Debasis Kundu
Department of Mathematics and Statistics
Indian Institute of Technology Kanpur

Part of this work is going to appear in Technometrics.
What is a Progressive Censoring?

Associated Problems

Existing Methods

Proposed Solutions

Some Open Problems

References
What is a Progressive Censoring?
OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems
OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems
- Existing Methods
OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems
- Existing Methods
- Proposed Solutions
OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems
- Existing Methods
- Proposed Solutions
- Some Open Problems
OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems
- Existing Methods
- Proposed Solutions
- Some Open Problems
- References
Progressive Censoring

- Put $n$ items on test.
- Prefix $m$, $R_1, \ldots, R_m$, such that
  \[ R_1 + \ldots + R_m + m = n \]
- At the $i$-th failure time say $X_{i:n}$ remove $R_i$ items from the remaining items.
- Stop the experiment at $X_{m:n}$. 
1st failure remove \( R_1 \)
2nd failure remove \( R_2 \)
m-th failure remove \( R_m \)

0 \( X_{1:n} \) \( X_{2:n} \) \( X_{m:n} \)
ASSOCIATED PROBLEMS?
ASSOCIATED PROBLEMS?

- Inference on the Lifetime Distribution.
ASSOCIATED PROBLEMS?

- Inference on the Lifetime Distribution.
- Optimal Censoring Plans
EXISTING METHODS
EXISTING METHODS

- Parametric Approach
EXISTING METHODS

- Parametric Approach
- Frequentist Solution
EXISTING METHODS

- Parametric Approach
- Frequentist Solution
- Find MLEs or Some Other Estimators
**Existing Methods**

- Parametric Approach
- Frequentist Solution
- Find MLEs or Some Other Estimators
- Find Exact or Asymptotic Distributions
EXISTING METHODS

- Parametric Approach
- Frequentist Solution
- Find MLEs or Some Other Estimators
- Find Exact or Asymptotic Distributions
- Find Optimal Censoring Plans Using E or D Optimality
PROPOSED METHODS

Parametric Approach
Assume Weibull Lifetime Distributions
Bayesian Solution
Obtain Bayes Estimates and Credible Intervals using MCMC
Propose New Optimal Censoring Plans
PROPOSED METHODS

- Parametric Approach

Assume Weibull Lifetime Distributions
Bayesian Solution
Obtain Bayes Estimates and Credible Intervals using
MCMC
Propose New Optimal Censoring Plans
PROPOSED METHODS

- Parametric Approach
- Assume Weibull Lifetime Distributions
PROPOSED METHODS

- Parametric Approach
- Assume Weibull Lifetime Distributions
- Bayesian Solution
PROPOSED METHODS

- Parametric Approach
- Assume Weibull Lifetime Distributions
- Bayesian Solution
- Obtain Bayes Estimates and Credible Intervals using MCMC
PROPOSED METHODS

- Parametric Approach
- Assume Weibull Lifetime Distributions
- Bayesian Solution
- Obtain Bayes Estimates and Credible Intervals using MCMC
- Propose New Optimal Censoring Plans
PRIORS AND POSTERIORS:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
- Approximate Bayes Estimates (Lindleys' Approximations)
- Posteriors are Log-Concave
- Posteriors are Approximated
- Bayes Estimates and Credible Intervals are Obtained Using MCMC
PRIORS AND POSTERIORS:

- No Conjugate Priors Exist
PRIORS AND POSTERIORS:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
PRIORS AND POSTERIORS:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
- Approximate Bayes Estimates (Lindleys’ Approximations)
PRIORS AND POSTERIORS:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
- Approximate Bayes Estimates (Lindley’s Approximations)
- Posteriors are Log-Concave
Priors and Posteriors:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
- Approximate Bayes Estimates (Lindley’s Approximations)
- Posteriors are Log-Concave
- Posteriors are Approximated
No Conjugate Priors Exist

Assume the Shape and Scale Parameters have Independent Gamma Priors

Approximate Bayes Estimates (Lindleys’ Approximations)

Posterior is Log-Concave

Posterior is Approximated

Bayes Estimates and Credible Intervals are Obtained Using MCMC
Posterior Density Function, Approximate Posterior Density Function and the Generated MCMC Samples
OPTIMAL CENSORING PLANS:

What is an Optimal Censoring Plan?

For fixed $m$ and $n$, the choice of $R_1, \ldots, R_m$ which provides the maximum Information regarding the unknown parameters.

What is the meaning of Information?

Trace or Determinant of the Fisher Information matrix. Not Scale Invariant.

The variance of the $p$-th percentile estimator Criterion depends on $p$. – p.10/17
What is an Optimal Censoring Plan?

- For fixed $m$ and $n$, the choice of $R_1; \ldots; R_m$ which provides the maximum Information regarding the unknown parameters.

- What is the meaning of Information? Trace or Determinant of the Fisher Information matrix. Not scale invariant.

- The variance of the $p$-th percentile estimator. Criterion depends on $p$.
**Optimal Censoring Plans:**

- What is an Optimal Censoring Plan?
- For fixed $m$ and $n$, the choice of $R_1, \ldots, R_m$ which provides the maximum Information regarding the unknown parameters.

**Information**
- Trace or Determinant of the Fisher Information matrix.
- Not scale invariant.
- The variance of the $p$-th percentile estimator.
- Criterion depends on $p$. – p.10/17
What is an Optimal Censoring Plan?

For fixed $m$ and $n$, the choice of $R_1, \ldots, R_m$ which provides the maximum Information regarding the unknown parameters.

What is the meaning of Information?
What is an Optimal Censoring Plan?

For fixed $m$ and $n$, the choice of $R_1, \ldots, R_m$ which provides the maximum Information regarding the unknown parameters.

What is the meaning of Information?

Trace or Determinant of the Fisher Information matrix
**Optimal Censoring Plans:**

- What is an Optimal Censoring Plan?
- For fixed $m$ and $n$, the choice of $R_1, \ldots, R_m$ which provides the maximum Information regarding the unknown parameters
- What is the meaning of Information?
- Trace or Determinant of the Fisher Information matrix
- Not Scale Invariant
OPTIMAL CENSORING PLANS:

- What is an Optimal Censoring Plan?
- For fixed $m$ and $n$, the choice of $R_1, \ldots, R_m$ which provides the maximum Information regarding the unknown parameters
- What is the meaning of Information?
- Trace or Determinant of the Fisher Information matrix
- Not Scale Invariant
- The variance of the $p$-th percentile estimator
What is an Optimal Censoring Plan?

For fixed $m$ and $n$, the choice of $R_1, \ldots, R_m$ which provides the maximum *Information* regarding the unknown parameters.

What is the meaning of *Information*?

Trace or Determinant of the Fisher Information matrix.

Not Scale Invariant.

The variance of the $p$-th percentile estimator.

Criterion depends on $p$. 
Frequentist Approach:

Criterion 1:

\[ C_1(P) = f(V(P) \ln T_p) \]

Criterion 2:

\[ C_2(P) = R_1^0 V(C(P)) \ln T_p dW(p) \]

\[ = R_1^0 V(C(P)) \ln T_p dW(p) \]
Frequentist Approach:

\[
C_1(P) = f_V(P)(\ln T_p)g_f_V(C)(\ln T_p)g_p
\]

\[
C_2(P) = R_1^0 V(P)(\ln T_p)dW(p)R_1^0 V(C)(\ln T_p)dW(p)
\]
Frequentist Approach:

Criterion 1:

\[ C_1(P) = \frac{\{V(P)(\ln T_p)\}}{\{V(C)(\ln T_p)\}} \]
Frequentist Approach:

Criterion 1:

\[ C_1(P) = \frac{\left\{ V(P)(\ln T_p) \right\} }{\left\{ V(C)(\ln T_p) \right\}} , \]

Criterion 2:

\[ C_2(P) = \frac{\int_0^1 V(P)(\ln T_p) dW(p) }{\int_0^1 V(C)(\ln T_p) dW(p) } , \]
INFORMATION MEASURES

Bayesian Approach

Criterion 1:

\[ C_1(P) = \mathbb{E}_{\text{data}} \mathbb{f}_{\text{posterior}}(P) \ln \mathbb{T}_{\text{p}} \]

Criterion 2:

\[ C_2(P) = \mathbb{E}_{\text{data}} \mathbb{R}_{10} \mathbb{V}_{\text{posterior}}(P) \ln \mathbb{T}_{\text{p}} \]
INFORMATION MEASURES

- Bayesian Approach
Bayesian Approach

Criterion 1:

\[ C_1(P) = \frac{E_{\text{data}} \{ V_{\text{posterior}}(P)(\ln T_p) \}}{E_{\text{data}} \{ V_{\text{posterior}}(C)(\ln T_p) \}}, \]
**INFORMATION MEASURES**

- Bayesian Approach
- Criterion 1:

\[
C_1(P) = \frac{E_{\text{data}} \{ V_{\text{posterior}}(P) (\ln T_p) \}}{E_{\text{data}} \{ V_{\text{posterior}}(C) (\ln T_p) \}},
\]

- Criterion 2:

\[
C_2(P) = \frac{E_{\text{data}} \int_0^1 V_{\text{posterior}}(P) (\ln T_p) dW(p)}{E_{\text{data}} \int_0^1 V_{\text{posterior}}(C) (\ln T_p) dW(p)},
\]
OPTIMAL CENSORING PLANS:

Choose $P$ so that $C_1$ or $C_2$ is maximum.
Optimal Censoring Plans:

- Choose $P$ so that $C_1$ or $C_2$ is maximum
SOME OPEN PROBLEMS:

- Find a discrete optimization algorithm
- Extend it to other lifetime distributions
- Extend it to other censoring plans
Some Open Problems:

- Find a discrete optimization algorithm
SOME OPEN PROBLEMS:

- Find a discrete optimization algorithm
- Extend it to other lifetime distributions
SOME OPEN PROBLEMS:

- Find a discrete optimization algorithm
- Extend it to other lifetime distributions
- Extend it to other censoring plans


REFERENCES:


Thank You