Texture Modeling

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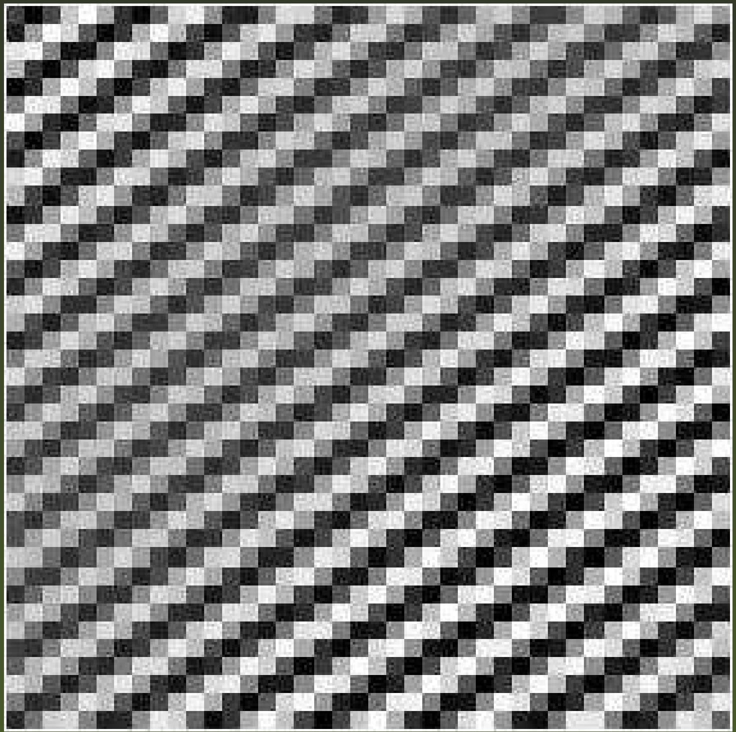
Part of this work is from the Ph.D. work of Mr. Anurag Prasad

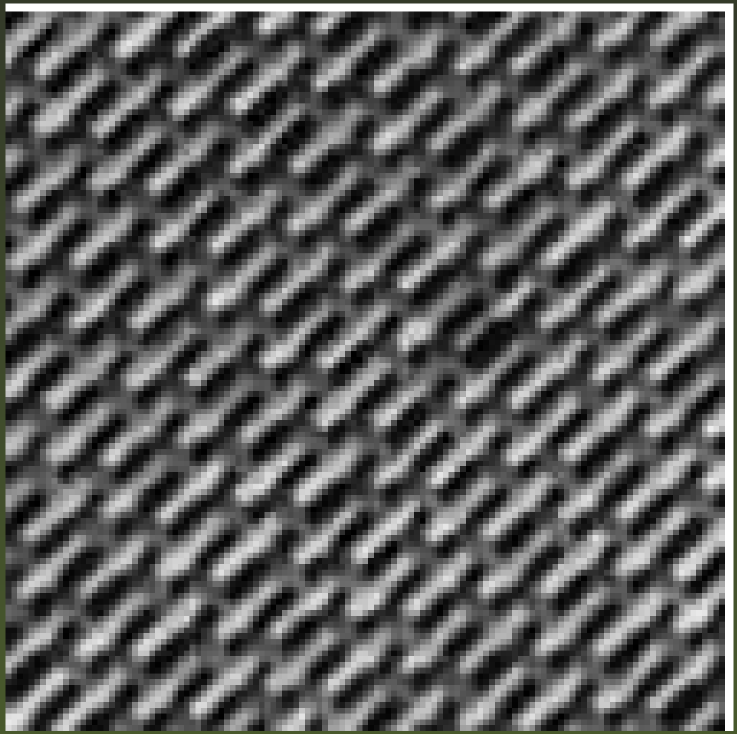
Outline of the presentation

- Some common textures (Pictures)
- What is a texture?
- How to quantify a texture?
- Some existing statistical literature
- Major statistical problems
- Some remedies
- Some related problems in SSP

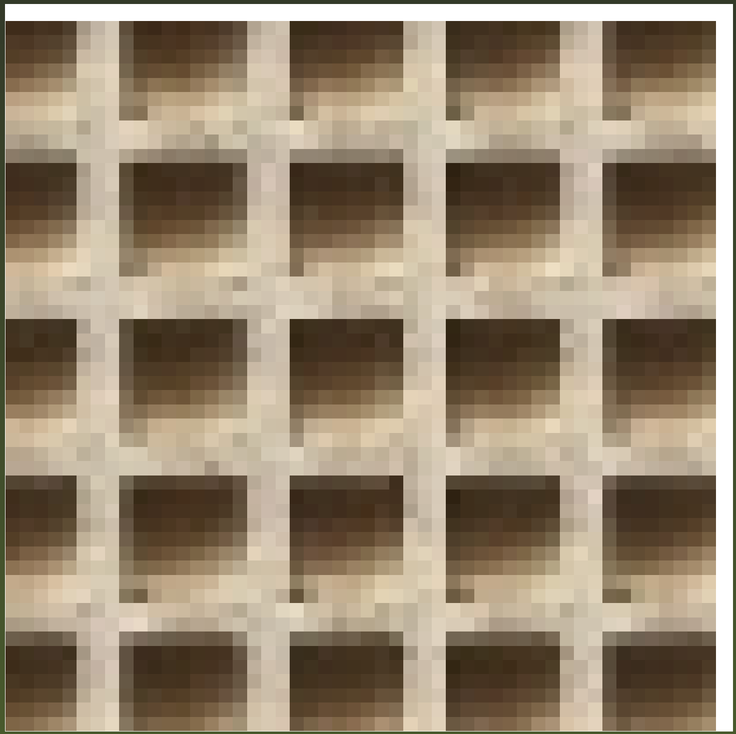
Grey Texture



Grey Texture



Colored Texture



What is a texture?

- There is no exact definition of texture.
- It is a human perception.
- It is generally a visual property of a surface representing the special information contained in a object surfaces.
- As a statistician we consider it as a realization of a random field.
- Our aim is simply to produce visually indiscernible copy of a given image using modeling technique.

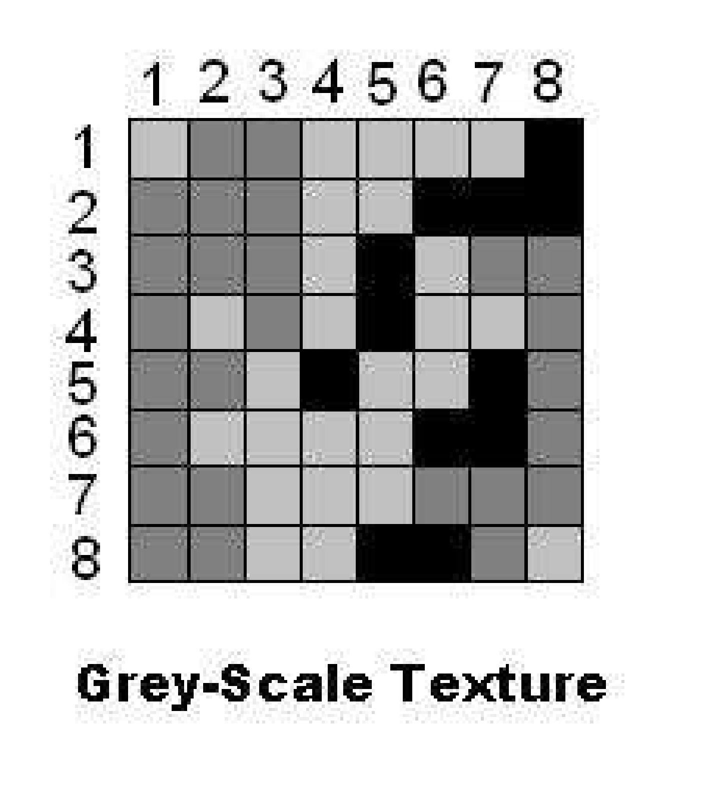
What is the necessity?

- Academic interest.
- It is possible to create in the computer
- It can be used for compression purpose

Grey texture: Quantification

When we talk about a grey texture, we refer to a digitized texture image on a lattice, $i.e.$ of size $M \times N$. We denote by (i, j), the (i, j)-th pixel of the image. It is assumed that each pixel of the image is described by a grey level taking on K possible values. Usually K = 256.

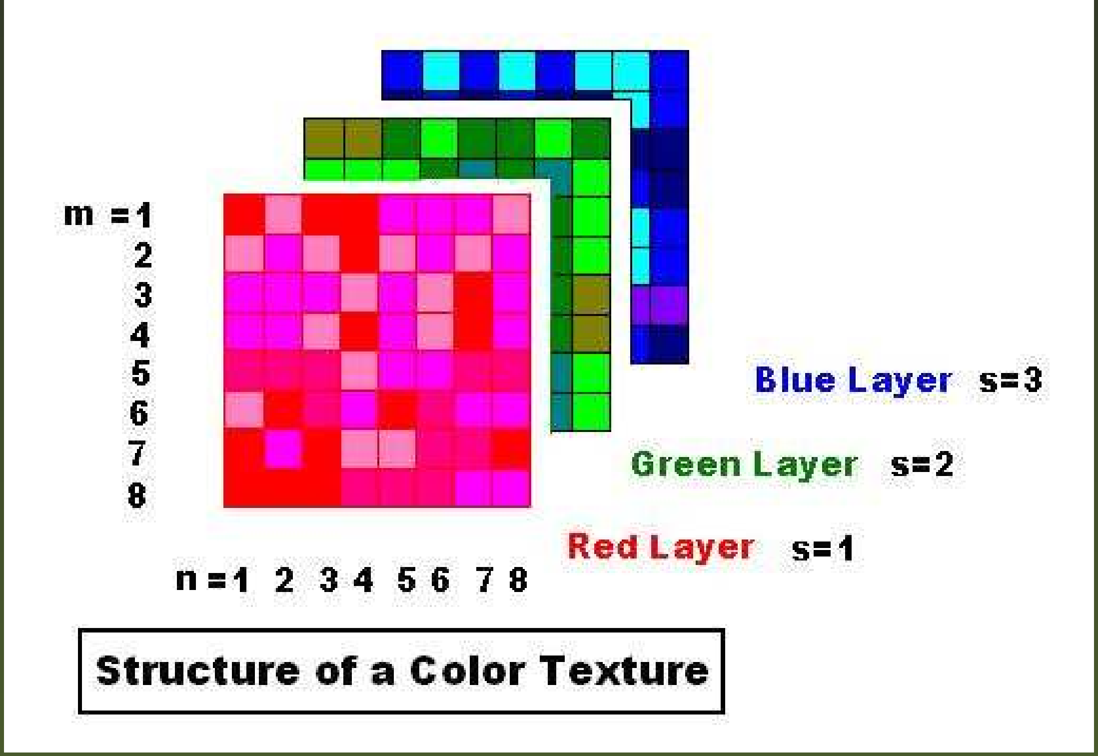
Grey Texture



Color texture: Quantification

The basic idea of the color texture is based on the assumptions that any visible color is a linear combination of three basic colors: Red, Green and Blue (RGB). Therefore, when we talk about a color texture, we refer to a digitized texture image on three $M \times N$ different lattices correspond to Red, Green and Blue respectively.

Color Texture



A STATISTICAL MODEL

$$Y(m,n) = Z(m,n) + X(m,n),$$

$$Z(m,n) = \sum_{k=1}^{p} \{A_k \cos(m\lambda_k + n\mu_k) + B_k \sin(m\lambda_k + n\mu_k)\}$$

- $Z(m,n)$ is the Deterministic part (Signal)
- $X(m,n)$ is the noise part
- $Y(m,n)$ Noise corrupted signal

The model was originally proposed by Francos, R.A., Meiri,A.Z. and Porat, B. (1993).

Aim

The main aim is to extract the deterministic component from the noisy component. It basically means from the given observations

- Estimate A_k's and B_k;s
- Estimate λ_k's and μ_k's
- Estimate p

Natural Estimates

The most intuitive estimates are the LSEs and they can be obtained by minimizing with respect to the unknown parameters

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\left[Y(m,n)-\sum_{k=1}^{p}\{f(A_k,B_k,\lambda_k,\mu_k)\}\right]^2$$

PROBLEMS

- The problem is highly non-linear in nature.
- Non-linear optimization method is required to compute the LSEs.
- LSEs can be obtained by solving a $2 \times p$ dimensional optimization problem.
- Extremely precise initial guesses are required.
- In practice p can be 65-70 or even more.

Theoretical Properties

Under the stationarity assumptions on the errors and under mild restrictions on the parameters the LSEs are consistent and asymptotically normal. The interesting properties of the LSEs are the convergence rates of the amplitudes and the frequencies.

$$M^{\frac{1}{2}}(\widehat{A}_k - A_k) \longrightarrow N(0, *)$$

$$N^{\frac{1}{2}}(\widehat{B}_k - B_k) \longrightarrow N(0, *)$$

$$M^{\frac{3}{2}}N^{\frac{1}{2}}(\widehat{\lambda}_k - \lambda_k) \longrightarrow N(0, *)$$

$$M^{\frac{1}{2}}N^{\frac{3}{2}}(\widehat{\mu}_k - \mu_k) \longrightarrow N(0, *)$$

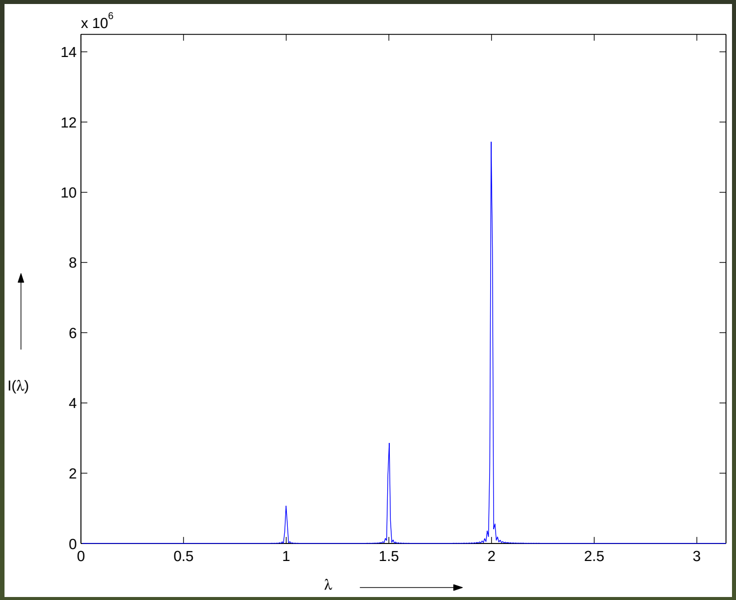
Alternative Estimates

The alternative estimates can be obtained by maximizing the periodogram function at the Fourier frequencies

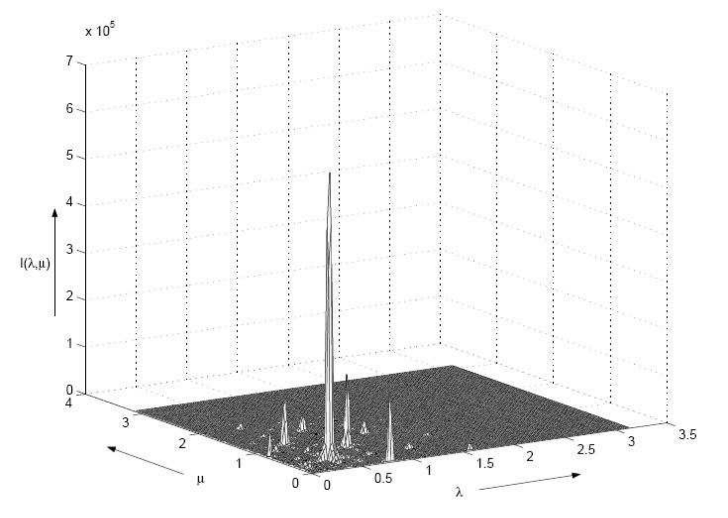
$$I(\lambda, \mu) = \frac{1}{MN} \left| \sum_{m=1}^{M} \sum_{n=1}^{N} Y(m,n) e^{i(m\lambda + n\mu)} \right|^2$$

It gives some idea about the location of the peaks and the number of peaks.

One dimensional periodogram



Two dimensional periodogram



Alternative Estimates

Reduce the problem to one dimensional problem

Data Matrix

$$\begin{bmatrix} Y(1,1) & \dots & Y(1,N) \\ \vdots & \dots & \vdots \\ Y(M,1) & \dots & Y(M,N) \end{bmatrix}$$

$$Y_1(m) = \sum_{n=1}^{N} Y(m,n) = \sum_{n=1}^{N} Z(m,n) + \sum_{n=1}^{N} X(m,n)$$

$$Y_2(n) = \sum_{m=1}^{M} Y(m,n) = \sum_{m=1}^{M} Z(m,n) + \sum_{m=1}^{M} X(m,n)$$

$$Y_1(m) = \sum_{k=1}^{p} C_k \cos(m\lambda_k) + D_k \sin(n\lambda_k) + X_1(m),$$

$$Y_2(n) = \sum_{k=1}^{p} U_k \cos(n\mu_k) + V_k \sin(n\mu_k) + X_2(n),$$

where

$$X_1(m) = \sum_{n=1}^{N} X(m,n), \quad X_2(m) = \sum_{m=1}^{M} X(m,n).$$

Unfortunately both have larger asymptotic variances than the LSEs

Sequential Estimates

Obtain $\widehat{A}_1$, $\widehat{B}_1$, $\widehat{\lambda}_1$, $\widehat{\mu}_1$ by minimizing

$$\sum_{m=1}^{M}\sum_{n=1}^{N}[Y(m,n) - f(A_1, B_1, \lambda_1, \mu_1)\}]^2$$

Replace $Y(m,n)$ by
$Y(m,n) - f(\widehat{A}_1, \widehat{B}_1, \widehat{\lambda}_1, \widehat{\mu}_1)$

Continue the process

Properties

If $k \leq p$

$$M^{\frac{1}{2}}(\widehat{A}_k - A_k) \longrightarrow N(0, *)$$

$$N^{\frac{1}{2}}(\widehat{B}_k - B_k) \longrightarrow N(0, *)$$

$$M^{\frac{3}{2}}N^{\frac{1}{2}}(\widehat{\lambda}_k - \lambda_k) \longrightarrow N(0, *)$$

$$M^{\frac{1}{2}}N^{\frac{3}{2}}(\widehat{\mu}_k - \mu_k) \longrightarrow N(0, *)$$

If $k > p$

$$\widehat{A}_k \longrightarrow 0 \quad \text{and} \quad \widehat{B}_k \longrightarrow 0.$$

Theorem

Let's assume $p = 1$. We need the following result.

THEOREM: Suppose $(\widetilde{\lambda}, \widetilde{\mu})$ are the consistent estimates of (λ, μ) and

$$\widehat{\lambda} = \widetilde{\lambda} + \frac{12}{M^2}\left[\frac{P_{MN}^{(\lambda)}}{Q_{MN}}\right], \quad \widehat{\mu} = \widetilde{\mu} + \frac{12}{N^2}\left[\frac{P_{MN}^{(\mu)}}{Q_{MN}}\right],$$

$$P_{MN}^{(\lambda)} = \sum_{m=1}^{M}\sum_{n=1}^{N}\left(m - \frac{M}{2}\right) Y(m,n) e^{-\widetilde{\lambda} m - \widetilde{\mu} n},$$

$$P_{MN}^{(\mu)} = \sum_{m=1}^{M}\sum_{n=1}^{N}\left(n - \frac{N}{2}\right) Y(m,n) e^{-\widetilde{\lambda} m - \widetilde{\mu} n},$$

Theorem (contd.)

$$Q_{MN} = \sum_{m=1}^{M} \sum_{n=1}^{N} Y(m,n) e^{-\widetilde{\lambda} m - \widetilde{\mu} n}.$$

If

$$(\widetilde{\lambda} - \lambda) = O_p(M^{-1-\delta_1} N^{-\delta_2})$$

and

$$(\widetilde{\mu} - \mu) = O_p(M^{-\delta_3} N^{-1-\delta_4})$$

for $\delta_i \in (0, 1/2)$

Theorem (contd.)

If $\delta_1 \leq \frac{1}{4}$

$$(\widehat{\lambda} - \lambda) = O_p(M^{-1-2\delta_1} N^{-\delta_2})$$

If $\delta_4 \leq \frac{1}{4}$

$$(\widehat{\mu} - \mu) = O_p(M^{-\delta_3} N^{-1-2\delta_4})$$

If $\delta_1 > \frac{1}{4}$ and $\delta_4 > \frac{1}{4}$

$$M^{\frac{3}{2}} N^{\frac{1}{2}} (\widehat{\lambda} - \lambda) \longrightarrow N(0, *)$$

$$M^{\frac{1}{2}} N^{\frac{3}{2}} (\widehat{\mu} - \mu) \longrightarrow N(0, *)$$

ALGORITHM

Start with the periodogram estimator of λ, say $\widetilde{\lambda}$ and

$$\widetilde{\lambda} - \lambda = O_p(M^{-1}N^{-1/2})$$

- Step 1: $M_1 = M^{0.8}$, $N_1 = N$.
- Step 2: $M_2 = M^{0.9}$, $N_2 = N$.
- Step 3: $M_3 = M$, $N_3 = N$.

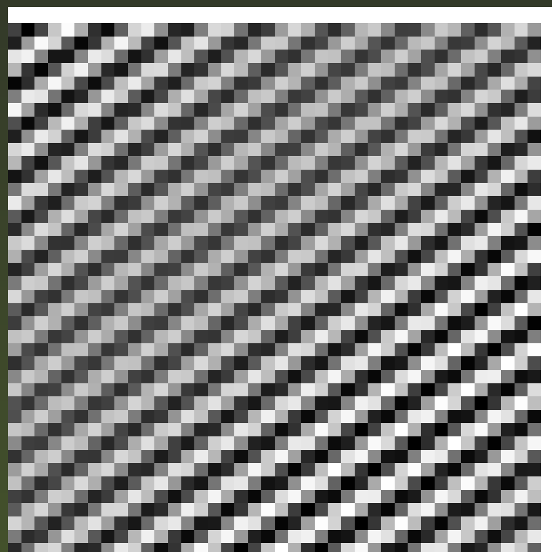
ALGORITHM

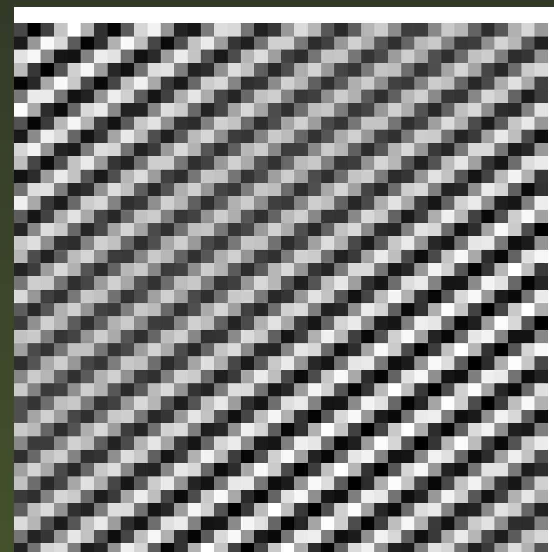
Start with the periodogram estimator of μ, say $\widetilde{\mu}$ and

$$\widetilde{\mu} - \mu = O_p(M^{-1/2}N^{-1})$$

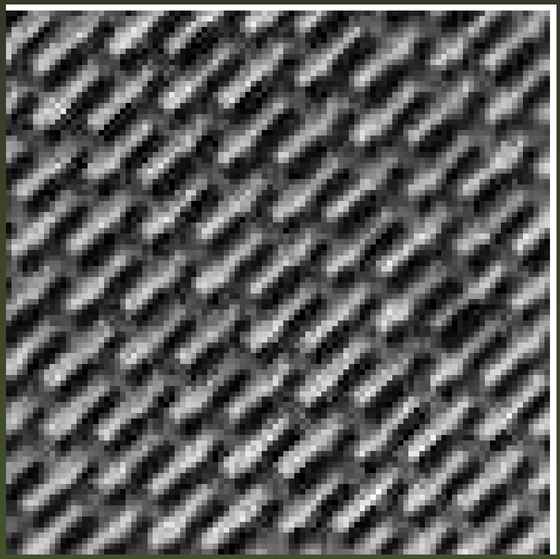
- Step 1: $M_1 = M$, $N_1 = N^{0.8}$.
- Step 2: $M_2 = M$, $N_2 = N^{0.9}$.
- Step 3: $M_3 = M$, $N_3 = N$.

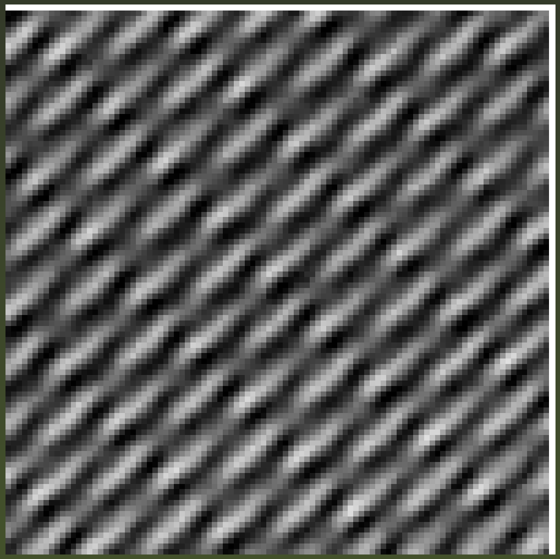
Actual and Estimated



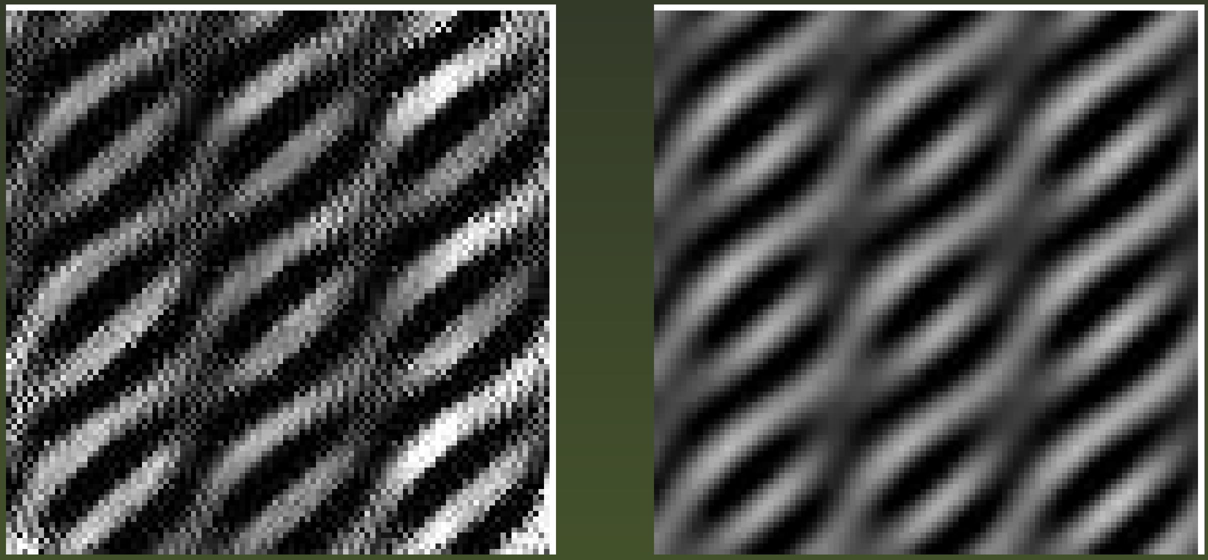


Actual and Estimated





Actual and Estimated



Colored Texture: A model

$$Y(m,n,s) = Z(m,n,s) + X(m,n,s),$$

$$Z(m,n,s) = \sum_{k=1}^{p} \{A_k \cos(u \cdot \theta_k) + B_k \sin(u \cdot \theta_k)\}.$$

Here

$$u = (m,n,s), \quad \theta_k = (\lambda_k, \mu_k, \nu_k).$$

$i.e.$

$$u \cdot \theta_k = m\lambda_k + n\mu_k + s\nu_k$$

Colored Texture: A model

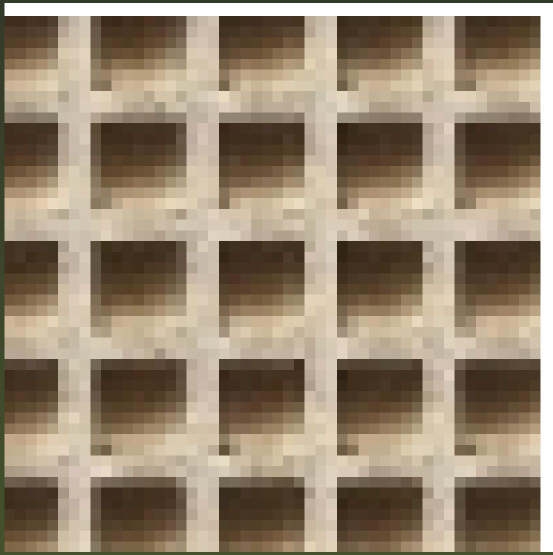
- $Z(m, n, s)$ is the Deterministic part (Signal)
- $X(m, n, s)$ is the noise part
- $Y(m, n, s)$ Noise corrupted signal

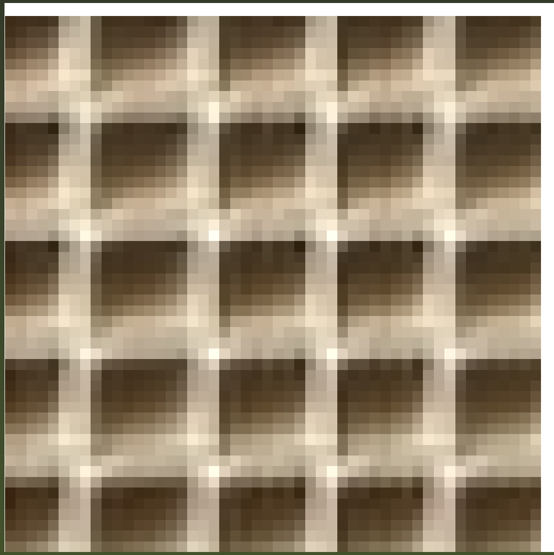
Problem remains the same: Estimate the unknown parameters.

Colored Texture: Estimation

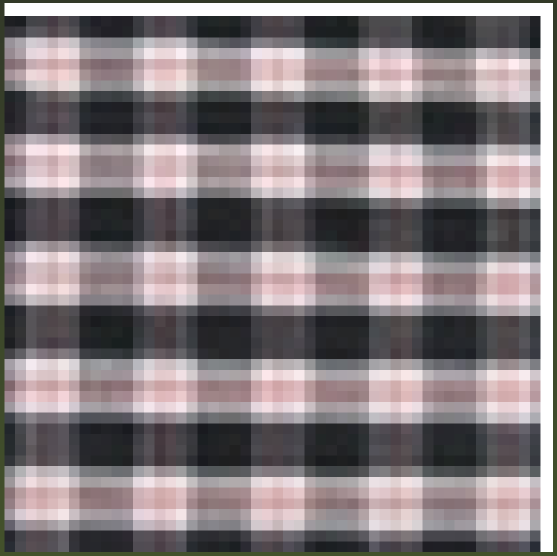
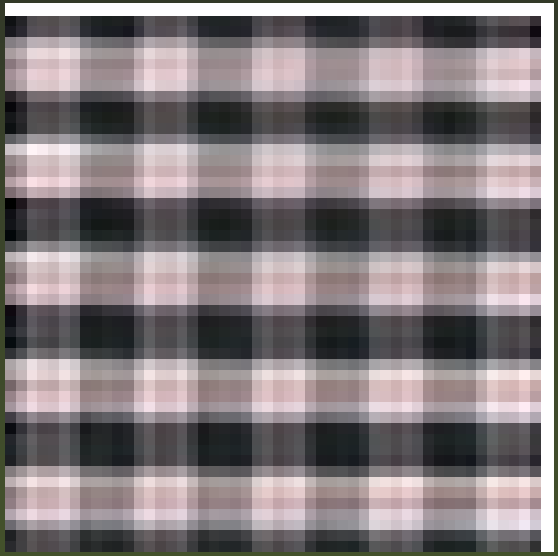
- We have used sequential estimation procedure
- Asymptotics are slightly different as S is constant
- It is possible to use the two dimensional estimation procedure

Actual and Estimated

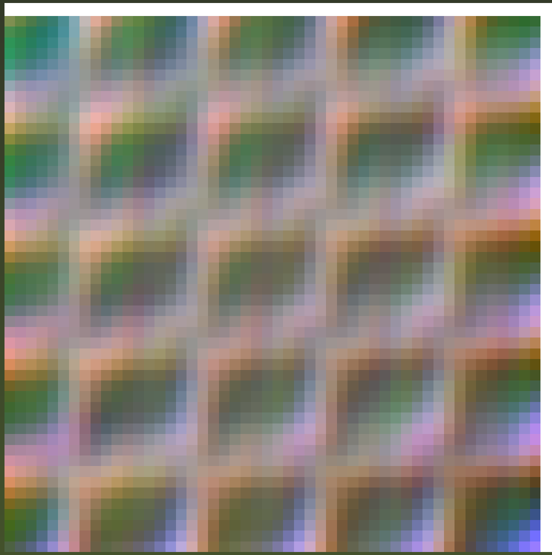




Actual and Estimated

Initial Estimates



Few Important References

- Francos, R.A., Meiri,A.Z. and Porat, B. (1993), IEEE Trans. SP
- Rao, C.R., Zhao, L. and Zhou, B. (1994), IEEE Trans. SP
- Bansal, N.K., Hamedani, G.G. and Zhang, Z. (1999) SPL
- Zhang, H. and Mandrekar, V. (2001), JTA
- Grim, J. and Haindl, M. (2003), CSDA
- Bai, Z.D. *et al.* (2003) JSPI

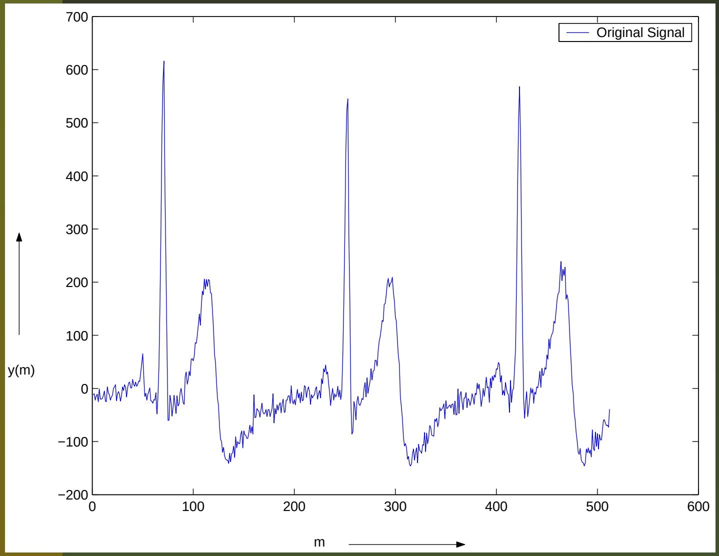
Some related problems in SSP

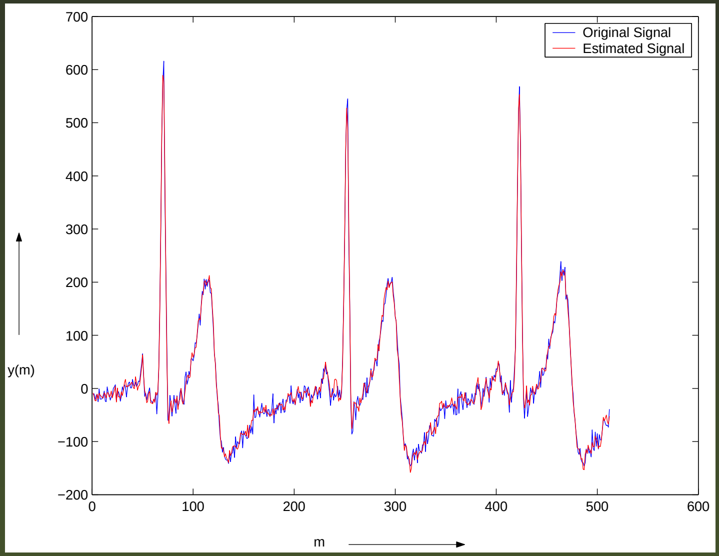
Sum of Sinusoidal Model

$$y(t) = \sum_{k=1}^{p} \{A_k \cos(\omega_k t) + B_k \sin(\omega_k t)\} + e(t)$$

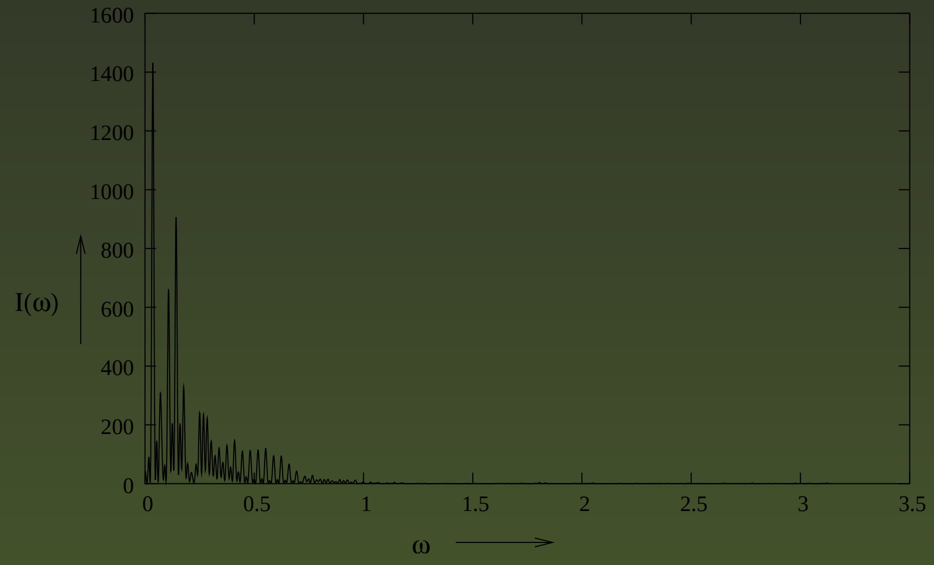
- Sequential procedure can be used.
- Asymptotic results can be established
- Modified Newton-Raphson Method

ECG SIGNALS

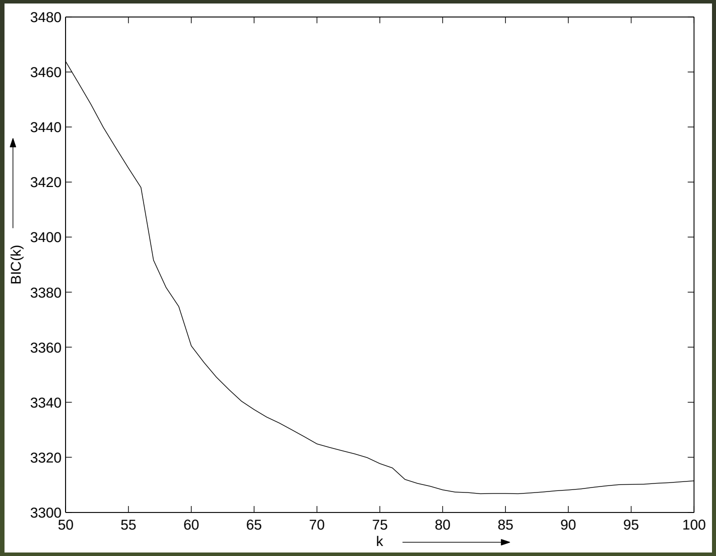




ECG Signals: Periodogram



ECG Signals: BIC



Chirp Signal

$$y(t) = \sum_{k=1}^{p} \mu_k(t) + e(t)$$

where

$$\mu_k(t) = \{A_k \cos(\alpha_k t + \beta_k t^2) + B_k \sin(\alpha_k t + \beta_k t^2)\}$$

- Asymptotic results can be established
- Sequential Procedure can not be applied
- Required efficient estimation procedure

Thank You