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Muhammad Aslam,[1] Debasis Kundu,[2] Chi- Hyuck Jun,[3] and Munir Ahmad[4]

Time Truncated Group Acceptance Sampling Plans for Generalized Exponential Distribution

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ABSTRACT: Two group acceptance sampling plans are considered for a two-parameter generalized exponential distribution when the life-test is truncated at a pre-specified time. It is assumed that the shape parameter of the generalized exponential distribution is known. The design parameters such as the number of groups and the acceptance number are obtained by satisfying the producer's and consumer's risks at the specified quality levels in terms of medians, under the assumption that the termination time and the number of items in each group are pre-fixed. Examples are provided for illustrative purposes.

KEYWORDS: group acceptance sampling plan, operating characteristic, consumer and producer's risks, truncated life-test

Introduction

Due to the highly competitive global market, quality of products definitely plays one of the most important roles for any industry today. For this reason, Statistical Quality Control plays a significant role for the success or failure of an industry. Acceptance sampling plans are an essential tool in Statistical Quality Control. It is very clear that in many situations it may not be possible to perform hundred percent inspection. On the other hand, if nothing is tested, desired quality cannot be assured. Acceptance sampling plan is a "middle path" between hundred percent inspection and no inspection at all.

The acceptance sampling plan requires a decision of accepting or rejecting a lot of products based on a random sample collected from the lot. An acceptance sampling plan is the plan that specifies the minimum sample size required to be used along with the acceptance and non-acceptance criteria for the lot. So the acceptance sampling plan specifies the number of units, say n, to be used for testing, and the acceptance number c, such that if there are at most c failures out of n items then the lot is accepted, otherwise it is rejected. For a given acceptance sampling plan, the consumer's and producer's risks are the probabilities that a bad lot is accepted and a good lot is rejected, respectively. Usually, with every acceptance sampling plan, the associated consumer's and producer's risks are also provided. Extensive work has been done on acceptance sampling plans since their inception. Several text books and papers are available which provide different acceptance sampling plans for different distribution functions, see, for example, Stephens [1], Squeglia [2], Tsai and Wu [3], Kantam, Rosaiah, and Rao [4], Aslam [5], and the references cited therein.

AQ: #1

Sometimes, to reduce testing time, group acceptance sampling plans have been used. In this case the total number of items (n) to be tested is divided into equal-sized groups according to the number of available experimental testers, see, for example, Pascual and Meeker [6] or Vleck, Hendricks, and Zaretsky [7]. There are r items in each group, and there are a total of "g" groups, so that $n=rg$. The items in each group are tested independently and under identical environmental conditions. Moreover, all the testers run simultaneously. The experiment is stopped at a pre-specified time T. If "c" is the acceptance number for this experiment, then a lot is accepted if the recorded number of failures in each group is less than c during the experimental time T.

The standard approach to handle this problem is to assume a parametric model for the lifetime distribution and then derive the minimum sample size n needed to ensure certain mean or median life of the items under investigation. It is further assumed that the experimental time and the number of items in each group are prefixed in advance. Since $n=rg$, determining n is equivalent to determining g. Moreover, for any group acceptance sampling plan, in addition to g, c, and T, there will be another parameter, say θ_m, where θ_m is the specified mean or median life, which acts as a quality parameter for the lifetime distribution under consideration. Since the generalized exponential distribution is a skewed distribution, as suggested by Gupta [8], we have used the median as the quality parameter. The decision upon acceptance of lot can be related to a hypothesis testing. The null hypothesis is "lot median is greater than or equal to a specified quantity" and the alternative hypothesis is "lot median is smaller than a specified quantity." On the basis of the observed number of failures in a sample, if the null hypothesis has failed to reject, then the lot is accepted as a good lot, which will ensure a certain quality of the products

Recently group acceptance sampling plans have received some attention. Variable sampling plans for the Weibull distribution have been considered by Jun, Balamurali, and Lee [9]. Recently Aslam and Jun [10] provided extensive tables for time truncated group acceptance sampling plans for the Weibull-distributed lifetime distribution. Although the Weibull distribution has been used very extensively in modeling lifetime data, in the last few years it is observed that the generalized exponential distribution can be used as an alternative. It has several desirable properties and in many cases it may provide a better fit than the Weibull distribution.

The main aim of this paper is to develop time truncated group

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[1]Dept. of Statistics, Forman Christian College University, Lahore 54000, Pakistan, e-mail: aslam_ravian@hotmail.com
[2]Dept. of Mathematics and Statistics, Indian Institute of Technology Kanpur, Kanpur 208016, India, e-mail: kundu@iitk.ac.in
[3]Dept. of Industrial and Management Engineering, POSTECH, Pohang 790-784, Korea, e-mail: chjun@postech.ac.kr
[4]Dept. of Statistics, National College of Business Administration and Economics, Lahore 54000, Pakistan, e-mail: drmunir@ncbae.edu.pk

acceptance sampling plans for the generalized exponential distribution when the corresponding shape parameter is known, and to compare the results of two possible plans with each other. In this manuscript we present a methodology to find the minimum number of groups and the acceptance number required to ensure a specified median life of the items under study. It is further assumed that the life testing will be stopped at a pre-determined time T, if more than c failures does not occur in any group. Otherwise, the experiment is stopped as soon as a pre-determined time T is reached or the number of observed failures in any group is more than c, whichever is earlier.

The rest of the paper is organized as follows. In the section Generalized Exponential Distribution, we describe briefly the generalized exponential distribution. The group acceptance sampling plans are provided in the section Group Acceptance Sampling Plans. An improved group acceptance sampling plan is provided in the section Improved Group Acceptance Sampling Plans. In the last section, concluding remarks are given.

Generalized Exponential Distribution

In this section we provide a brief review of the two-parameter generalized exponential distribution. The two-parameter generalized exponential distribution was originally introduced by Gupta and Kundu [11] as a possible alternative to the well known Weibull and Gamma distributions. It is a special case of a more general exponential Weibull distribution proposed by Mudholkar and Srivastava [12].

The two-parameter generalized exponential distribution has the following cumulative distribution function (CDF)

$$F(x;\alpha,\lambda) = (1 - e^{-(x/\lambda)})^{\alpha}; \quad \text{for} \quad x > 0 \tag{1}$$

Here $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters, respectively. It may be noted that when $\alpha = 1$, it coincides with the exponential distribution. Therefore, as the name suggests, this is a generalization of the exponential distribution, as are the Weibull and gamma distributions, but in a different way. It is observed that the PDF of a generalized exponential distribution is a decreasing function or an unimodal function if $0 < \alpha \leq 1$ or $\alpha > 1$, respectively. The hazard function of the generalized exponential distribution is a decreasing function if $\alpha < 1$ and an increasing function for $\alpha > 1$. In this respect, it is very similar to those of the Weibull and gamma distributions. It is also observed in different studies that the generalized exponential distribution might fit better than the Weibull or gamma distribution in some cases. In different studies it has been shown that for certain ranges of the parameter values, it is extremely difficult to distinguish between generalized exponential and Weibull, gamma, log-normal or generalized Rayleigh distributions. AQ: #2

Note that the generation of random deviates from the generalized exponential distribution is quite straightforward, and a very simple graphical technique can be used to assess the goodness of fit of the generalized exponential distribution. From Eq 1, it can be easily observed that

$$[F(x;\alpha,\lambda)]^{1/\alpha} = 1 - e^{-x/\lambda} \Rightarrow -\ln[1 - (F(x;\alpha,\lambda))^{1/\alpha}] = \frac{x}{\lambda} \tag{2}$$

Therefore, if the value of the shape parameter is known, the plot of $g(\hat{F}(x))$ against x is linear, where

$$g(y) = -\ln(1 - y^{1/\alpha}) \tag{3}$$

and $\hat{F}(x)$ is the empirical distribution function. If α is unknown, which is usually the case, plot $g(\hat{F}(x))$ for different values of α, and the generalized exponential distribution can be used if the plot is linear for some value of α. This method can be very useful for data analysis purposes. The readers are referred to the recent review article by Gupta and Kundu [13] for a current account on the generalized exponential distribution.

From now on a generalized exponential distribution with shape and scale parameters α and λ will be denoted by $\text{GE}(\alpha,\lambda)$. If $X \approx \text{GE}(\alpha,\lambda)$, then the mean and variance of X can be expressed as

$$E(X) = \lambda[\psi(\alpha+1) - \psi(1)], \quad V(X) = \lambda^2[\psi'(\alpha+1) - \psi'(1)] \tag{4}$$

Here $\psi(.)$ and $\psi'(.)$ are the digamma and polygamma functions respectively. Both the mean and the variance are increasing functions of λ.

The pth percentile of $\text{GE}(\alpha,\lambda)$, say $\theta_p = F_{\text{GE}}^{-1}(p;\alpha,\lambda)$ is given by

$$\theta_p = -\lambda \ln(1 - p^{1/\alpha}) \tag{5}$$

Therefore, the median of $\text{GE}(\alpha,\lambda)$ becomes

$$\theta_m = -\lambda \ln\left[1 - \left(\frac{1}{2}\right)^{1/\alpha}\right] \tag{6}$$

It is important to note that a generalized exponential distribution is a skewed one, therefore it is preferable to use the median life to develop acceptance plans rather than the mean life. Hence, unless otherwise stated, we treat θ_m as the quality parameter. From Eq 6 it is clear that for fixed $\alpha = \alpha_0$, $\theta_m \geq \theta_m^0 \Leftrightarrow \lambda \geq \lambda_m^0$, where

$$\lambda_m^0 = \frac{\theta_m^0}{-\ln\left[1 - \left(\frac{1}{2}\right)^{1/\alpha_0}\right]} \tag{7}$$

Note that λ_m^0 also depends on α_0; for brevity we do not make it explicit. Now we develop the acceptance sampling plans for the generalized exponential distribution to ensure that the median lifetime under study exceeds a pre-determined quality provided by the consumer, say θ_m^0, equivalently that λ exceeds λ_m^0

Group Acceptance Sampling Plans

In this section we provide group acceptance sampling plans under the assumption that the lifetime distribution of items follows a two-parameter generalized exponential distribution with the CDF (1) and with known shape parameter α. In a group acceptance sampling plan, the test terminates at a pre-specified time T, and the number of failures in each group is noted. On the basis of the number of failures, a lower confidence limit on the median life is formed. Equivalently, on the basis of the number of failures, it is then desired to establish a specified median life to ensure certain quality of the product, with a given probability of at least $-\beta$. AQ: #3

The acceptance or rejection of the lot is equivalent to the acceptance or rejection of the hypothesis on the quality parameter namely $H_0: \theta_m \geq \theta_m^0$. In this proposed group acceptance sampling plan, the decision to fail to reject H_0 takes place if and only if the number of failures in each group at the end of the time point T does

TABLE 1—*Group sampling plans for generalized exponential distribution when* $\alpha=2$.

		$r=5$						$r=10$					
		$a=0.5$			$a=1.0$			$a=0.5$			$a=1.0$		
β	$\theta_m/\theta_m^0=r_2$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	170	3	0.9810	44	4	0.9820	10	3	0.9650	3	5	0.9753
	4	5	1	0.9805	2	2	0.9939	2	1	0.9672	1	2	0.9718
	6	5	1	0.9956	1	1	0.9891	2	1	0.9924	1	1	0.9560
	8	5	1	0.9985	1	1	0.9961	2	1	0.9974	1	1	0.9834
	10	2	0	0.9651	1	1	0.9983	1	0	0.9651	1	1	0.9925
0.10	2	281	3	0.9689	73	4	0.9703	57	4	0.9824	5	5	0.9592
	4	7	1	0.9728	4	2	0.9878	3	1	0.9511	1	2	0.9718
	6	7	1	0.9939	2	1	0.9782	3	1	0.9886	1	1	0.9560
	8	7	1	0.9979	2	1	0.9921	3	1	0.9961	1	1	0.9834
	10	2	0	0.9651	2	1	0.9965	1	0	0.9651	1	1	0.9925
0.05	2	366	3	0.9596	95	4	0.9615	73	4	0.9775	16	6	0.9810
	4	9	1	0.9651	5	2	0.9848	3	1	0.9511	2	3	0.9929
	6	9	1	0.9921	2	1	0.9782	3	1	0.9886	1	1	0.9560
	8	9	1	0.9974	2	1	0.9921	3	1	0.9961	1	1	0.9834
	10	9	1	0.9989	2	1	0.9965	3	1	0.9983	1	1	0.9925
0.01	2	-	-	-	-	-	-	112	4	0.9657	25	6	0.9705
	4	68	2	0.9945	7	2	0.9787	11	2	0.9902	3	3	0.9894
	6	14	1	0.9878	3	1	0.9675	5	1	0.9810	2	2	0.9920
	8	14	1	0.9959	3	1	0.9882	5	1	0.9935	2	2	0.9982
	10	14	1	0.9984	3	1	0.9948	5	1	0.9972	2	2	0.9995

Note: The cells with hyphens (-) indicate that g and c are found to be very large.

not exceed c, the acceptance number. To test the hypothesis $H_0: \theta_m \geq \theta_m^0$, the group acceptance sampling plan takes the following form:

- **Step 1:** Select the number of groups g and allocate predefined r (group size) items to each group, so that the sample size of the lot will be $n=rg$.
- **Step 2:** Select the acceptance number c and the experiment time T.
- **Step 3:** Perform the experiment for the g groups simultaneously and record the number of failures for each group.
- **Step 4:** Accept the lot if there are not more than c failures in each and every group, otherwise reject the lot.

The ordinary single acceptance sampling plan is a special case of the proposed plan. The proposed plan reduces to the ordinary acceptance sampling plan if the number of testers (r) becomes 1.

The consumers demands that the lot acceptance probability should be smaller than the specified consumer's risk β if its quality is not good enough and the producer wants lot rejection probability to be smaller than the specified producer's risk γ if its quality is good. Note also that we can express the quality level of a product in terms of the ratio of its median lifetime to the specified median lifetime θ_m/θ_m^0. The proposed approach of finding the design parameters is to satisfy the following two inequalities for the operating characteristic function L simultaneously:

$$L(p|\theta_m/\theta_m^0 = r_1) \leq \beta \tag{8}$$

$$L(p|\theta_m/\theta_m^0 = r_2) \geq 1-\gamma \tag{9}$$

where:

r_1=ratio at the consumer's risk (it is assumed to be 1 here), and

r_2=ratio at the producer's risk.

It is convenient to write the experiment time as the multiple θ_m^0 such that $T=a\theta_m^0$ for a constant a. So, the probability of failure of an item before time T is given by

$$p = [1-\exp(-a\ \ln(1-(1/2)^{1/\alpha})/\theta_m/\theta_m^0)]^\alpha \tag{10}$$

Let p_1 be the failure probability corresponding to the consumer's risk and p_2 be the failure probability corresponding to the producer's risk. For given α, β, γ, r, a, and r_2, we need to find g and c, that satisfy the following two inequalities simultaneously:

$$\left[\sum_{i=0}^{c}\binom{r}{i}p_1^i(1-p_1)^{r-i}\right]^g \leq \beta \tag{11}$$

$$\left[\sum_{i=0}^{c}\binom{r}{i}p_2^i(1-p_2)^{r-i}\right]^g \geq 1-\gamma \tag{12}$$

The design parameters for the proposed plans for different values of the shape parameters are constructed. Tables 1 and 2 are constructed for different values of number of testers and different experiment times when α=2 and 3, respectively. The producer's risk (γ) is chosen as 5 %. The probability of acceptance is also calculated for these values.

From Tables 1 and 2, we can see that as the median ratio is increased, the design parameters are decreased. We observed that as the shape value increases from 2 to 3, the plan generally requires a smaller number of groups.

Example 1

Suppose that a manufacturer wants to use the proposed group sampling plan for the inspection of incoming lots of light bulbs. Multi-item tests with group size of five will be used. Suppose also that the

TABLE 2—*Group sampling plans for generalized exponential distribution when* $\alpha=3$.

		$r=5$						$r=10$					
		$a=0.5$			$a=1.0$			$a=0.5$			$a=1.0$		
β	$\theta_m/\theta_m^0=r_2$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	42	2	0.9835	7	3	0.9789	6	2	0.9753	2	4	0.9723
	4	7	1	0.9977	1	1	0.9888	2	1	0.9971	1	1	0.9551
	6	2	0	0.9814	1	1	0.9985	1	0	0.9814	1	1	0.9935
	8	2	0	0.9917	1	0	0.9716	1	0	0.9917	1	1	0.9986
	10	2	0	0.9956	1	0	0.9845	1	0	0.9956	1	0	0.9693
0.10	2	69	2	0.9731	12	3	0.9641	10	2	0.9592	3	4	0.9588
	4	12	1	0.9961	2	1	0.9777	4	1	0.9943	1	1	0.9551
	6	3	0	0.9723	2	1	0.9970	2	0	0.9632	1	1	0.9935
	8	3	0	0.9876	1	0	0.9716	2	0	0.9835	1	1	0.9986
	10	3	0	0.9935	1	0	0.9845	2	0	0.9913	1	1	0.9996
0.05	2	89	2	0.9654	15	3	0.9554	45	3	0.9885	7	5	0.9851
	4	15	1	0.9951	2	1	0.9777	5	1	0.9928	1	1	0.9551
	6	4	0	0.9632	2	1	0.9970	2	0	0.9632	1	1	0.9935
	8	4	0	0.9835	1	0	0.9716	2	0	0.9835	1	1	0.9986
	10	4	0	0.9913	1	0	0.9845	2	0	0.9913	1	0	0.9693
0.01	2	1512	3	0.9894	145	4	0.9837	69	3	0.9825	10	5	0.9788
	4	23	1	0.9925	3	1	0.9668	7	1	0.9900	2	2	0.9917
	6	23	1	0.9992	3	1	0.9955	7	1	0.9989	2	1	0.9871
	8	6	0	0.9754	3	1	0.9990	3	0	0.9754	2	1	0.9971
	10	6	0	0.9870	2	0	0.9693	3	0	0.9870	1	0	0.9693

life of this product follows a generalized exponential distribution with shape parameter 3. It is known that the specified median life of interest is 1000 h. The test time was specified as 500 h. It is required that the consumer's risk is 25 % if the true median life is 1000 and the producer's risk is 5 % if the true median is 2000. As $\alpha=3$, $r=5$, $\beta=0.25$, $a=0.5$, and $r_2=2$, it is found from Table 2 that $(g,c)=(42,2)$. So, a sample of 210 items is drawn and allocated to 42 groups with 5 items. If no more than two failures occur in each of the 42 groups, then the lot will be accepted.

Example 2

Suppose that an experimenter wants to adopt the group sampling plan with $r=10$ to decide about the acceptance or the rejection of the submitted lot of products. The specified median life of the product is $\theta_m^0=1000$ and the test duration is 1000 h. The producer's risk is $\gamma=0.05$ at $\theta_m/\theta_m^0=2$ and the consumer's risk is $\beta=0.10$. Now we consider data set given by Wood [14] which is the failure time in hours of a software, which represents the time from the starting of the execution of the software until a software failure is experienced. We have the following values: 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, and 5218. Aslam et al. [15] showed that the generalized exponential distribution provides a good fit to this software data. According to them, the maximum likelihood estimators of α and λ are 2.65 and 0.6547, respectively. Let us assume that $\hat{\alpha}=3$. As $r=10$ and $a=1$, the design parameters can be found as $g=3$ and $c=4$ from Table 2. This sampling plan is operated as: Take a sample of size 30 and allocate 10 items to 3 groups. Accept the lot if no more than four failures are recorded from each of three groups before 1000 h, but reject it otherwise.

Improved Group Acceptance Sampling Plans

In the above proposed group acceptance sampling plan, the lot of products is rejected if the number of failures in any group is greater than the acceptance number. In this section, we made a new group acceptance sampling plan by relaxing this condition to allow possibly more failures in some groups when accepting a lot. The new plan is as follows:

- **Step 1:** Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be $n=gr$.
- **Step 2:** Select the acceptance number c $(c\leq r)$ for a group and the experiment time T.
- **Step 3:** Perform the experiment for the g groups simultaneously and record the number of failures for each group.
- **Step 4:** Accept the lot if the number of failures is smaller than or equal to c from at least k groups $(k\leq g)$. Otherwise, truncate the experiment and reject the lot.

This plan is characterized by three parameters g, k, and c. Here we introduce an additional parameter k such that $k\leq g$ as compared with the original plan in the section Group Acceptance Sampling Plans. It should be noted that if k=g, the new plan reduces to the original group plan mentioned in the said section. We call it the improved group acceptance sampling plan because it turns out to have better performance in terms of the sample size as compared with the original plan.

The lot acceptance probability for the improved group acceptance sampling plan is given by

$$L(p)=\sum_{j=k}^{g}\binom{g}{j}Q^j(1-Q)^{g-j} \tag{13}$$

where:

Q=probability that c or fewer failures are observed in a group

TABLE 3—*Improved group sampling plans for generalized exponential distribution when* α=2 (r=5).

		a=0.5				a=1.0			
β	$\theta_m/\theta_m^0=r_2$	g	k	c	$L(p_0)$	g	k	c	$L(p_0)$
0.25	2	40	39	2	0.9932	5	4	2	0.9616
	4	5	3	2	0.9725	1	1	1	0.9576
	6	3	2	0	0.9937	↑	↑	↑	0.9891
	8	↑	↑	↑	0.9979	↑	↑	↑	0.9961
0.10	2	57	58	2	0.9861	20	19	3	0.9885
	4	4	3	0	0.9504	3	2	1	0.9948
	6	↑	↑	↑	0.9879	2	1	0	0.9744
	8	↑	↑	↑	0.9958	↑	↑	↑	0.9905
0.05	2	70	69	2	0.9802	24	23	3	0.9837
	4	9	9	1	0.9651	4	3	1	0.9898
	6	5	4	0	0.9804	2	2	1	0.9782
	8	↑	↑	↑	0.9931	↑	↑	↑	0.9921
0.01	2	98	97	2	0.9632	33	32	3	0.9702
	4	21	20	1	0.9969	5	4	1	0.9835
	6	7	6	0	0.9613	3	3	1	0.9675
	8	↑	↑	↑	0.9860	3	2	0	0.9735

Note: The upward arrow (↑) indicates that the same value of the parameter as in the upward cell applies to the corresponding cell.

$$Q = \left[\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i} \right] \tag{14}$$

Here, p is the probability that an item fails by time T, which is given by Eq 10. The purpose is to find the three parameters g, k, and c such that the following inequalities are satisfied:

$$\sum_{j=k}^{g} \binom{g}{j} Q_0^j (1-Q_0)^{g-j} \geq 1-\gamma \tag{15}$$

$$\sum_{j=k}^{g} \binom{g}{j} Q_1^j (1-Q_1)^{g-j} \leq \beta \tag{16}$$

Tables 3–6 provide design parameters of the improved plan according to several values for shape parameters, number of testers, median quality level, and producer's and consumer's risks. From these tables we can see that as the quality level of a product increases the number of groups and acceptance number decrease for all the parameters. As the shape parameter is changing from 2 to 3, decreasing numbers of groups are required for testing.

Example 3

Suppose that a manufacturer wants to use the improved group sampling plan for the inspection of incoming lots of bulbs. Multi-item testers with group size of 5 will be used. Suppose also that the life of this product follows a generalized exponential distribution with shape parameter of 3. It is known that the specified median life of interest is 1000 h. The test time was specified as 500 h. It is required that the consumer's risk is 25 % if the true median life is 1000 and the producer's risk is 5 % if the true median is 2000. As γ=0.05,

TABLE 4—*Improved group sampling plans for generalized exponential distribution when* α=2 (r=10).

		a=0.5				a=1.0			
β	$\theta_m/\theta_m^0=r_2$	g	k	c	$L(p_0)$	g	k	c	$L(p_0)$
0.25	2	7	6	2	0.9848	3	3	5	0.9753
	4	2	1	0	0.9858	1	1	2	0.9718
	6	↑	↑	↑	0.9918	1	1	1	0.9560
	8	↑	↑	↑	0.9972	↑	↑	↑	0.9834
0.10	2	10	9	2	0.9692	5	4	4	0.9851
	4	3	3	1	0.9511	1	1	2	0.9718
	6	3	2	0	0.9768	1	1	1	0.9560
	8	↑	↑	↑	0.9918	↑	↑	↑	0.9834
0.05	2	12	11	2	0.9565	6	5	4	0.9782
	4	3	3	1	0.9511	2	1	1	0.9771
	6	3	2	0	0.9768	1	1	1	0.9560
	8	↑	↑	↑	0.9918	↑	↑	↑	0.9834
0.01	2	45	44	3	0.9887	8	7	4	0.9614
	4	7	6	1	0.9946	3	2	2	0.9977
	6	4	3	0	0.9564	2	2	2	0.9920
	8	↑	↑	↑	0.9842	2	1	0	0.9658

Note: The upward arrow (↑) indicates that the same value of the parameter as in the upward cell applies to the corresponding cell.

TABLE 5—*Improved group sampling plans for generalized exponential distribution when* $\alpha=3$ $(r=5)$.

		$a=0.5$				$a=1.0$			
β	$\theta_m/\theta_m^0=r_2$	g	k	c	$L(p_0)$	g	k	c	$L(p_0)$
0.25	2	14	13	1	0.9896	5	4	2	0.9897
	4	4	3	0	0.9953	1	1	1	0.9888
	6	↑	↑	↑	0.9995	↑	↑	↑	0.9985
	8	↑	↑	↑	0.9999	1	1	0	0.9716
0.10	2	20	19	1	0.9791	7	6	2	0.9793
	4	5	4	0	0.9924	2	1	0	0.9738
	6	↑	↑	↑	0.9991	↑	↑	↑	0.9964
	8	↑	↑	↑	0.9998	1	1	0	0.9716
0.05	2	24	23	1	0.9706	8	7	2	0.9730
	4	6	5	0	0.9888	2	2	1	0.9777
	6	↑	↑	↑	0.9987	↑	↑	↑	0.9970
	8	↑	↑	↑	0.9997	1	1	0	0.9992
0.01	2	198	197	2	0.9971	11	10	2	0.9504
	4	9	8	0	0.9746	3	3	1	0.9668
	6	↑	↑	↑	0.9970	3	2	0	0.9895
	8	↑	↑	↑	0.9994	↑	↑	↑	0.9976

Note: The upward arrow (↑) indicates that the same value of the parameter as in the upward cell applies to the corresponding cell.

$\beta=0.25$, $a=0.5$, and $r_2=2$, it is found from Table 5 that $(g,k,c)=(1,13,14)$. So, a sample of 30 items is drawn and allocated to three groups. If there is no more than one failure from at least 13 groups out of 14, then the lot will be accepted.

We may compare the two plans considered in the sections Group Acceptance Sampling Plans and Improved Group Acceptance Sampling Plans in terms of the sample size required to accept or to reject a lot. Table 7 compares the sample size required for the original plan versus the improved plan according to various parameter values. We can see from Table 7 that the sample size is in the improved group acceptance sampling plan is quite smaller than the original plan particularly as the quality ratio is lower. The savings in the sample size seem to be larger as the shape parameter is smaller.

Example 4

Suppose that an experimenter wants to adopt the improved group sampling plan with $r=10$ to decide about the acceptance or the rejection of the submitted lot of products. The specified median life of the product is $\theta_m^0=1000$ and the test duration is 1000 h. The producer's risk is $\gamma=0.05$, $\beta=0.10$, and $\theta_m/\theta_m^0=2$. For the same data given in example 2, let us assume that $\hat{\alpha}=3$. As $r=10$, $a=1$, and $\hat{\alpha}=3$, the design parameters can be found from Table 6 that $(g,k,c)=(3,2,3)$. This sampling plan is operated as: Take a sample size of 30 and allocate 10 items to three groups. Accept the lot if no more than three failures are recorded from at least three out of four groups before 1000 h, but reject it, otherwise. Although this improved plan requires the same sample size as the original plan for this example, the acceptance probability becomes larger.

Conclusions

We proposed two group acceptance sampling plans for truncated life tests when the lifetime of a product follows the generalized exponential distribution. We first considered the original group sam-

TABLE 6—*Improved group sampling plans for generalized exponential distribution when* $\alpha=3$ $(r=10)$.

		$a=0.5$				$a=1.0$			
β	$\theta_m/\theta_m^0=r_2$	g	k	c	$L(p_0)$	g	k	c	$L(p_0)$
0.25	2	5	4	1	0.9816	2	1	2	0.9544
	4	3	2	0	0.9910	1	1	1	0.9551
	6	↑	↑	↑	0.9990	↑	↑	↑	0.9935
	8	↑	↑	↑	0.9998	↑	↑	↑	0.9986
0.10	2	7	6	1	0.9935	3	2	3	0.9880
	4	3	2	0	0.9910	1	1	1	0.9551
	6	↑	↑	↑	0.9990	↑	↑	↑	0.9935
	8	↑	↑	↑	0.9998	↑	↑	↑	0.9986
0.05	2	8	7	1	0.9528	4	3	3	0.9771
	4	4	3	0	0.9826	1	1	1	0.9551
	6	↑	↑	↑	0.9980	↑	↑	↑	0.9935
	8	↑	↑	↑	0.9996	↑	↑	↑	0.9986
0.01	2	29	28	2	0.9935	5	4	3	0.9635
	4	5	4	0	0.9721	2	2	2	0.9917
	6	↑	↑	↑	0.9967	2	1	0	0.9863
	8	↑	↑	↑	0.9993	↑	↑	↑	0.9969

Note: The upward arrow (↑) indicates that the same value of the parameter as in the upward cell applies to the corresponding cell.

TABLE 7—*Comparison of sample sizes between two group sampling plans.*

		$r=5$		$r=10$	
β	$\theta_m/\theta_m^0=r_2$	Improved Plan	Original Plan	Improved Plan	Original Plan
$\alpha=2$, $a=0.5$					
0.25	2	200	850	70	100
	4	15	25	20	20
0.10	2	285	1405	100	540
	4	20	35	30	30
0.05	2	350	1830	120	730
	4	45	45	30	30
0.01	2	490	-	450	1120
	4	105	340	70	110
$\alpha=3$, $a=0.5$					
0.25	2	70	210	50	60
	4	25	35	30	20
0.10	2	100	345	70	100
	4	25	60	30	40
0.05	2	120	445	80	450
	4	30	75	40	50
0.01	2	990	7560	290	690
	4	45	115	50	70

Note: The cell with hyphens (-) indicates that the sample size required is very large.

pling plan, where we reject the lot if more than c failures are recorded in any of the g groups. Then, we proposed an improved group sampling plan by relaxing the condition of the acceptance in the original plan. Under the improved plan the lot is accepted if the number of failures per group is smaller than the specified acceptance number in k out of g groups ($k \le g$). It turns out that the improved group sampling plan reduces the sample size as compared with the original group sampling plan at the same condition. So, the improved plan meets the economic criteria in life testing because cost and time of the experiment are directly attached with the number of items being tested for possible rejection of the submitted product. We advice the industrial practitioner and the experimenter to adopt the improved plan in order to save the cost and time of the experiment to reach the same decision as the original plan. The improved plan can be further studied for many other distributions and for various quality and reliability characteristics as future research.

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References

[1] Stephens, K. S., *The Handbook of Applied Acceptance Sampling Plans, Procedures and Principles*, ASQ Quality Press, Milwaukee, WI, 2001.

[2] Squeglia, N. L., *Zero Acceptance Number Sampling Plans*, 5th ed., ASQ Quality Press, Milwaukee, WI, 2008.

[3] Tsai, T. R. and Wu, S. J., "Acceptance Sampling Based on Truncated Life Test for Generalized Rayleigh Distribution," *J. Appl. Stat.*, Vol. 33, 2006, pp. 595–600.

[4] Kantam, R. R. L., Rosaiah, K., and Srinivas Rao, G., "Acceptance Sampling Plans Based on Life Tests: Log-Logistic Model," *J. Appl. Stat.*, Vol. 28, 2001, pp. 121–128.

[5] Aslam, M., "Double Acceptance Sampling Based on Truncated Life-Tests in Rayleigh Distribution," *Eur. J. Sci. Res.*, Vol. 17, 2007, pp. 605–611.

[6] Pascual, F. G. and Meeker, W. Q., "The Modified Sudden Death Test: Planning Life-Test with Limited Number of Test Positions," *J. Test. Eval.*, Vol. 26, 1998, pp. 434–443.

[7] Vlcek, B. L., Hendricks, R.C., and Zaretsky, E. V., *Monte Carlo Simulations of Sudden Death Bearing Testing*, NASA, Hanover, MD, 2003.

[8] Gupta, S. S., "Life Test Plans for Normal and Log-Normal Distributions," *Technometrics*, Vol. 4, 1962, pp. 151–160.

[9] Jun, C.-H., Balamurali, S., and Lee, S. H., "Variable Sampling Plans for Weibull Distribution Lifetimes Under Sudden Death Testing," *IEEE Trans. Reliab.*, Vol. 55, 2006, pp. 53–58.

[10] Aslam, M. and Jun, C. H., "A Group Acceptance Sampling Plan for Truncated Life Test Having Weibull Distribution," *J. Appl. Stat.*, Vol. 36, 2009, pp. 1021–1027. AQ: #4

[11] Gupta, R. D. and Kundu, D., "Generalized Exponential Distribution," *Australian and New Zealand Journal of Statistics*, Vol. 41, 1999, pp. 173–188.

[12] Mudholkar, G. S. and Srivastava, D. K., "Exponentiated Weibull Family for Analyzing Bathtub Failure Data," *IEEE Trans. Reliab.*, Vol. 42, 1993, pp. 299–302.

[13] Gupta, R. D. and Kundu, D., "Generalized Exponential Distribution: Existing Methods and Recent Developments," *J. Stat. Plan. Infer.*, Vol. 137, 2007, pp. 3537–3547.

[14] Wood, A., "Predicting Software Reliability," *IEEE Trans. Software Eng.*, Vol. 22, 1996, pp. 69–77.

[15] Aslam, M., Kundu, D., and Ahmad, M., "Time Truncated Acceptance Sampling Plan for Generalized Exponential Distribution," *J. Appl. Stat.*, Vol. 37, 2010, pp. 555–566.

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