stresses to cause failure, are independently and identically distributed random variables with distribution function $F(.)$. The reliability of system can be obtained by equation (3). The estimation of survival probability when the stress and strength variates follow two parameters Burr-XII distribution is not paid much attention. Therefore, an attempt is made here to study the estimation of reliability in multicomponent stress-strength model with reference to two parameters Burr-XII distribution. The rest of the paper is organized as follows. In Section 2, we discussed the research methodology and procedure for expression of $R_{s,k}$. The asymptotic distribution and confidence interval of equation (3) is obtained using the MLE. The results of small sample comparisons made through Monte Carlo simulations are in Section 3. Some findings are discussed in Section 4.

2. Research Methodology for Maximum Likelihood Estimator of $R_{s,k}$

Let $X \sim BD(\alpha_1, \beta)$ and $Y \sim BD(\alpha_2, \beta)$ are independently distributed with unknown shape parameters α_1 and α_2 and common shape parameter β respectively. The reliability in multicomponent stress-strength for two parameter Burr-XII distribution using (3) we get

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \int_0^\infty \left[\left(1+y^\beta\right)^{-\alpha_1}\right]^i \left[1-\left(1+y^\beta\right)^{-\alpha_1}\right]^{k-i} \alpha_2 \beta y^{\beta-1} \left(1+y^\beta\right)^{-(\alpha_2+1)} dy$$

$$= \sum_{i=s}^{k} \binom{k}{i} \nu \int_0^1 t^{i+\nu-1} (1-t)^{k-i} \, dt, \quad \text{where } t = \left(1+y^\beta\right)^{-\alpha_1}, \ \nu = \frac{\alpha_2}{\alpha_1}$$

$$= \sum_{i=s}^{k} \binom{k}{i} \nu \, \mathrm{B}(\nu+i, k-i+1).$$

After the simplification we get

$$R_{s,k} = \nu \sum_{i=s}^{k} \frac{k!}{i!} \left[\prod_{j=i}^{k} (\nu + j)\right]^{-1} \quad \text{since } k \text{ and } i \text{ are integers.} \tag{4}$$

The probability in (4) is called reliability in a multicomponent stress-strength model. It is important to note that the MLE (maximum likelihood estimation) of $R_{s,k}$ depends on the MLE of α_1 and α_2. Therefore, we need to find the MLE of the later before the MLE of first one. Similarly, to derive the MLE of α_1 and α_2 we need the MLE of β. Let us assume that $X_1 < X_2 < \ldots < X_n$ is random sample from $BD(\alpha_1, \beta)$ and $Y_1 < Y_2 < \ldots < Y_m$ is a random sample from $BD(\alpha_2, \beta)$, then the log-likelihood function (LLF) of these observed sample is

$$L(\alpha_1, \alpha_2, \beta) = (m+n)\ln\beta + n\ln\alpha_1 + m\ln\alpha_2 + (\beta-1)\left[\sum_{i=1}^{n} \ln x_i + \sum_{j=1}^{m} \ln y_j\right]$$
$$-(\alpha_1+1)\sum_{i=1}^{n} \ln\left(1+x_i^\beta\right) - (\alpha_2+1)\sum_{j=1}^{m} \ln\left(1+y_j^\beta\right) \tag{5}$$

The MLE of β, α_1 and α_2; say $\hat{\beta}, \hat{\alpha}_1$ and $\hat{\alpha}_2$ respectively, can be obtained as the solution of

$$\frac{\partial \mathrm{L}}{\partial \beta} = 0 \Rightarrow \frac{m+n}{\beta} + \left[\sum_{i=1}^{n} \ln x_i + \sum_{j=1}^{m} \ln y_j\right] - (\alpha_1+1)\sum_{i=1}^{n} \frac{x_i^\beta \ln x_i}{\left(1+x_i^\beta\right)} - (\alpha_2+1)\sum_{j=1}^{m} \frac{y_j^\beta \ln y_j}{\left(1+y_j^\beta\right)} = 0 \tag{6}$$

$$\frac{\partial \text{ L}}{\partial \alpha_1} = 0 \ \Rightarrow \frac{n}{\alpha_1} - \sum_{i=1}^{n} \ln\left(1 + x_i^{\beta}\right) = 0 \,, \tag{7}$$

$$\frac{\partial \text{ L}}{\partial \alpha_2} = 0 \ \Rightarrow \frac{m}{\alpha_2} - \sum_{j=1}^{m} \ln\left(1 + y_j^{\beta}\right) = 0 \tag{8}$$

From (7) and (8), we obtain

$$\hat{\alpha}_1(\beta) \ \Rightarrow \frac{n}{\sum_{i=1}^{n} \ln\left(1 + x_i^{\beta}\right)} \text{ and } \hat{\alpha}_2(\beta) \ \Rightarrow \frac{m}{\sum_{j=1}^{m} \ln\left(1 + y_j^{\beta}\right)} \tag{9}$$

Putting the values of $\hat{\alpha}_1(\beta)$ and $\hat{\alpha}_2(\beta)$ into (6), we obtain

$$\frac{m+n}{\beta} + \left[\sum_{i=1}^{n} \ln x_i + \sum_{j=1}^{m} \ln y_j\right] - \frac{n}{\sum_{k=1}^{n} \ln\left(1 + x_k^{\beta}\right)} \sum_{i=1}^{n} \frac{x_i^{\beta} \ln x_i}{\left(1 + x_i^{\beta}\right)} - \frac{m}{\sum_{k=1}^{m} \ln\left(1 + y_k^{\beta}\right)} \sum_{j=1}^{m} \frac{y_j^{\beta} \ln y_j}{\left(1 + y_j^{\beta}\right)}$$

$$- \sum_{i=1}^{n} \frac{x_i^{\beta} \ln x_i}{\left(1 + x_i^{\beta}\right)} - \sum_{j=1}^{m} \frac{y_j^{\beta} \ln y_j}{\left(1 + y_j^{\beta}\right)} = 0 \tag{10}$$

Therefore, $\hat{\beta}$ can be obtained using the following non-linear equation (11)

$$h(\beta) = \beta \tag{11}$$

$$\text{Where } h(\beta) = \frac{(m+n) + \sum_{i=1}^{n} \frac{\ln x_i^{\beta}}{\left(1 + x_i^{\beta}\right)} + \sum_{j=1}^{m} \frac{\ln y_j^{\beta}}{\left(1 + y_j^{\beta}\right)}}{\frac{n}{\sum_{k=1}^{n} \ln\left(1 + x_k^{\beta}\right)} \sum_{i=1}^{n} \frac{x_i^{\beta} \ln x_i}{\left(1 + x_i^{\beta}\right)} + \frac{m}{\sum_{k=1}^{m} \ln\left(1 + y_k^{\beta}\right)} \sum_{j=1}^{m} \frac{y_j^{\beta} \ln y_j}{\left(1 + y_j^{\beta}\right)}} \tag{12}$$

Because $\hat{\beta}$ is a fixed point solution of the non-linear (11), therefore, it can be obtained by using a simple iterative procedure as

$$h(\beta_{(j)}) = \beta_{(j+1)} \,, \tag{13}$$

where $\beta_{(j)}$ is the j^{th} iteration of $\hat{\beta}$. It should be noted that during the simulation process when the difference between $\beta_{(j)}$ and $\beta_{(j+1)}$ is sufficiently small then stop the iterative process. Once we obtain $\hat{\beta}$, the parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ can be obtained from (9) as $\hat{\alpha}_1(\hat{\beta})$ and $\hat{\alpha}_2(\hat{\beta})$ respectively. The MLE of $R_{s,k}$ becomes

$$\hat{R}_{s,k} = \hat{v} \sum_{i=s}^{k} \frac{k!}{i!} \left[\prod_{j=i}^{k} (\hat{v} + j)\right]^{-1} \text{ where } \hat{v} = \frac{\hat{\alpha}_2}{\hat{\alpha}_1}. \tag{14}$$

To obtain the asymptotic confidence interval for $R_{s,k}$, we proceed as follows:

The asymptotic variance of the MLE is given by

$$\mathrm{V}(\hat{\alpha}_1)=\left[E\left(-\frac{\partial^2 L}{\partial \alpha_1^2}\right)\right]^{-1}=\frac{\alpha_1^2}{n} \text{ and } \mathrm{V}(\hat{\alpha}_2)=\left[E\left(-\frac{\partial^2 L}{\partial \alpha_2^2}\right)\right]^{-1}=\frac{\alpha_2^2}{m} \tag{15}$$

The asymptotic variance (AV) of an estimate of $R_{s,k}$ which is a function of two independent statistics (say) α_1,α_2 is given by Rao (1973).

$$\mathrm{AV}(\hat{\mathrm{R}}_{s,k})=\mathrm{V}(\hat{\alpha}_1)\left(\frac{\partial \mathrm{R}_{s,k}}{\partial \alpha_1}\right)^2+\mathrm{V}(\hat{\alpha}_2)\left(\frac{\partial \mathrm{R}_{s,k}}{\partial \alpha_2}\right)^2. \tag{16}$$

Thus from Equation (16), the asymptotic variance of $\hat{\mathrm{R}}_{s,k}$ can be obtained.

To avoid the difficulty of derivation of $R_{s,k}$, we obtain $\hat{\mathrm{R}}_{s,k}$ and their derivatives for (*s, k*)=(1,3) and (2,4) separately, they are given by

$$\hat{R}_{1,3}=\frac{\hat{\nu}(\hat{\nu}^2+6\hat{\nu}+11)}{(1+\hat{\nu})(2+\hat{\nu})(3+\hat{\nu})} \text{ and } \hat{R}_{2,4}=\frac{\hat{\nu}(\hat{\nu}^2+9\hat{\nu}+26)}{(2+\hat{\nu})(3+\hat{\nu})(4+\hat{\nu})}.$$

$$\frac{\partial \hat{R}_{1,3}}{\partial \alpha_1}=\frac{-6\hat{\nu}(3\hat{\nu}^2+12\hat{\nu}+11)}{\alpha_1\left[(1+\hat{\nu})(2+\hat{\nu})(3+\hat{\nu})\right]^2} \text{ and } \frac{\partial \hat{R}_{1,3}}{\partial \alpha_2}=\frac{6(3\hat{\nu}^2+12\hat{\nu}+11)}{\alpha_2\left[(1+\hat{\nu})(2+\hat{\nu})(3+\hat{\nu})\right]^2}.$$

$$\frac{\partial \hat{R}_{2,4}}{\partial \alpha_1}=\frac{-24\hat{\nu}(3\hat{\nu}^2+18\hat{\nu}+26)}{\alpha_1\left[(2+\hat{\nu})(3+\hat{\nu})(4+\hat{\nu})\right]^2} \text{ and } \frac{\partial \hat{R}_{2,4}}{\partial \alpha_2}=\frac{24(3\hat{\nu}^2+18\hat{\nu}+26)}{\alpha_2\left[(2+\hat{\nu})(3+\hat{\nu})(4+\hat{\nu})\right]^2}.$$

Therefore, $\mathrm{AV}(\hat{\mathrm{R}}_{1,3})=\dfrac{36\hat{\nu}^2(3\hat{\nu}^2+12\hat{\nu}+11)^2}{\left[(1+\hat{\nu})(2+\hat{\nu})(3+\hat{\nu})\right]^4}\left(\dfrac{1}{n}+\dfrac{1}{m}\right)$ and

$$\mathrm{AV}(\hat{\mathrm{R}}_{2,4})=\frac{\left\{24\hat{\nu}(3\hat{\nu}^2+18\hat{\nu}+26)\right\}^2}{\left[(2+\hat{\nu})(3+\hat{\nu})(4+\hat{\nu})\right]^4}\left(\frac{1}{n}+\frac{1}{m}\right)$$

As $n\to\infty, m\to\infty$, $\dfrac{\hat{R}_{s,k}-R_{s,k}}{\mathrm{AV}(\hat{\mathrm{R}}_{s,k})}\xrightarrow{d}\mathrm{N}(0,1)$,

where $\hat{R}_{s,k}\mp 1.96\sqrt{\mathrm{AV}(\hat{\mathrm{R}}_{s,k})}$ is the asymptotic 95% confidence interval (C.I) of system reliability $R_{s,k}$ and asymptotic 95% C.I for $R_{1,3}$ is given by

$$\hat{R}_{1,3}\mp 1.96\frac{6\hat{\nu}(3\hat{\nu}^2+12\hat{\nu}+11)}{\left[(1+\hat{\nu})(2+\hat{\nu})(3+\hat{\nu})\right]^2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)}, \text{ where } \hat{\nu}=\hat{\alpha}_2/\hat{\alpha}_1.$$

The asymptotic 95% confidence interval for $R_{2,4}$ is given by

$$\hat{R}_{2,4} \mp 1.96 \frac{\{24\hat{v}(3\hat{v}^2+18\hat{v}+26)\}}{[(2+\hat{v})(3+\hat{v})(4+\hat{v})]^2} \sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)}, \text{ where } \hat{v}=\hat{\alpha}_2/\hat{\alpha}_1.$$

3. Results and Data Analysis

3.1. Results from simulation study

Suppose 3000 random sample of size 10(5)30 each from stress and strength populations are generated for (α_1,α_2)=(3.0,1.5), (2.5,1.5), (2.0,1.5), (1.5,1.5), (1.5,2.0), (1.5,2.5) and (1.5,3.0) as proposed by of Bhattacharyya and Johnson (1974). The MLE of shape parameter β is estimated by iterative method and using β the shape parameters α_1 and α_2 are estimated from (9). These ML estimators of α_1 and α_2 are then substituted in v to get the multicomponent reliability for (*s, k*) = (1, 3), (2, 4). The average bias and average mean square error (MSE) of the reliability estimates over the 3000 replications are given in Tables 1 and 2. Average confidence length and coverage probability of the simulated 95% confidence intervals of $R_{s,k}$ are given in Tables 3 and 4. The true value of reliability in multicomponent stress-strength with the given combinations of (α_1,α_2) for (*s, k*) = (1, 3) are 0.543, 0.599, 0.668, 0.750, 0.822, 0.869, 0.900 and for (*s, k*) = (2, 4) are 0.390, 0.443, 0.510, 0.600, 0.688, 0.752, 0.800. Thus the true value of reliability in multicomponent stress-strength increases as α_2 increases for a fixed α_1 whereas reliability in multicomponent stress-strength decreases as α_1 increases for a fixed α_2 in both the cases of (*s, k*). Therefore, the true value of reliability is increases as v increases and vice versa. The average bias and average MSE are decreases as sample size increases for both cases of estimation in reliability. Also the bias is negative in all the combinations of the parameters in both situations of (*s, k*). It proofs the consistency property of the MLE of $R_{s,k}$. Whereas, among the parameters the absolute bias and MSE are increases as α_1 increases for a fixed α_2 in both the cases of (*s, k*) and the absolute bias and MSE are decreases as α_2 increases for a fixed α_1 in both the cases of (*s, k*). From these tables, it is clear that the as sample size decreases, the length of C.I is also decreases and coverage probability in all cases is than 0.95 which shows the performance of C.I using the Burr type XII distribution is quite good for various combinations of the parameters. .

Whereas, among the parameters we observed the same phenomenon for average length and average coverage probability that we observed in case of average bias and MSE.

3.2. Data Analysis

In this sub section we analyze two real data sets and demonstrate how the proposed methods can be used in practice. Both datasets were discussed by Zimmer *et al*. (1998) and Lie *et al*. (2010) for the Burr XII reliability analysis. Lie et al. (2010) studied the validity of the model for both data sets and they showed that Burr-XII distribution fits quite well for both the data sets. These data sets are reproduced here for easy reference.

(X): 0.19, 0.78, 0.96, 0.31, 2.78, 3.16, 4.15, 4.67, 4.85, 6.50, 7.35, 8.01, 8.27, 12.06, 31.75, 32.52, 33.91, 36.71 and 72.89.

(Y): 0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1 and 53.0.

We use the iterative procedure to obtain the MLE of β using (11) and MLEs of α_1 and α_2 are obtained by substituting MLE of β in (9). The final estimates for real data sets are $\hat{\alpha}_1$= 0.287835, $\hat{\alpha}_2$= 0.232784 and $\hat{\beta}$=1.799809. Base on estimates of α_1 and α_2 the MLE of $R_{s,k}$ become $\hat{R}_{1,3}$=0.689914 and $\hat{R}_{2,4}$= 0.533462. The 95% confidence intervals for $R_{1,3}$ become (0.505449, 0.874379) and for $R_{2,4}$ become (0.337642, 0.729282).

4. Conclusions

In this paper, we have studied the multicomponent stress-strength reliability for two parameters Burr-XII distribution when both of stress, strength variates follows the same population. Also, we have estimated asymptotic confidence interval for multicomponent stress-strength reliability. The simulation results indicates that the average bias and average MSE are decreases as sample size increases for both methods of estimation in reliability. Among the parameters the absolute bias and MSE are increases (decreases) as α_1 increases (α_2 increases) in both the cases of (*s, k*). From the length of C.I and trend in sample show the performance of the proposed procedure using the Burr-XII distribution is quite good. Further, the coverage probability is quite close to given value in all sets of parameters. The real example shows that the proposed procedure can be used in real world to estimate the reliability of multicomponent stress-strength using the Burr-XII distribution.

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Table 1. Average bias of the simulated estimates of $R_{s,k}$

(s,k)	(n,m)	(α_1,α_2)						
		(3.0,1.5)	(2.5,1.5)	(2.0,1.5)	(1.5,1.5)	(1.5,2.0)	(1.5,2.5)	(1.5,3.0)
(1,3)	(10,10)	-0.01920	-0.01870	-0.01801	-0.01718	-0.01600	-0.01437	-0.01278
	(15,15)	-0.00851	-0.00802	-0.00761	-0.00750	-0.00739	-0.00692	-0.00636
	(20,20)	-0.01009	-0.00977	-0.00934	-0.00887	-0.00818	-0.00729	-0.00644
	(25,25)	-0.00688	-0.00648	-0.00604	-0.00567	-0.00523	-0.00468	-0.00416
	(30,30)	-0.00623	-0.00573	-0.00519	-0.00469	-0.00418	-0.00366	-0.00321
(2,4)	(10,10)	-0.00684	-0.00613	-0.00557	-0.00578	-0.00650	-0.00692	-0.00720
	(15,15)	-0.00100	-0.00015	0.00056	0.00050	-0.00029	-0.00102	-0.00167
	(20,20)	-0.00428	-0.00391	-0.00357	-0.00359	-0.00384	-0.00391	-0.00393
	(25,25)	-0.00244	-0.00195	-0.00148	-0.00134	-0.00152	-0.00167	-0.00181
	(30,30)	-0.00241	-0.00185	-0.00129	-0.00095	-0.00089	-0.00089	-0.00096

Table 2. Average MSE of the simulated estimates of $R_{s,k}$

(s,k)	(n,m)	(α_1,α_2)						
		(3.0,1.5)	(2.5,1.5)	(2.0,1.5)	(1.5,1.5)	(1.5,2.0)	(1.5,2.5)	(1.5,3.0)
(1,3)	(10,10)	0.02255	0.02208	0.02040	0.01680	0.01264	0.00932	0.00688
	(15,15)	0.01426	0.01394	0.01284	0.01050	0.00775	0.00562	0.00408
	(20,20)	0.01052	0.01030	0.00952	0.00783	0.00579	0.00420	0.00305

	(25,25)	0.00812	0.00794	0.00730	0.00594	0.00433	0.00309	0.00222
	(30,30)	0.00694	0.00679	0.00625	0.00509	0.00370	0.00264	0.00189
(2,4)	(10,10)	0.01813	0.01973	0.02091	0.02079	0.01905	0.01646	0.01385
	(15,15)	0.01163	0.01272	0.01359	0.01361	0.01244	0.01070	0.00894
	(20,20)	0.00831	0.00911	0.00978	0.00991	0.00915	0.00795	0.00670
	(25,25)	0.00652	0.00716	0.00769	0.00776	0.00711	0.00612	0.00511
	(30,30)	0.00556	0.00612	0.00658	0.00667	0.00612	0.00527	0.00441

Table 3. Average confidence length of the simulated 95% confidence intervals of $R_{s,k}$

(s,k)	(n,m)	(α_1,α_2)						
		(3.0,1.5)	(2.5,1.5)	(2.0,1.5)	(1.5,1.5)	(1.5,2.0)	(1.5,2.5)	(1.5,3.0)
(1,3)	(10,10)	0.49488	0.49693	0.48356	0.44167	0.37697	0.31785	0.26800
	(15,15)	0.41815	0.41985	0.40833	0.37224	0.31686	0.26624	0.22360
	(20,20)	0.36808	0.36877	0.35742	0.32387	0.27358	0.22820	0.19036
	(25,25)	0.33213	0.33279	0.32257	0.29219	0.24655	0.20534	0.17100
	(30,30)	0.30428	0.30508	0.29597	0.26842	0.22682	0.18913	0.15764
(2,4)	(10,10)	0.44993	0.47705	0.49861	0.50273	0.47623	0.43648	0.39421
	(15,15)	0.37719	0.40067	0.41989	0.42475	0.40389	0.37090	0.33517
	(20,20)	0.33238	0.35293	0.36954	0.37302	0.35351	0.32339	0.29105
	(25,25)	0.29916	0.31786	0.33312	0.33668	0.31942	0.29232	0.26309
	(30,30)	0.27335	0.29062	0.30489	0.30865	0.29342	0.26900	0.24245

Table 4. Average coverage probability of the simulated 95% confidence intervals of $R_{s,k}$

(s,k)	(n,m)	(α_1,α_2)						
		(3.0,1.5)	(2.5,1.5)	(2.0,1.5)	(1.5,1.5)	(1.5,2.0)	(1.5,2.5)	(1.5,3.0)
(1,3)	(10,10)	0.89300	0.90167	0.91000	0.92400	0.93200	0.93100	0.92833
	(15,15)	0.92167	0.92833	0.93367	0.94033	0.94133	0.94133	0.94100
	(20,20)	0.93100	0.93333	0.94033	0.94300	0.94600	0.94200	0.94333
	(25,25)	0.93233	0.93800	0.94367	0.94700	0.94800	0.94600	0.94500
	(30,30)	0.92500	0.93167	0.93333	0.93800	0.94033	0.93967	0.93867
(2,4)	(10,10)	0.90733	0.90567	0.90767	0.91067	0.91433	0.92200	0.92767
	(15,15)	0.92200	0.92600	0.92967	0.93467	0.93400	0.93333	0.93267
	(20,20)	0.93300	0.93633	0.93667	0.93967	0.94033	0.93900	0.93933
	(25,25)	0.93500	0.93667	0.94167	0.94400	0.94233	0.93933	0.93967
	(30,30)	0.92800	0.92767	0.93233	0.93300	0.93500	0.93367	0.93267