

HOME WORK 2
MTH 311
PROBABILITY THEORY - I

- 1 Let $\Omega = \mathbb{R}^2$ and A_n the interior of a circle with center at $((-1)^n/n, 0)$ and radius 1. Find the $\liminf A_n$ and $\limsup A_n$.
- 2 Let $\{x_n\}$ be a sequence of real numbers and let $A_n = (-\infty, x_n)$. What is the connection between the $\limsup x_n$ and $\limsup A_n$ (similarly for \liminf).
- 3 Suppose $\Omega = \{0, 1, 2\}$ and $\mathcal{C} = \{\{0\}\}$, enumerate the class of all σ -field containing \mathcal{C} and find $\sigma(\mathcal{C})$.
- 4 Show that the intersection of two σ -fields is a σ -field. What about arbitrary collections of σ -fields?
- 5 Let Ω the set of integers. Define

$$\mathcal{A} = \{A \subset \Omega : A \text{ or } A^c \text{ is finite}\}$$

Show that \mathcal{A} is a field but not a σ -field.

- 6 For any countably infinite set Ω , the collection of its finite subsets and their complements forms a field \mathcal{F} . If we define $P(E)$ on \mathcal{F} to be 0 or 1 according as E is finite or not, then show that P is finitely additive but not countable additive.
- 7 Prove that for any arbitrary subsets of $A_1 \in \mathcal{F}, \dots, A_n \in \mathcal{F}$

$$P(A_1 \cup A_2 \dots \cup A_n) = \sum_{i=1}^n P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i A_j) + \dots + (-1)^n P(A_1 \dots A_n).$$

Using the above result show that if A_i and A_j are pairwise disjoint then

$$P(A_1 \cup A_2 \dots \cup A_n) = \sum_{i=1}^n P(A_i).$$

- 8 Let $f : \Omega \rightarrow \Omega'$ and \mathcal{C} be the class of subsets of Ω' . Show that

$$\sigma(f^{-1}(\mathcal{C})) = f^{-1}(\sigma(\mathcal{C})),$$

where $f^{-1}(\mathcal{C}) = \{f^{-1}(A); A \in \mathcal{C}\}$