

ON PROGRESSIVE CENSORING

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Part of this work is going to appear in Technometrics.

OUTLINE OF THE TALK

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- What is a Progressive Censoring?

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- Associated Problems

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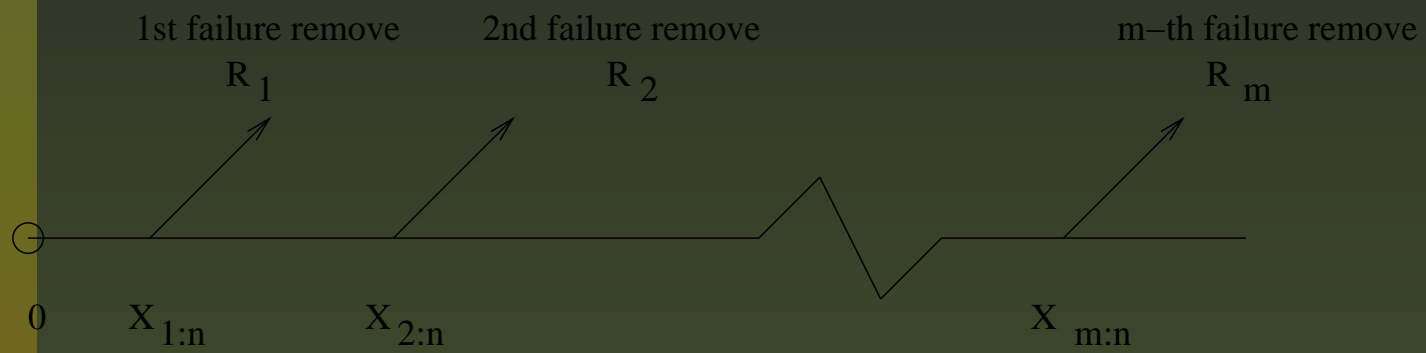
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- References

PROGRESSIVE CENSORING

- Put n items on test.
- Prefix m, R_1, \dots, R_m , such that

$$R_1 + \dots + R_m + m = n$$

- At the i -th failure time say $X_{i:n}$ remove R_i items from the remaining items.
- Stop the experiment at $X_{m:n}$.



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- Propose New Optimal Censoring Plans

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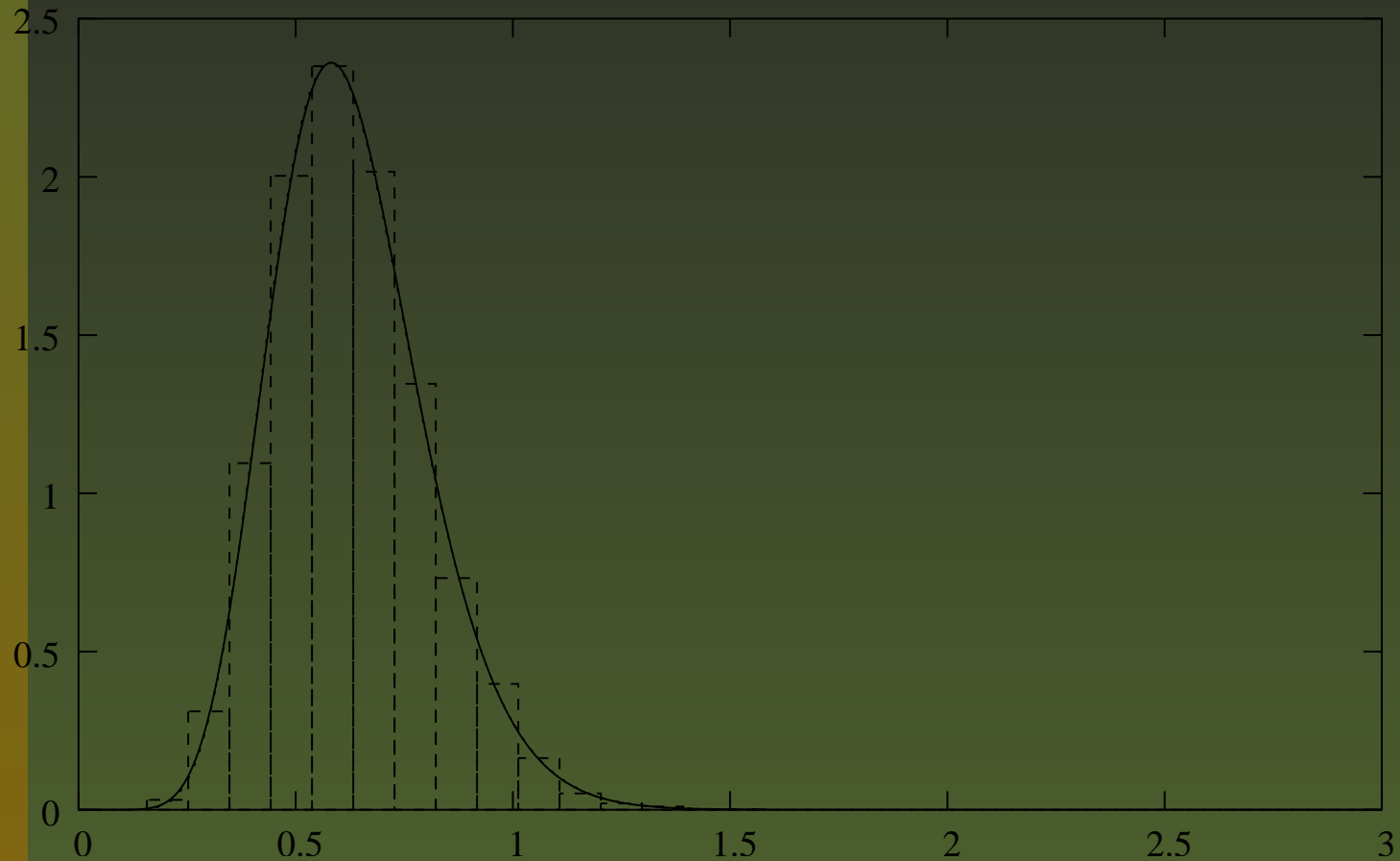
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- Bayes Estimates and Credible Intervals are Obtained Using MCMC



Posterior Density Function, Approximate Posterior Density Function and the Generated MCMC Samples

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- Criterion depends on p

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- Criterion 2:

$$C_2(P) = \frac{\int_0^1 V_{(P)}(\ln T_p) dW(p)}{\int_0^1 V_{(C)}(\ln T_p) dW(p)},$$

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- Choose P so that C_1 or C_2 is maximum

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Thank You