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2 Time Truncated Group Acceptance Sampling 3 Plans for Generalized Exponential Distribution

4

5 **ABSTRACT:** Two group acceptance sampling plans are considered for a two-parameter generalized exponential distribution when the life-test is
6 truncated at a pre-specified time. It is assumed that the shape parameter of the generalized exponential distribution is known. The design parameters
7 such as the number of groups and the acceptance number are obtained by satisfying the producer's and consumer's risks at the specified quality levels
8 in terms of medians, under the assumption that the termination time and the number of items in each group are pre-fixed. Examples are provided for
9 illustrative purposes.

10 **KEYWORDS:** group acceptance sampling plan, operating characteristic, consumer and producer's risks, truncated life-test

11 Introduction

12 Due to the highly competitive global market, quality of products
13 definitely plays one of the most important roles for any industry
14 today. For this reason, Statistical Quality Control plays a significant
15 role for the success or failure of an industry. Acceptance sampling
16 plans are an essential tool in Statistical Quality Control. It is very
17 clear that in many situations it may not be possible to perform hun-
18 dred percent inspection. On the other hand, if nothing is tested, de-
19 sired quality cannot be assured. Acceptance sampling plan is a
20 "middle path" between hundred percent inspection and no inspec-
21 tion at all.

22 The acceptance sampling plan requires a decision of accepting
23 or rejecting a lot of products based on a random sample collected
24 from the lot. An acceptance sampling plan is the plan that specifies
25 the minimum sample size required to be used along with the accep-
26 tance and non-acceptance criteria for the lot. So the acceptance
27 sampling plan specifies the number of units, say n , to be used for
28 testing, and the acceptance number c , such that if there are at most
29 c failures out of n items then the lot is accepted, otherwise it is
30 rejected. For a given acceptance sampling plan, the consumer's and
31 producer's risks are the probabilities that a bad lot is accepted and a
32 good lot is rejected, respectively. Usually, with every acceptance
33 sampling plan, the associated consumer's and producer's risks are
34 also provided. Extensive work has been done on acceptance sam-
35 pling plans since their inception. Several text books and papers are
36 available which provide different acceptance sampling plans for
37 different distribution functions, see, for example, Stephens [1],
38 Squeglia [2], Tsai and Wu [3], Kantam, Rosaiah, and Rao [4],
39 Aslam [5], and the references cited therein.

AQ: #1 40 Sometimes, to reduce testing time, group acceptance sampling

plans have been used. In this case the total number of items (n) to
be tested is divided into equal-sized groups according to the num-
ber of available experimental testers, see, for example, Pascual and
Meeker [6] or Vleck, Hendricks, and Zaretsky [7]. There are r items
in each group, and there are a total of " g " groups, so that $n = rg$. The
items in each group are tested independently and under identical
environmental conditions. Moreover, all the testers run simulta-
neously. The experiment is stopped at a pre-specified time T . If " c "
is the acceptance number for this experiment, then a lot is accepted
if the recorded number of failures in each group is less than c dur-
ing the experimental time T .

The standard approach to handle this problem is to assume a
parametric model for the lifetime distribution and then derive the
minimum sample size n needed to ensure certain mean or median
life of the items under investigation. It is further assumed that the
experimental time and the number of items in each group are pre-
fixed in advance. Since $n = rg$, determining n is equivalent to deter-
mining g . Moreover, for any group acceptance sampling plan, in
addition to g , c , and T , there will be another parameter, say θ_m ,
where θ_m is the specified mean or median life, which acts as a qual-
ity parameter for the lifetime distribution under consideration.
Since the generalized exponential distribution is a skewed distribu-
tion, as suggested by Gupta [8], we have used the median as the
quality parameter. The decision upon acceptance of lot can be re-
lated to a hypothesis testing. The null hypothesis is "lot median is
greater than or equal to a specified quantity" and the alternative
hypothesis is "lot median is smaller than a specified quantity." On
the basis of the observed number of failures in a sample, if the null
hypothesis has failed to reject, then the lot is accepted as a good lot,
which will ensure a certain quality of the products

Recently group acceptance sampling plans have received some
attention. Variable sampling plans for the Weibull distribution have
been considered by Jun, Balamurali, and Lee [9]. Recently Aslam
and Jun [10] provided extensive tables for time truncated group ac-
ceptance sampling plans for the Weibull-distributed lifetime distri-
bution. Although the Weibull distribution has been used very exten-
sively in modeling lifetime data, in the last few years it is observed
that the generalized exponential distribution can be used as an al-
ternative. It has several desirable properties and in many cases it
may provide a better fit than the Weibull distribution.

The main aim of this paper is to develop time truncated group

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82 acceptance sampling plans for the generalized exponential distribu-
 83 tion when the corresponding shape parameter is known, and to
 84 compare the results of two possible plans with each other. In this
 85 manuscript we present a methodology to find the minimum number
 86 of groups and the acceptance number required to ensure a specified
 87 median life of the items under study. It is further assumed that the
 88 life testing will be stopped at a pre-determined time T , if more than
 89 c failures does not occur in any group. Otherwise, the experiment is
 90 stopped as soon as a pre-determined time T is reached or the num-
 91 ber of observed failures in any group is more than c , whichever is
 92 earlier.

93 The rest of the paper is organized as follows. In the section Gen-
 94 eralized Exponential Distribution, we describe briefly the general-
 95 ized exponential distribution. The group acceptance sampling plans
 96 are provided in the section Group Acceptance Sampling Plans. An
 97 improved group acceptance sampling plan is provided in the sec-
 98 tion Improved Group Acceptance Sampling Plans. In the last sec-
 99 tion, concluding remarks are given.

100 Generalized Exponential Distribution

101 In this section we provide a brief review of the two-parameter gen-
 102 eralized exponential distribution. The two-parameter generalized
 103 exponential distribution was originally introduced by Gupta and
 104 Kundu [11] as a possible alternative to the well known Weibull and
 105 Gamma distributions. It is a special case of a more general expo-
 106 nential Weibull distribution proposed by Mudholkar and Srivastava
 107 [12].

108 The two-parameter generalized exponential distribution has the
 109 following cumulative distribution function (CDF)

$$110 \quad F(x; \alpha, \lambda) = (1 - e^{-(x/\lambda)})^\alpha; \quad \text{for } x > 0 \quad (1)$$

111 Here $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters, respec-
 112 tively. It may be noted that when $\alpha = 1$, it coincides with the expo-
 113 nential distribution. Therefore, as the name suggests, this is a gen-
 114 eralization of the exponential distribution, as are the Weibull and
 115 gamma distributions, but in a different way. It is observed that the
 116 PDF of a generalized exponential distribution is a decreasing func-
 117 tion or an unimodal function if $0 < \alpha \leq 1$ or $\alpha > 1$, respectively. The
 118 hazard function of the generalized exponential distribution is a de-
 119 creasing function if $\alpha < 1$ and an increasing function for $\alpha > 1$. In
 120 this respect, it is very similar to those of the Weibull and gamma
 121 distributions. It is also observed in different studies that the gener-
 122 alized exponential distribution might fit better than the Weibull or
 123 gamma distribution in some cases. In different studies it has been
 124 shown that for certain ranges of the parameter values, it is ex-
 125 tremely difficult to distinguish between generalized exponential
 126 and Weibull, gamma, log-normal or generalized Rayleigh distribu-
 127 tions.

128 Note that the generation of random deviates from the general-
 129 ized exponential distribution is quite straightforward, and a very
 130 simple graphical technique can be used to assess the goodness of fit
 131 of the generalized exponential distribution. From Eq 1, it can be
 132 easily observed that

$$133 \quad [F(x; \alpha, \lambda)]^{1/\alpha} = 1 - e^{-x/\lambda} \Rightarrow -\ln[1 - (F(x; \alpha, \lambda))^{1/\alpha}] = \frac{x}{\lambda} \quad (2)$$

134 Therefore, if the value of the shape parameter is known, the plot of
 135 $g(\hat{F}(x))$ against x is linear, where

$$g(y) = -\ln(1 - y^{1/\alpha}) \quad (3) \quad 136$$

and $\hat{F}(x)$ is the empirical distribution function. If α is unknown,
 which is usually the case, plot $g(\hat{F}(x))$ for different values of α , and
 the generalized exponential distribution can be used if the plot is
 linear for some value of α . This method can be very useful for data
 analysis purposes. The readers are referred to the recent review ar-
 ticle by Gupta and Kundu [13] for a current account on the gener-
 alized exponential distribution.

From now on a generalized exponential distribution with shape
 and scale parameters α and λ will be denoted by $GE(\alpha, \lambda)$. If X
 $\approx GE(\alpha, \lambda)$, then the mean and variance of X can be expressed as

$$E(X) = \lambda[\psi(\alpha + 1) - \psi(1)], \quad V(X) = \lambda^2[\psi'(\alpha + 1) - \psi'(1)] \quad (4) \quad 147$$

Here $\psi(\cdot)$ and $\psi'(\cdot)$ are the digamma and polygamma functions
 respectively. Both the mean and the variance are increasing func-
 tions of λ .

The p th percentile of $GE(\alpha, \lambda)$, say $\theta_p = F_{GE}^{-1}(p; \alpha, \lambda)$ is given by

$$\theta_p = -\lambda \ln(1 - p^{1/\alpha}) \quad (5) \quad 152$$

Therefore, the median of $GE(\alpha, \lambda)$ becomes

$$\theta_m = -\lambda \ln \left[1 - \left(\frac{1}{2} \right)^{1/\alpha} \right] \quad (6) \quad 154$$

It is important to note that a generalized exponential distribution is
 a skewed one, therefore it is preferable to use the median life to
 develop acceptance plans rather than the mean life. Hence, unless
 otherwise stated, we treat θ_m as the quality parameter. From Eq 6 it
 is clear that for fixed $\alpha = \alpha_0$, $\theta_m \geq \theta_m^0 \Leftrightarrow \lambda \geq \lambda_m^0$, where

$$\lambda_m^0 = \frac{\theta_m^0}{-\ln \left[1 - \left(\frac{1}{2} \right)^{1/\alpha_0} \right]} \quad (7) \quad 160$$

Note that λ_m^0 also depends on α_0 ; for brevity we do not make it
 explicit. Now we develop the acceptance sampling plans for the
 generalized exponential distribution to ensure that the median life-
 time under study exceeds a pre-determined quality provided by the
 consumer, say θ_m^0 , equivalently that λ exceeds λ_m^0

Group Acceptance Sampling Plans

In this section we provide group acceptance sampling plans under
 the assumption that the lifetime distribution of items follows a two-
 parameter generalized exponential distribution with the CDF (1)
 and with known shape parameter α . In a group acceptance sam-
 pling plan, the test terminates at a pre-specified time T , and the
 number of failures in each group is noted. On the basis of the num-
 ber of failures, a lower confidence limit on the median life is
 formed. Equivalently, on the basis of the number of failures, it is
 then desired to establish a specified median life to ensure certain
 quality of the product, with a given probability of at least $-\beta$.

The acceptance or rejection of the lot is equivalent to the accep-
 tance or rejection of the hypothesis on the quality parameter
 namely $H_0: \theta_m \geq \theta_m^0$. In this proposed group acceptance sampling
 plan, the decision to fail to reject H_0 takes place if and only if the
 number of failures in each group at the end of the time point T does

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TABLE 1—Group sampling plans for generalized exponential distribution when $\alpha=2$.

β	$\theta_m/\theta_m^0=r_2$	$r=5$						$r=10$					
		$a=0.5$			$a=1.0$			$a=0.5$			$a=1.0$		
		g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	170	3	0.9810	44	4	0.9820	10	3	0.9650	3	5	0.9753
	4	5	1	0.9805	2	2	0.9939	2	1	0.9672	1	2	0.9718
	6	5	1	0.9956	1	1	0.9891	2	1	0.9924	1	1	0.9560
	8	5	1	0.9985	1	1	0.9961	2	1	0.9974	1	1	0.9834
	10	2	0	0.9651	1	1	0.9983	1	0	0.9651	1	1	0.9925
0.10	2	281	3	0.9689	73	4	0.9703	57	4	0.9824	5	5	0.9592
	4	7	1	0.9728	4	2	0.9878	3	1	0.9511	1	2	0.9718
	6	7	1	0.9939	2	1	0.9782	3	1	0.9886	1	1	0.9560
	8	7	1	0.9979	2	1	0.9921	3	1	0.9961	1	1	0.9834
	10	2	0	0.9651	2	1	0.9965	1	0	0.9651	1	1	0.9925
0.05	2	366	3	0.9596	95	4	0.9615	73	4	0.9775	16	6	0.9810
	4	9	1	0.9651	5	2	0.9848	3	1	0.9511	2	3	0.9929
	6	9	1	0.9921	2	1	0.9782	3	1	0.9886	1	1	0.9560
	8	9	1	0.9974	2	1	0.9921	3	1	0.9961	1	1	0.9834
	10	9	1	0.9989	2	1	0.9965	3	1	0.9983	1	1	0.9925
0.01	2	-	-	-	-	-	-	112	4	0.9657	25	6	0.9705
	4	68	2	0.9945	7	2	0.9787	11	2	0.9902	3	3	0.9894
	6	14	1	0.9878	3	1	0.9675	5	1	0.9810	2	2	0.9920
	8	14	1	0.9959	3	1	0.9882	5	1	0.9935	2	2	0.9982
	10	14	1	0.9984	3	1	0.9948	5	1	0.9972	2	2	0.9995

Note: The cells with hyphens (-) indicate that g and c are found to be very large.

182 not exceed c , the acceptance number. To test the hypothesis
 183 $H_0: \theta_m \geq \theta_m^0$, the group acceptance sampling plan takes the follow-
 184 ing form:

- 185 • **Step 1:** Select the number of groups g and allocate pre-
 186 defined r (group size) items to each group, so that the sample
 187 size of the lot will be $n=rg$.
- 188 • **Step 2:** Select the acceptance number c and the experiment
 189 time T .
- 190 • **Step 3:** Perform the experiment for the g groups simulta-
 191 neously and record the number of failures for each group.
- 192 • **Step 4:** Accept the lot if there are not more than c failures in
 193 each and every group, otherwise reject the lot.

194 The ordinary single acceptance sampling plan is a special case
 195 of the proposed plan. The proposed plan reduces to the ordinary
 196 acceptance sampling plan if the number of testers (r) becomes 1.

197 The consumers demands that the lot acceptance probability
 198 should be smaller than the specified consumer's risk β if its quality
 199 is not good enough and the producer wants lot rejection probability
 200 to be smaller than the specified producer's risk γ if its quality is
 201 good. Note also that we can express the quality level of a product in
 202 terms of the ratio of its median lifetime to the specified median
 203 lifetime θ_m/θ_m^0 . The proposed approach of finding the design pa-
 204 rameters is to satisfy the following two inequalities for the operat-
 205 ing characteristic function L simultaneously:

206
$$L(p|\theta_m/\theta_m^0=r_1) \leq \beta \tag{8}$$

207
$$L(p|\theta_m/\theta_m^0=r_2) \geq 1 - \gamma \tag{9}$$

208 where:

- 209 r_1 =ratio at the consumer's risk (it is assumed to be 1 here), and
- 210 r_2 =ratio at the producer's risk.

It is convenient to write the experiment time as the multiple θ_m^0
 such that $T=a\theta_m^0$ for a constant a . So, the probability of failure of an
 item before time T is given by

$$p = [1 - \exp(-a \ln(1 - (1/2)^{1/\alpha}/\theta_m/\theta_m^0))]^\alpha \tag{10}$$

Let p_1 be the failure probability corresponding to the consumer's
 risk and p_2 be the failure probability corresponding to the produc-
 er's risk. For given $\alpha, \beta, \gamma, r, a$, and r_2 , we need to find g and c , that
 satisfy the following two inequalities simultaneously:

$$\left[\sum_{i=0}^c \binom{r}{i} p_1^i (1-p_1)^{r-i} \right]^g \leq \beta \tag{11}$$

$$\left[\sum_{i=0}^c \binom{r}{i} p_2^i (1-p_2)^{r-i} \right]^g \geq 1 - \gamma \tag{12}$$

The design parameters for the proposed plans for different val-
 ues of the shape parameters are constructed. Tables 1 and 2 are con-
 structed for different values of number of testers and different ex-
 periment times when $\alpha=2$ and 3, respectively. The producer's risk
 (γ) is chosen as 5 %. The probability of acceptance is also calcu-
 lated for these values.

From Tables 1 and 2, we can see that as the median ratio is in-
 creased, the design parameters are decreased. We observed that as
 the shape value increases from 2 to 3, the plan generally requires a
 smaller number of groups.

Example 1

Suppose that a manufacturer wants to use the proposed group sam-
 pling plan for the inspection of incoming lots of light bulbs. Multi-
 item tests with group size of five will be used. Suppose also that the

TABLE 2—Group sampling plans for generalized exponential distribution when $\alpha=3$.

β	$\theta_m/\theta_m^0=r_2$	$r=5$						$r=10$					
		$a=0.5$			$a=1.0$			$a=0.5$			$a=1.0$		
		g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	42	2	0.9835	7	3	0.9789	6	2	0.9753	2	4	0.9723
	4	7	1	0.9977	1	1	0.9888	2	1	0.9971	1	1	0.9551
	6	2	0	0.9814	1	1	0.9985	1	0	0.9814	1	1	0.9935
	8	2	0	0.9917	1	0	0.9716	1	0	0.9917	1	1	0.9986
	10	2	0	0.9956	1	0	0.9845	1	0	0.9956	1	0	0.9693
0.10	2	69	2	0.9731	12	3	0.9641	10	2	0.9592	3	4	0.9588
	4	12	1	0.9961	2	1	0.9777	4	1	0.9943	1	1	0.9551
	6	3	0	0.9723	2	1	0.9970	2	0	0.9632	1	1	0.9935
	8	3	0	0.9876	1	0	0.9716	2	0	0.9835	1	1	0.9986
	10	3	0	0.9935	1	0	0.9845	2	0	0.9913	1	1	0.9996
0.05	2	89	2	0.9654	15	3	0.9554	45	3	0.9885	7	5	0.9851
	4	15	1	0.9951	2	1	0.9777	5	1	0.9928	1	1	0.9551
	6	4	0	0.9632	2	1	0.9970	2	0	0.9632	1	1	0.9935
	8	4	0	0.9835	1	0	0.9716	2	0	0.9835	1	1	0.9986
	10	4	0	0.9913	1	0	0.9845	2	0	0.9913	1	0	0.9693
0.01	2	1512	3	0.9894	145	4	0.9837	69	3	0.9825	10	5	0.9788
	4	23	1	0.9925	3	1	0.9668	7	1	0.9900	2	2	0.9917
	6	23	1	0.9992	3	1	0.9955	7	1	0.9989	2	1	0.9871
	8	6	0	0.9754	3	1	0.9990	3	0	0.9754	2	1	0.9971
	10	6	0	0.9870	2	0	0.9693	3	0	0.9870	1	0	0.9693

235 life of this product follows a generalized exponential distribution
 236 with shape parameter 3. It is known that the specified median life of
 237 interest is 1000 h. The test time was specified as 500 h. It is required
 238 that the consumer's risk is 25 % if the true median life is 1000 and
 239 the producer's risk is 5 % if the true median is 2000. As $\alpha=3$, r
 240 $=5$, $\beta=0.25$, $a=0.5$, and $r_2=2$, it is found from Table 2 that (g, c)
 241 $=(42, 2)$. So, a sample of 210 items is drawn and allocated to 42
 242 groups with 5 items. If no more than two failures occur in each of
 243 the 42 groups, then the lot will be accepted.

244 **Example 2**

245 Suppose that an experimenter wants to adopt the group sampling
 246 plan with $r=10$ to decide about the acceptance or the rejection of
 247 the submitted lot of products. The specified median life of the prod-
 248 uct is $\theta_m^0=1000$ and the test duration is 1000 h. The producer's risk
 249 is $\gamma=0.05$ at $\theta_m/\theta_m^0=2$ and the consumer's risk is $\beta=0.10$. Now we
 250 consider data set given by Wood [14] which is the failure time in
 251 hours of a software, which represents the time from the starting of
 252 the execution of the software until a software failure is experienced.
 253 We have the following values: 519, 968, 1430, 1893, 2490, 3058,
 254 3625, 4422, and 5218. Aslam et al. [15] showed that the generalized
 255 exponential distribution provides a good fit to this software data.
 256 According to them, the maximum likelihood estimators of α and λ
 257 are 2.65 and 0.6547, respectively. Let us assume that $\alpha=3$. As r
 258 $=10$ and $a=1$, the design parameters can be found as $g=3$ and c
 259 $=4$ from Table 2. This sampling plan is operated as: Take a sample
 260 of size 30 and allocate 10 items to 3 groups. Accept the lot if no
 261 more than four failures are recorded from each of three groups be-
 262 fore 1000 h, but reject it otherwise.

263 **Improved Group Acceptance Sampling Plans**

264 In the above proposed group acceptance sampling plan, the lot of
 265 products is rejected if the number of failures in any group is greater

than the acceptance number. In this section, we made a new group
 acceptance sampling plan by relaxing this condition to allow possi-
 bly more failures in some groups when accepting a lot. The new
 plan is as follows:

- **Step 1:** Select the number of groups g and allocate pre-
 defined r items to each group so that the sample size for a lot
 will be $n=gr$.
- **Step 2:** Select the acceptance number c ($c \leq r$) for a group
 and the experiment time T .
- **Step 3:** Perform the experiment for the g groups simulta-
 neously and record the number of failures for each group.
- **Step 4:** Accept the lot if the number of failures is smaller
 than or equal to c from at least k groups ($k \leq g$). Otherwise,
 truncate the experiment and reject the lot.

This plan is characterized by three parameters g , k , and c . Here
 we introduce an additional parameter k such that $k \leq g$ as compared
 with the original plan in the section Group Acceptance Sampling
 Plans. It should be noted that if $k=g$, the new plan reduces to the
 original group plan mentioned in the said section. We call it the
 improved group acceptance sampling plan because it turns out to
 have better performance in terms of the sample size as compared
 with the original plan.

The lot acceptance probability for the improved group accep-
 tance sampling plan is given by

$$L(p) = \sum_{j=k}^g \binom{g}{j} Q^j (1-Q)^{g-j} \tag{13}$$

where:
 Q = probability that c or fewer failures are observed in a group

TABLE 3—Improved group sampling plans for generalized exponential distribution when $\alpha=2$ ($r=5$).

β	$\theta_m/\theta_m^0=r_2$	$a=0.5$				$a=1.0$			
		g	k	c	$L(p_0)$	g	k	c	$L(p_0)$
0.25	2	40	39	2	0.9932	5	4	2	0.9616
	4	5	3	2	0.9725	1	1	1	0.9576
	6	3	2	0	0.9937	↑	↑	↑	0.9891
	8	↑	↑	↑	0.9979	↑	↑	↑	0.9961
0.10	2	57	58	2	0.9861	20	19	3	0.9885
	4	4	3	0	0.9504	3	2	1	0.9948
	6	↑	↑	↑	0.9879	2	1	0	0.9744
	8	↑	↑	↑	0.9958	↑	↑	↑	0.9905
0.05	2	70	69	2	0.9802	24	23	3	0.9837
	4	9	9	1	0.9651	4	3	1	0.9898
	6	5	4	0	0.9804	2	2	1	0.9782
	8	↑	↑	↑	0.9931	↑	↑	↑	0.9921
0.01	2	98	97	2	0.9632	33	32	3	0.9702
	4	21	20	1	0.9969	5	4	1	0.9835
	6	7	6	0	0.9613	3	3	1	0.9675
	8	↑	↑	↑	0.9860	3	2	0	0.9735

Note: The upward arrow (↑) indicates that the same value of the parameter as in the upward cell applies to the corresponding cell.

293
$$Q = \left[\sum_{i=0}^c \binom{r}{i} p^i (1-p)^{r-i} \right] \quad (14)$$

294 Here, p is the probability that an item fails by time T , which is given
 295 by Eq 10. The purpose is to find the three parameters g , k , and c
 296 such that the following inequalities are satisfied:

297
$$\sum_{j=k}^g \binom{g}{j} Q_0^j (1-Q_0)^{g-j} \geq 1 - \gamma \quad (15)$$

298
$$\sum_{j=k}^g \binom{g}{j} Q_1^j (1-Q_1)^{g-j} \leq \beta \quad (16)$$

299 Tables 3–6 provide design parameters of the improved plan ac-
 300 cording to several values for shape parameters, number of testers,

median quality level, and producer’s and consumer’s risks. From 301
 these tables we can see that as the quality level of a product in- 302
 creases the number of groups and acceptance number decrease for 303
 all the parameters. As the shape parameter is changing from 2 to 3, 304
 decreasing numbers of groups are required for testing. 305

Example 3 306

Suppose that a manufacturer wants to use the improved group sam- 307
 pling plan for the inspection of incoming lots of bulbs. Multi-item 308
 testers with group size of 5 will be used. Suppose also that the life 309
 of this product follows a generalized exponential distribution with 310
 shape parameter of 3. It is known that the specified median life of 311
 interest is 1000 h. The test time was specified as 500 h. It is required 312
 that the consumer’s risk is 25 % if the true median life is 1000 and 313
 the producer’s risk is 5 % if the true median is 2000. As $\gamma=0.05$, 314

TABLE 4—Improved group sampling plans for generalized exponential distribution when $\alpha=2$ ($r=10$).

β	$\theta_m/\theta_m^0=r_2$	$a=0.5$				$a=1.0$			
		g	k	c	$L(p_0)$	g	k	c	$L(p_0)$
0.25	2	7	6	2	0.9848	3	3	5	0.9753
	4	2	1	0	0.9858	1	1	2	0.9718
	6	↑	↑	↑	0.9918	1	1	1	0.9560
	8	↑	↑	↑	0.9972	↑	↑	↑	0.9834
0.10	2	10	9	2	0.9692	5	4	4	0.9851
	4	3	3	1	0.9511	1	1	2	0.9718
	6	3	2	0	0.9768	1	1	1	0.9560
	8	↑	↑	↑	0.9918	↑	↑	↑	0.9834
0.05	2	12	11	2	0.9565	6	5	4	0.9782
	4	3	3	1	0.9511	2	1	1	0.9771
	6	3	2	0	0.9768	1	1	1	0.9560
	8	↑	↑	↑	0.9918	↑	↑	↑	0.9834
0.01	2	45	44	3	0.9887	8	7	4	0.9614
	4	7	6	1	0.9946	3	2	2	0.9977
	6	4	3	0	0.9564	2	2	2	0.9920
	8	↑	↑	↑	0.9842	2	1	0	0.9658

Note: The upward arrow (↑) indicates that the same value of the parameter as in the upward cell applies to the corresponding cell.

TABLE 5—Improved group sampling plans for generalized exponential distribution when $\alpha=3$ ($r=5$).

β	$\theta_m/\theta_m^0=r_2$	$\alpha=0.5$				$\alpha=1.0$			
		g	k	c	$L(p_0)$	g	k	c	$L(p_0)$
0.25	2	14	13	1	0.9896	5	4	2	0.9897
	4	4	3	0	0.9953	1	1	1	0.9888
	6	↑	↑	↑	0.9995	↑	↑	↑	0.9985
	8	↑	↑	↑	0.9999	1	1	0	0.9716
0.10	2	20	19	1	0.9791	7	6	2	0.9793
	4	5	4	0	0.9924	2	1	0	0.9738
	6	↑	↑	↑	0.9991	↑	↑	↑	0.9964
	8	↑	↑	↑	0.9998	1	1	0	0.9716
0.05	2	24	23	1	0.9706	8	7	2	0.9730
	4	6	5	0	0.9888	2	2	1	0.9777
	6	↑	↑	↑	0.9987	↑	↑	↑	0.9970
	8	↑	↑	↑	0.9997	1	1	0	0.9992
0.01	2	198	197	2	0.9971	11	10	2	0.9504
	4	9	8	0	0.9746	3	3	1	0.9668
	6	↑	↑	↑	0.9970	3	2	0	0.9895
	8	↑	↑	↑	0.9994	↑	↑	↑	0.9976

Note: The upward arrow (↑) indicates that the same value of the parameter as in the upward cell applies to the corresponding cell.

315 $\beta=0.25$, $\alpha=0.5$, and $r_2=2$, it is found from Table 5 that (g, k, c)
 316 $= (1, 13, 14)$. So, a sample of 30 items is drawn and allocated to
 317 three groups. If there is no more than one failure from at least 13
 318 groups out of 14, then the lot will be accepted.

319 We may compare the two plans considered in the sections Group
 320 Acceptance Sampling Plans and Improved Group Acceptance Sam-
 321 pling Plans in terms of the sample size required to accept or to re-
 322 ject a lot. Table 7 compares the sample size required for the original
 323 plan versus the improved plan according to various parameter val-
 324 ues. We can see from Table 7 that the sample size is in the improved
 325 group acceptance sampling plan is quite smaller than the original
 326 plan particularly as the quality ratio is lower. The savings in the
 327 sample size seem to be larger as the shape parameter is smaller.

328 *Example 4*

329 Suppose that an experimenter wants to adopt the improved group
 330 sampling plan with $r=10$ to decide about the acceptance or the re-

jection of the submitted lot of products. The specified median life 331
 of the product is $\theta_m^0=1000$ and the test duration is 1000 h. The pro- 332
 ducer's risk is $\gamma=0.05$, $\beta=0.10$, and $\theta_m/\theta_m^0=2$. For the same data 333
 given in example 2, let us assume that $\alpha=3$. As $r=10$, $\alpha=1$, and $\hat{\alpha}$ 334
 $=3$, the design parameters can be found from Table 6 that (g, k, c) 335
 $= (3, 2, 3)$. This sampling plan is operated as: Take a sample size of 336
 30 and allocate 10 items to three groups. Accept the lot if no more 337
 than three failures are recorded from at least three out of four 338
 groups before 1000 h, but reject it, otherwise. Although this im- 339
 proved plan requires the same sample size as the original plan for 340
 this example, the acceptance probability becomes larger. 341

342 **Conclusions**

343 We proposed two group acceptance sampling plans for truncated 343
 life tests when the lifetime of a product follows the generalized ex- 344
 ponential distribution. We first considered the original group sam- 345

TABLE 6—Improved group sampling plans for generalized exponential distribution when $\alpha=3$ ($r=10$).

β	$\theta_m/\theta_m^0=r_2$	$\alpha=0.5$				$\alpha=1.0$			
		g	k	c	$L(p_0)$	g	k	c	$L(p_0)$
0.25	2	5	4	1	0.9816	2	1	2	0.9544
	4	3	2	0	0.9910	1	1	1	0.9551
	6	↑	↑	↑	0.9990	↑	↑	↑	0.9935
	8	↑	↑	↑	0.9998	↑	↑	↑	0.9986
0.10	2	7	6	1	0.9935	3	2	3	0.9880
	4	3	2	0	0.9910	1	1	1	0.9551
	6	↑	↑	↑	0.9990	↑	↑	↑	0.9935
	8	↑	↑	↑	0.9998	↑	↑	↑	0.9986
0.05	2	8	7	1	0.9528	4	3	3	0.9771
	4	4	3	0	0.9826	1	1	1	0.9551
	6	↑	↑	↑	0.9980	↑	↑	↑	0.9935
	8	↑	↑	↑	0.9996	↑	↑	↑	0.9986
0.01	2	29	28	2	0.9935	5	4	3	0.9635
	4	5	4	0	0.9721	2	2	2	0.9917
	6	↑	↑	↑	0.9967	2	1	0	0.9863
	8	↑	↑	↑	0.9993	↑	↑	↑	0.9969

Note: The upward arrow (↑) indicates that the same value of the parameter as in the upward cell applies to the corresponding cell.

TABLE 7—Comparison of sample sizes between two group sampling plans.

β	$\theta_m/\theta_m^0=r_2$	$r=5$		$r=10$	
		Improved Plan	Original Plan	Improved Plan	Original Plan
$\alpha=2, a=0.5$					
0.25	2	200	850	70	100
	4	15	25	20	20
0.10	2	285	1405	100	540
	4	20	35	30	30
0.05	2	350	1830	120	730
	4	45	45	30	30
0.01	2	490	-	450	1120
	4	105	340	70	110
$\alpha=3, a=0.5$					
0.25	2	70	210	50	60
	4	25	35	30	20
0.10	2	100	345	70	100
	4	25	60	30	40
0.05	2	120	445	80	450
	4	30	75	40	50
0.01	2	990	7560	290	690
	4	45	115	50	70

Note: The cell with hyphens (-) indicates that the sample size required is very large.

346 pling plan, where we reject the lot if more than c failures are
 347 corded in any of the g groups. Then, we proposed an improved
 348 group sampling plan by relaxing the condition of the acceptance in
 349 the original plan. Under the improved plan the lot is accepted if the
 350 number of failures per group is smaller than the specified accep-
 351 tance number in k out of g groups ($k \leq g$). It turns out that the im-
 352 proved group sampling plan reduces the sample size as compared
 353 with the original group sampling plan at the same condition. So, the
 354 improved plan meets the economic criteria in life testing because
 355 cost and time of the experiment are directly attached with the num-
 356 ber of items being tested for possible rejection of the submitted
 357 product. We advice the industrial practitioner and the experimenter
 358 to adopt the improved plan in order to save the cost and time of the
 359 experiment to reach the same decision as the original plan. The im-
 360 proved plan can be further studied for many other distributions and
 361 for various quality and reliability characteristics as future research.

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