

ON PROGRESSIVE CENSORING

Debasis Kundu
Department of Mathematics and Statistics
Indian Institute of Technology Kanpur

Part of this work is going to appear in Technometrics.

OUTLINE OF THE TALK

OUTLINE OF THE TALK

- What is a Progressive Censoring?

OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems

OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems
- Existing Methods

OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems
- Existing Methods
- Proposed Solutions

OUTLINE OF THE TALK

- What is a Progressive Censoring?
- Associated Problems
- Existing Methods
- Proposed Solutions
- Some Open Problems

OUTLINE OF THE TALK

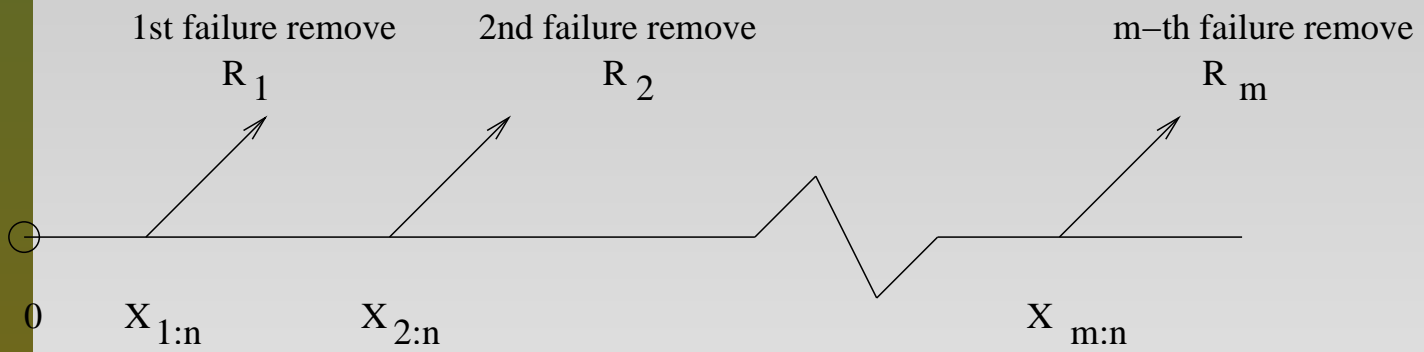
- What is a Progressive Censoring?
- Associated Problems
- Existing Methods
- Proposed Solutions
- Some Open Problems
- References

PROGRESSIVE CENSORING

- Put n items on test.
- Prefix m, R_1, \dots, R_m , such that

$$R_1 + \dots + R_m + m = n$$

- At the i -th failure time say $X_{i:n}$ remove R_i items from the remaining items.
- Stop the experiment at $X_{m:n}$.



ASSOCIATED PROBLEMS?

ASSOCIATED PROBLEMS?

- Inference on the Lifetime Distribution.

ASSOCIATED PROBLEMS?

- Inference on the Lifetime Distribution.
- Optimal Censoring Plans

EXISTING METHODS

EXISTING METHODS

- Parametric Approach

EXISTING METHODS

- Parametric Approach
- Frequentist Solution

EXISTING METHODS

- Parametric Approach
- Frequentist Solution
- Find MLEs or Some Other Estimators

EXISTING METHODS

- Parametric Approach
- Frequentist Solution
- Find MLEs or Some Other Estimators
- Find Exact or Asymptotic Distributions

EXISTING METHODS

- Parametric Approach
- Frequentist Solution
- Find MLEs or Some Other Estimators
- Find Exact or Asymptotic Distributions
- Find Optimal Censoring Plans Using E or D Optimality

PROPOSED METHODS

PROPOSED METHODS

- Parametric Approach

PROPOSED METHODS

- Parametric Approach
- Assume Weibull Lifetime Distributions

PROPOSED METHODS

- Parametric Approach
- Assume Weibull Lifetime Distributions
- Bayesian Solution

PROPOSED METHODS

- Parametric Approach
- Assume Weibull Lifetime Distributions
- Bayesian Solution
- Obtain Bayes Estimates and Credible Intervals using MCMC

PROPOSED METHODS

- Parametric Approach
- Assume Weibull Lifetime Distributions
- Bayesian Solution
- Obtain Bayes Estimates and Credible Intervals using MCMC
- Propose New Optimal Censoring Plans

PRIORS AND POSTERIORS:

PRIORS AND POSTERIORIS:

- No Conjugate Priors Exist

PRIORS AND POSTERIORIS:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors

PRIORS AND POSTERIORIS:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
- Approximate Bayes Estimates (Lindleys' Approximations)

PRIORS AND POSTERIORIS:

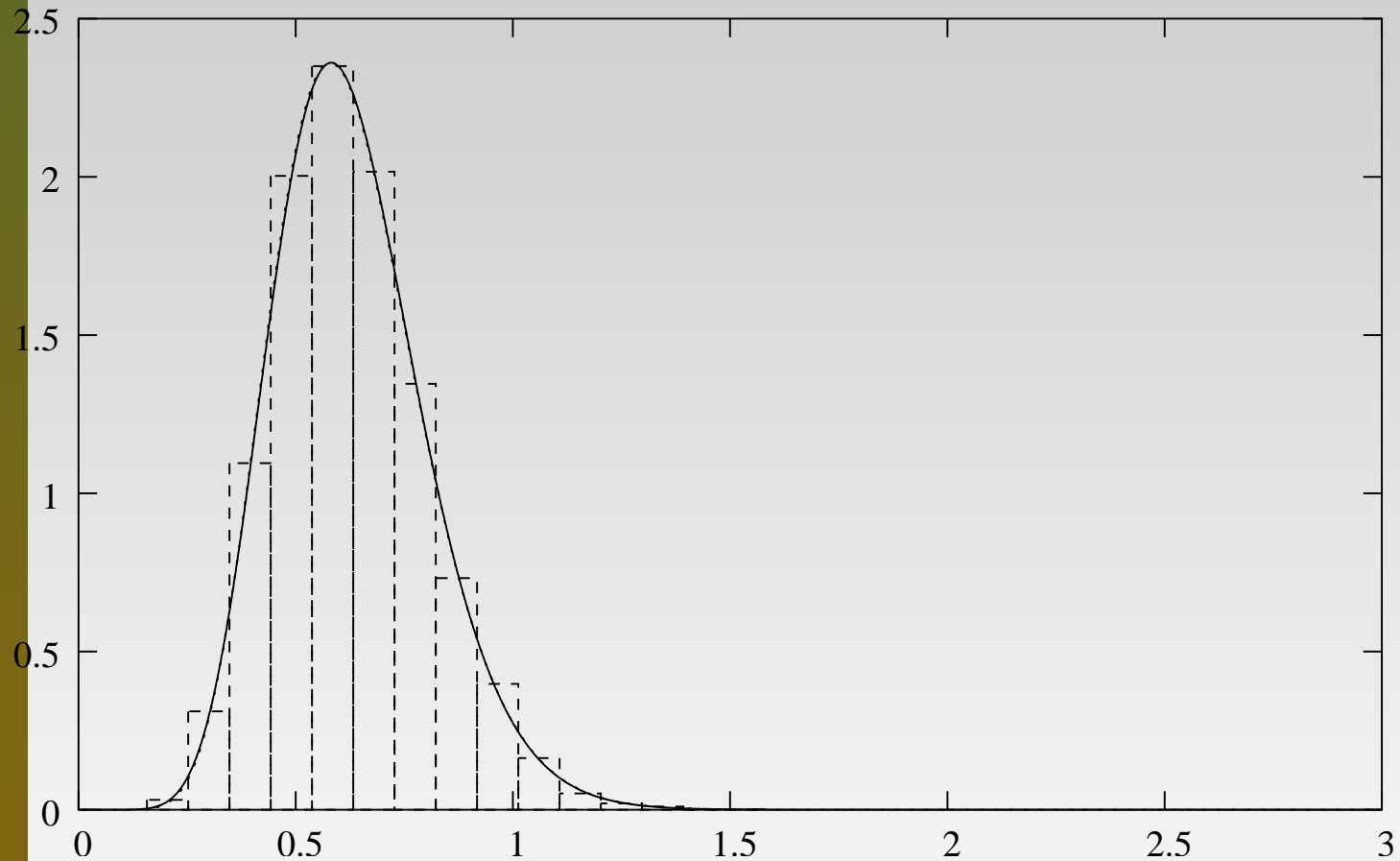
- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
- Approximate Bayes Estimates (Lindleys' Approximations)
- Posteriors are Log-Concave

PRIORS AND POSTERIOR:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
- Approximate Bayes Estimates (Lindleys' Approximations)
- Posteriors are Log-Concave
- Posteriors are Approximated

PRIORS AND POSTERIORIS:

- No Conjugate Priors Exist
- Assume the Shape and Scale Parameters have Independent Gamma Priors
- Approximate Bayes Estimates (Lindleys' Approximations)
- Posteriors are Log-Concave
- Posteriors are Approximated
- Bayes Estimates and Credible Intervals are Obtained Using MCMC



Posterior Density Function, Approximate Posterior Density Function and the Generated MCMC Samples

OPTIMAL CENSORING PLANS:

OPTIMAL CENSORING PLANS:

- What is an Optimal Censoring Plan?

OPTIMAL CENSORING PLANS:

- What is an Optimal Censoring Plan?
- For fixed m and n , the choice of R_1, \dots, R_m which provides the maximum *Information* regarding the unknown parameters

OPTIMAL CENSORING PLANS:

- What is an Optimal Censoring Plan?
- For fixed m and n , the choice of R_1, \dots, R_m which provides the maximum *Information* regarding the unknown parameters
- What is the meaning of *Information*?

OPTIMAL CENSORING PLANS:

- What is an Optimal Censoring Plan?
- For fixed m and n , the choice of R_1, \dots, R_m which provides the maximum *Information* regarding the unknown parameters
- What is the meaning of *Information*?
- Trace or Determinant of the Fisher Information matrix

OPTIMAL CENSORING PLANS:

- What is an Optimal Censoring Plan?
- For fixed m and n , the choice of R_1, \dots, R_m which provides the maximum *Information* regarding the unknown parameters
- What is the meaning of *Information*?
- Trace or Determinant of the Fisher Information matrix
- Not Scale Invariant

OPTIMAL CENSORING PLANS:

- What is an Optimal Censoring Plan?
- For fixed m and n , the choice of R_1, \dots, R_m which provides the maximum *Information* regarding the unknown parameters
- What is the meaning of *Information*?
- Trace or Determinant of the Fisher Information matrix
- Not Scale Invariant
- The variance of the p -th percentile estimator

OPTIMAL CENSORING PLANS:

- What is an Optimal Censoring Plan?
- For fixed m and n , the choice of R_1, \dots, R_m which provides the maximum *Information* regarding the unknown parameters
- What is the meaning of *Information*?
- Trace or Determinant of the Fisher Information matrix
- Not Scale Invariant
- The variance of the p -th percentile estimator
- Criterion depends on p

INFORMATION MEASURES

INFORMATION MEASURES

- Frequentist Approach:

INFORMATION MEASURES

- Frequentist Approach:
- Criterion 1:

$$C_1(P) = \frac{\{V_{(P)}(\ln T_p)\}}{\{V_{(C)}(\ln T_p)\}},$$

INFORMATION MEASURES

- Frequentist Approach:
- Criterion 1:

$$C_1(P) = \frac{\{V_{(P)}(\ln T_p)\}}{\{V_{(C)}(\ln T_p)\}},$$

- Criterion 2:

$$C_2(P) = \frac{\int_0^1 V_{(P)}(\ln T_p) dW(p)}{\int_0^1 V_{(C)}(\ln T_p) dW(p)},$$

INFORMATION MEASURES

INFORMATION MEASURES

- Bayesian Approach

INFORMATION MEASURES

- Bayesian Approach
- Criterion 1:

$$C_1(P) = \frac{E_{data} \{ V_{posterior(P)}(\ln T_p) \}}{E_{data} \{ V_{posterior(C)}(\ln T_p) \}},$$

INFORMATION MEASURES

- Bayesian Approach
- Criterion 1:

$$C_1(P) = \frac{E_{data} \{ V_{posterior(P)}(\ln T_p) \}}{E_{data} \{ V_{posterior(C)}(\ln T_p) \}},$$

- Criterion 2:

$$C_2(P) = \frac{E_{data} \int_0^1 V_{posterior(P)}(\ln T_p) dW(p)}{E_{data} \int_0^1 V_{posterior(C)}(\ln T_p) dW(p)},$$

OPTIMAL CENSORING PLANS:

OPTIMAL CENSORING PLANS:

- Choose P so that C_1 or C_2 is maximum

SOME OPEN PROBLEMS:

SOME OPEN PROBLEMS:

- Find a discrete optimization algorithm

SOME OPEN PROBLEMS:

- Find a discrete optimization algorithm
- Extend it to other lifetime distributions

SOME OPEN PROBLEMS:

- Find a discrete optimization algorithm
- Extend it to other lifetime distributions
- Extend it to other censoring plans

REFERENCES:

- Ahmad; A. Rao, M. B., Zhang, H. (2003), “ Periodic inspection plans: the case of Weibull distribution”, *Metrika*, vol. 58, 15–30.
- Balakrishnan, N. and Aggarwala, R. (2000), *Progressive Censoring: Theory, Methods and Applications*, Birkhäuser, Boston.
- Balakrishnan, N. (2007), “Progressive Censoring Methodology: An Appraisal (with discussions)”, *TEST*, vol. 16, 211-296.

REFERENCES:

- Kundu, D. (2008), “Bayesian Inference & Life Testing Plan for Weibull Distribution in Presence of Progressive Censoring”, (to appear in *Technometrics*)
- Ng, H.K.T., Chan, P.S. and Balakrishnan, N. (2004), “Optimal progressive censoring plans for the Weibull distribution”, *Technometrics*, vol. 46, no. 4, 470 - 481.
- Zhang, Y. and Meeker, W. Q. (2005), “Bayesian life test planning for the Weibull distribution with given shape parameter”, *Metrika*, vol. 61, 237 - 249.

Thank You