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Genetic algorithm and M-estimator based robust sequential estimation of parameters of nonlinear sinusoidal signals

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ABSTRACT

Estimation of parameters of nonlinear superimposed sinusoidal signals is an important problem in digital signal processing. In this paper, we consider the problem of estimation of parameters of real valued sinusoidal signals. We propose a real-coded genetic algorithm based robust sequential estimation procedure for estimation of signal parameters. The proposed sequential method is based on elitist generational genetic algorithm and robust M-estimation techniques. The method is particularly useful when there is a large number of superimposed sinusoidal components present in the observed signal and is robust with respect to presence of outliers in the data and impulsive heavy tail noise distributions. Simulations studies and real life signal analysis are performed to ascertain the performance of the proposed sequential procedure. It is observed that the proposed methods perform better than the usual non-robust methods of estimation.

1. Introduction

Sinusoidal models in various forms are used to describe and model many real life applications where periodic phenomena are present in a variety of signal processing applications and time series data analysis. The review work of Brillinger [1] presents some of the important real life applications from diverse areas where sinusoidal modeling is used in practice. The importance of multiple sinusoidal models in signal processing is highlighted in the signal processing literature (see for example, [2–4] and the references cited therein). Applications of sinusoidal modeling in various forms are found, among others, in speech signal processing ([5–11]), in biomedical signal processing ([12,13]), modeling of biological systems ([14,15]), radio location of distant objects ([16]) and also in communications and geophysical exploration by seismic waves processing.

A real valued superimposed sinusoidal signal model is given by

$$y_t = f_t\left(\underset{\sim}{\theta_0}\right) + \varepsilon_t; \quad f_t\left(\underset{\sim}{\theta_0}\right) = \sum_{k=1}^{p} (\alpha_k^0 \, Cos\, \omega_k^0 t + \beta_k^0 \, Sin\, \omega_k^0 t); \quad t = 1, \ldots, n, \tag{1}$$

where, $f_t\left(\underset{\sim}{\theta_0}\right)$ denotes the noise free superimposed sinusoidal signal and $\{\varepsilon_t\}$ is a sequence of additive observational random noise component. The number of superimposed signal components, p, is assumed to be known. $\underset{\sim}{\theta_0} = (\alpha_1^0, \beta_1^0, \omega_1^0, \ldots, \alpha_p^0, \beta_p^0, \omega_p^0)^T$ is the unknown true $3p$ dimensional parameter vector characterizing the signal with p components.

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$\left(\omega_1^0,\ldots,\omega_p^0\right)$ is the set of unknown frequencies of the signal and $\left(\alpha_1^0,\ldots,\alpha_p^0\right)$ and $\left(\beta_1^0,\ldots,\beta_p^0\right)$ are the corresponding amplitudes. α_k^0s and β_k^0s are arbitrary real numbers and ω_k^0s are distinct real numbers lying between $(0,\pi)$. The sequence of error random variables $\{\varepsilon_t\}$ may be assumed to be independently and identically distributed random variables with mean zero and finite variance or in general to be random variables from a stationary linear process. Given a sample of size n, the problem is to estimate the unknown signal parameter vector $\underset{\sim}{\theta_0}$.

A number of methods for estimation of the parameters of a superimposed sinusoidal model (1) have been proposed in the past and the literature on this subject is extensive ([2,17]). The most popular approaches include Gaussian maximum likelihood (or nonlinear least-squares) ([18]), Fourier transform (periodogram maxmizer) ([19,20]) and eigen decomposition (signal/noise subspace) ([21–23]).

The most intuitive and the most efficient estimator of the parameters is the non-linear least squares estimators obtained as

$$\underset{\sim}{\hat{\theta}}_{NLSE} = \arg\min_{\theta\sim} \sum_{t=1}^{n} \left(y_t - f_t\left(\underset{\sim}{\theta}\right)\right)^2. \tag{2}$$

The asymptotic theoretical properties of the nonlinear least squares estimators have been studied extensively under Gaussian and non-Gaussian noise setup ([18,19,24–29]). It is observed that the NLSE attains the Gaussian CRLB asymptotically for any white noise process, Gaussian or non-Gaussian ([27–29]). Asymptotic statistical theory pertaining to frequency estimation of model (1) using nonlinear least squares under Gaussian and non-Gaussian noise indicates that frequency may be estimated with extraordinary accuracy. The rates of convergence of the least squares estimators are $O_p(n^{-3/2})$ and $O_p(n^{-1/2})$, respectively, for the frequencies and amplitudes ([26]). Another popular method of parameter estimation is to find estimates of the frequencies by finding the maxima at the Fourier frequencies of the periodogram function $I(\omega)$, where

$$I(\omega) = \left|\sum_{t=1}^{n} y_t e^{-i\omega t}\right|^2. \tag{3}$$

Asymptotically the periodogram function has local maxima at the true frequencies. The periodogram estimators obtained under the condition that frequencies are Fourier frequencies, provide estimators with convergence rate of $O_p(n^{-1})$ ([30]). Furthermore, the estimators obtained by finding p local maxima of $I(\omega)$ achieve the best possible rate and are asymptotically equivalent to the LSEs and are referred to as the approximate least squares estimators.

Apart from the two above mentioned approaches, a number of non-iterative methods of parameter estimation have been proposed in the past. Notable among these are the Modified Forward Backward Linear Prediction (MFBLP) method [22], Estimation of Signal Parameters using Rotational Invariance Technique (ESPRIT) [31], Noise Space Decomposition method (NSD) [23].

It is interesting to observe that in many real life applications, p, the number of superimposed components in model (1) may be quite large. For example, in practice speech signals and ECG signals can have very large number of components ([11,13]). Further, in some real life applications, it is also observed that p increases with n, the number of samples. Increase in the number of samples in such situations, implies more varied patterns to be modeled and hence a natural increase in the number of sinusoidal components required to fit the data effectively. Estimation of the signal parameters in such a situation when a large number of superimposed sinusoidal components are present in the signal involves solving a high dimensional optimization problem. The problem of estimation of signal parameters of model (1) is well known to be numerically difficult, especially for high dimensional problems ([13]). The choice of initial guess can be very crucial in such a scenario and presence of several local minima of the error surface may often lead the iterative process to converge to a local optimum point rather than the global optimum.

Prasad et. al. [13] introduced a step-by-step sequential non-linear least squares procedure for estimation of the signal parameters. They observed that such a method works satisfactorily for parameter estimation when a large number of superimposed components are present and further proved that the estimators obtained using a sequential procedure are consistent. It is however well known that performance of least squares estimators, both sequential and non-sequential, deteriorates drastically when noise is heavy tailed and outliers are present in the data. In such a situation, rather than using a least square approach it is more appropriate to adopt a robust approach for estimation of signal parameters. Robust M-estimation based model parameter estimation technique seems to be the natural choice in such a situation.

The main purpose of this paper is to develop a computationally efficient robust estimation procedure for estimation of signal parameter when a large number of superimposed components are present and the contaminating noise is impulsive heavy tailed with possibility of outliers present in the observed signal data. In this paper, we propose a genetic algorithm based sequential robust M-estimation technique for estimation of signal parameters of sinusoidal signals. In recent times genetic algorithms have effectively been used for solving various complex problems. (See for example, [31–35] and the references cited therein).

The proposed technique has a number of advantages over the usually adopted standard estimation techniques like the nonlinear least squares or the periodogram maximizer estimation technique. The proposed procedure uses a sequential M-estimation approach and hence is robust to presence to heavy tailed noise and outliers; the sequential estimation approach makes estimation of parameters of large number of components possible in a fast and efficient manner. Furthermore, the proposed genetic algorithm based estimation procedure does not suffer from the drawbacks of non-stochastic optimization techniques.

The rest of the paper is organized as follows. In Section 2, we present the M-estimation technique for parameter estimation of sinusoidal signals. In Section 3, we present the proposed genetic search based sequential M-estimation algorithm. The empirical studies and real life signal analysis using the proposed algorithm will be presented in Section 4. Finally, the conclusions will be discussed in Section 5.

2. M-estimators of sinusoidal signal parameters

The non-linear least squares estimators (2) of parameters of sinusoidal signals are the most efficient estimators. However, performance of least squares estimators deteriorates drastically when the underlying noise is heavy tailed and outliers are present in the data. It is more appropriate, in such a situation, to adopt a robust approach for estimation of signal parameters. M-estimators are the most widely used class of estimators under such a robust approach. The M-estimator of $\underset{\sim}{\theta}_0$ for the real sinusoidal model (1) is given by

$$\hat{\underset{\sim}{\theta}}_M = \arg\min_{\underset{\sim}{\theta}} Q\left(\underset{\sim}{\theta}\right); \tag{4}$$

where,

$$Q\left(\underset{\sim}{\theta}\right) = \sum_{t=1}^{n} \rho\left(y_t - f_t\left(\underset{\sim}{\theta}\right)\right). \tag{5}$$

$\rho(\cdot)$ is some suitably chosen non-negative (usually convex) penalty function. The score function for solution of the M-estimator is

$$Q'\left(\underset{\sim}{\theta}\right) = \sum_{t=1}^{n} \psi\left(y_t - f_t\left(\underset{\sim}{\theta}\right)\right) f_t'\left(\underset{\sim}{\theta}\right) \tag{6}$$

with $\psi(\cdot) = \rho'(\cdot)$ and

$$f_t'\left(\underset{\sim}{\theta}\right) = \frac{\partial f_t}{\partial \underset{\sim}{\theta}} = \begin{pmatrix} Cos\,\omega_1 t \\ Sin\,\omega_1 t \\ -\alpha_1 t\, Sin\,\omega_1 t + \beta_1 t\, Cos\,\omega_1 t \\ \vdots \\ Cos\,\omega_p t \\ Sin\,\omega_p t \\ -\alpha_p t\, Sin\,\omega_p t + \beta_p t\, Cos\,\omega_p t \end{pmatrix}. \tag{7}$$

Note that

$$Q'\left(\hat{\underset{\sim}{\theta}}_M\right) = 0. \tag{8}$$

Observe that since $y_t = f_t\left(\underset{\sim}{\theta}\right) + \varepsilon_t$, the predicted value of y_t at $t = t_0$ using the M-estimator $\hat{\underset{\sim}{\theta}}_M$ is $\hat{y}_{t_0} = f_{t_0}\left(\hat{\underset{\sim}{\theta}}_M\right)$.

Corresponding to different choices of the ρ function, we get different M-estimators. We explore the following most widely used choices of the ρ function. Consider for example the Huber's function ([36–38])

$$\rho_h(z) = \begin{cases} \frac{z^2}{2}, & \text{if } |z| \leqslant c, \\ |z|c - \frac{c^2}{2}, & \text{if } |z| > c. \end{cases} \tag{9}$$

Using $\rho_h(\cdot)$ in (5) we get the Huber's M-estimator. Note that for $c \to \infty$ we get usual nonlinear least squares estimator and for $c \to 0$ we get the L_1 or the least absolute deviation (LAD) estimator. Thus by taking $\rho(z) = |z|$, for all z, we get LAD estimator.

Alternatively, suppose we use the Andrew's ρ_a function ([39]) given by

$$\rho_a(z) = \begin{cases} a(1 - \cos(z/a)) & \text{if } |z| \leqslant a\pi, \\ 2a, & \text{if } |z| > a\pi. \end{cases} \tag{10}$$

Andrew's M-estimator is obtained by choosing $\rho(\cdot)$ in (5) as (10).Ramsey's M-estimator ([40]) is obtained by using Ramsey's $\rho_r(\cdot)$ in (5). The Ramsey's ρ_r function is given by

$$\rho_r(z) = a^{-2}[1 - (\exp(-a|z|))(1 + a|z|)] \text{ for all } z. \tag{11}$$

Consistency of M-estimators under different classes of ρ functions and assumptions on the noise random variables have been studied in detail in [41].

3. Genetic algorithms based sequential M-estimation technique

In this section, we present the proposed genetic algorithms based sequential M-estimation algorithm for parameter estimation of sinusoidal signal model (1). Let us assume, without loss of generality, that for model (1)

$$\left((\alpha_1^0)^2 + (\beta_1^0)^2\right) > \left((\alpha_2^0)^2 + (\beta_2^0)^2\right) > \cdots > \left(\left(\alpha_p^0\right)^2 + \left(\beta_p^0\right)^2\right). \quad (12)$$

We propose the following sequential procedure for estimation of the signal parameters.

3.1. Step I: Obtain M-estimates of α_1^0, β_1^0 and ω_1^0

Consider the following single component sinusoidal model

$$y_t = \alpha^0\, Cos\, \omega^0 t + \beta^0\, Sin\, \omega^0 t + \varepsilon_t; \quad t = 1, \ldots, n. \quad (13)$$

Define, $\underset{\sim}{y} = (y_1, y_2, \ldots, y_n)^T$, $\underset{\sim}{\gamma} = (\alpha^0, \beta^0)^T$, $\underset{\sim}{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)^T$ and

$$X(\omega^0) = \begin{bmatrix} \cos(\omega^0) & \sin(\omega^0) \\ \vdots & \vdots \\ \cos(\omega^0 n) & \sin(\omega^0 n) \end{bmatrix}.$$

Model (13) can thus be written as $\underset{\sim}{y} = X(\omega^0)\,\underset{\sim}{\gamma} + \underset{\sim}{\varepsilon}$. We first obtain an initial estimate $\tilde{\omega}$ of the frequency ω^0 as the maxima of the periodogram function, i.e.

$$\tilde{\omega} = \arg\max_{\omega} \left| \frac{1}{n} \sum_{t=1}^{n} y_t e^{-i\omega t} \right|^2. \quad (14)$$

Using the initial estimate of ω^0 as $\tilde{\omega}$, we obtain an initial estimate of $\underset{\sim}{\gamma}$ as

$$\underset{\sim}{\tilde{\gamma}} = \left(\tilde{\alpha}, \tilde{\beta}\right)^T = \left(X(\tilde{\omega})^T X(\tilde{\omega})\right)^{-1} X(\tilde{\omega})^T \underset{\sim}{y}. \quad (15)$$

Consider now the function

$$Q_1\left(\underset{\sim}{\theta}\right) = \sum_{t=1}^{n} \rho(y_t - \alpha\, Cos\, \omega t - \beta\, Sin\, \omega t), \quad (16)$$

where, $\rho(\cdot)$ is one of the robust functions defined in Section 2. We propose an elitism based real-coded generational genetic algorithm for finding $\left(\hat{\alpha}, \hat{\beta}, \hat{\omega}\right)$ that minimizes (16). The solution $\left(\hat{\alpha}, \hat{\beta}, \hat{\omega}\right)$ are the M-estimates (for a particular chosen function $\rho(\cdot)$) of $(\alpha_1^0, \beta_1^0, \omega_1^0)$ for model (1).

In order to obtain $\left(\hat{\alpha}, \hat{\beta}, \hat{\omega}\right)$, we follow the following genetic algorithmic steps;

We first populate an initial population of possible solutions. While the binary coded chromosomal representation is the most widely used, the use of real valued chromosomes [42] in GAs offer a number of advantages in numerical function optimization over binary encodings [43]. In this paper, we have used a real-coded chromosomal string representation. The randomly initialized population is constructed in a reasonably large, predetermined, neighborhood of $\left(\tilde{\alpha}, \tilde{\beta}, \tilde{\omega}\right)$. Each individual of this population are members of the initial solution set. The members of the initial population are first evaluated for their fitness based on the fitness function (16), with $\rho(\cdot)$ as one of the robust functions defined in Section 2. Rather than using the raw fitness, we use a ranking based fitness function [44]. In this procedure, the chromosomes are assigned fitness according to their rank in the population, rather than their raw performance. According to the rank based fitness values of the chromosomes, a stochastic universal sampling rule [44] is applied for selection of the fit chromosomes, to be used for crossover and hence for generating chromosomes for the next generation of chromosomes.

Members selected from the current population using the selection operator, are next combined to produce new chromosomes by exchanging their genetic string material. We adopt the approach of discrete recombination based on the principle of a multipoint uniform crossover [45]. The crossover is applied on the two selected parents, according to a pre-assigned crossover probability.

Mutation operator is applied next to the new chromosomes produced by the crossover process. Mutation is considered to be the genetic operator that ensures that the probability of searching any given string will never be zero and thus has the effect of avoiding the possibility of convergence of the GA to a local optimum. In the present setup, with real-coded chromosomal representations, mutation can be achieved by either perturbing the gene values or random selection of new values within the allowed range. Wright [43] and Janikow and Michalewicz [46] demonstrate how real-coded GAs can take advantage of higher mutation rates than binary-coded GAs, increasing the level of possible exploration of the search space without adversely affecting the convergence characteristics.

We further adopt an elitist strategy [47] for populating the next generation of chromosomal strings. Elitism encourages the inclusion of highly fit genetic material, from earlier generations, in the subsequent generations. We deterministically allow a predetermined fraction of the most fit individuals to propagate through successive generations by replacing the same predetermined fraction of least fit individuals obtained after a new generation is formed after selection, crossover and mutation. The fractional difference between the number of chromosomes in the old population and the number of chromosomes produced by selection and recombination is the generation gap and is filled using the elitist approach.

We thus finally obtain the chromosomes to populate the next generation of individuals. For the real-coded chromosomal strings of the next generation, ranking based fitness values are assigned and a new set of chromosomes are selected for crossover; crossover and subsequent mutation is applied and elitism is used for filling the subsequent generation. And thus the process of stochastic optimization continues through subsequent generations.

We note here that since GA is a stochastic search procedure, it is difficult to formally specify its termination criteria as application of conventional termination criteria are inappropriate. Under a generational GA setup, it may so happen that fitness level, appropriately defined, of a population may remain static for a number of generations before a superior individual is found. In this paper, we follow the most commonly used approach of continuing the evolution process until a pre-determined number of generations have been completed. For each of the successive generations, we preserve the information regarding the most fit, i.e. the parameter vector that is the best solution for the optimization for obtaining the M-estimates, in that generation. The solution $\left(\hat{\alpha}, \hat{\beta}, \hat{\omega}\right)$ is the most fit individual evolving among all the generations, at the point when termination criterion is reached. The GA based sequential M-estimate of $(\alpha_1^0, \beta_1^0, \omega_1^0)$, say $\left(\hat{\alpha}_1(M)^0, \hat{\beta}_1(M)^0, \hat{\omega}_1(M)^0\right)$, is the solution $\left(\hat{\alpha}, \hat{\beta}, \hat{\omega}\right)$ obtained by minimizing (16) using the above GA procedure.

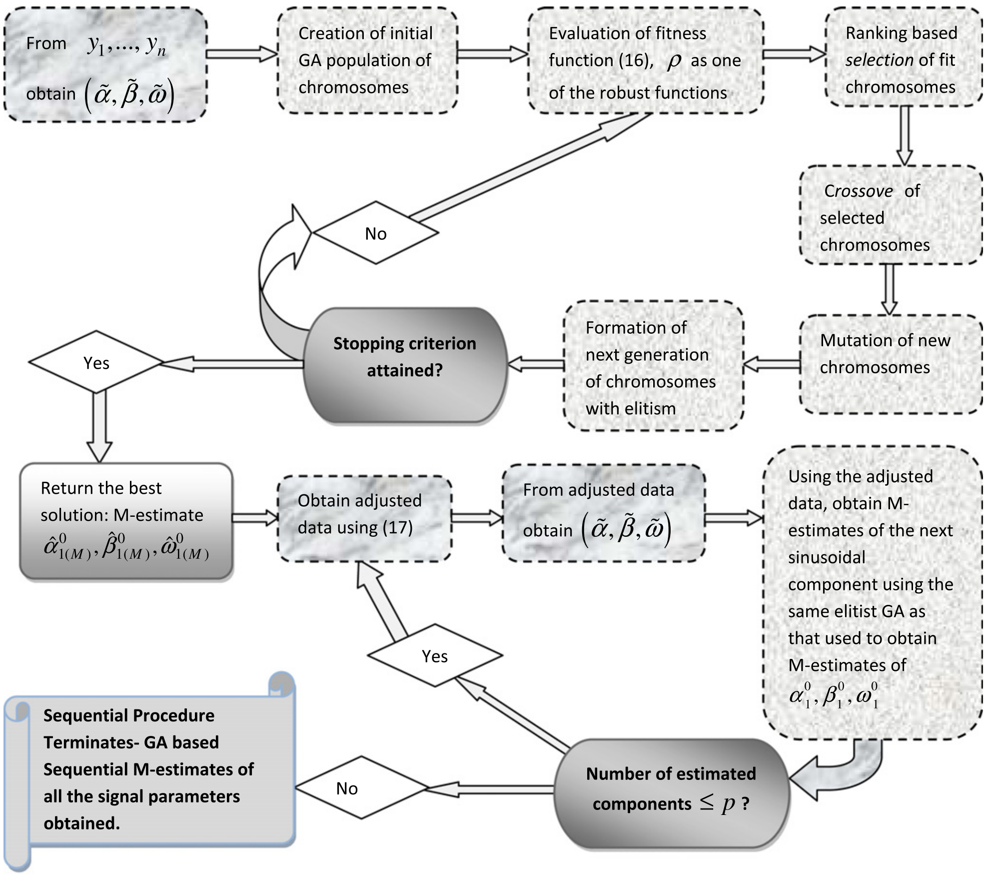

Fig. 1. Flow chart of the GA based sequential M-estimation procedure.

3.2. Step II: Adjust the data and obtain M-estimates of $\alpha_2^0, \beta_2^0, \omega_2^0$

Once the M-estimates of $(\alpha_1^0, \beta_1^0, \omega_1^0)$ are obtained as $\left(\hat{\alpha}^0_{1(M)}, \hat{\beta}^0_{1(M)}, \hat{\omega}^0_{1(M)}\right)$, we adjust the given data with respect to this estimated component as

$$y_t^{(1)} = y_t - \hat{\alpha}_1(M)^0\, Cos\left(\hat{\omega}_1(M)^0 t\right) - \hat{\beta}_1(M)^0\, Sin\left(\hat{\omega}_1(M)^0 t\right). \quad (17)$$

Table 1
Results for no outlier mixture normal noise.

Signal parameters	(σ_1, σ_2)	Methods						
			HUSM	ANSM	RASM	L1SM	LSE	PME
$\omega_1^0 = 0.4$	(0.6,0.01)	AV	0.4001	0.4001	0.4001	0.4001	0.4001	0.3995
		MSE	1.82E−07	1.91E−07	4.37E−07	5.89E−07	2.00E−07	1.59E−06
	(1.0,0.1)	AV	0.4001	0.4001	0.4001	0.4001	0.4002	0.3999
		MSE	4.20E−07	4.37E−07	9.21E−07	1.10E−06	5.41E−07	2.04E−06
$\omega_2^0 = 0.6$	(0.6,0.01)	AV	0.6000	0.6000	0.6000	0.6000	0.6000	0.6001
		MSE	7.03E−07	6.72E−07	3.45E−07	4.32E−07	6.97E−07	4.98E−07
	(1.0,0.1)	AV	0.6002	0.6002	0.6001	0.6001	0.6002	0.6004
		MSE	2.04E−06	1.93E−06	8.17E−07	9.03E−07	2.40E−06	3.39E−06
$\alpha_1^0 = 1.5$	(0.6,0.01)	AV	1.5131	1.5128	1.5157	1.5161	1.5101	1.5690
		MSE	4.79E−03	5.10E−03	1.20E−02	1.57E−02	5.05E−03	2.77E−02
	(1.0,0.1)	AV	1.5090	1.5086	1.5196	1.5226	1.5039	1.5222
		MSE	1.30E−02	1.33E−02	2.70E−02	3.01E−02	1.66E−02	4.48E−02
$\alpha_2^0 = 0.9$	(0.6,0.01)	AV	0.8932	0.8932	0.8933	0.8937	0.8931	0.8930
		MSE	3.21E−03	3.00E−03	1.20E−03	1.58E−03	3.21E−03	3.26E−03
	(1.0,0.1))	AV	0.8883	0.8887	0.8896	0.8862	0.8911	0. 8811
		MSE	6.99E−03	6.59E−03	2.76E−03	3.12E−03	7.62E−03	1.07E−02
$\beta_1^0 = 1.2$	(0.6,0.01)	AV	1.1792	1.1788	1.1729	1.1713	1.1800	1.0768
		MSE	5.82E−03	6.09E−03	1.63E02	2.27E−02	6.37E−03	4.23E−02
	(1.0,0.1)	AV	1.1921	1.1926	1.1781	1.1747	1.1969	1. 1453
		MSE	1.26E−02	1.31E−02	2.50E−02	2.97E−02	1.50E−02	4.49E−02
$\beta_2^0 = 1.2$	(0.6,0.01)	AV	0.2969	0.2970	0. 2935	0.2900	0.2980	0.3023
		MSE	8.83E−03	8.43E−03	4.05E−03	5.11E−03	8.83E−03	5.79E−O3
	(1.0,0.1)	AV	0.3248	0.3248	0.3075	0.3061	0.3255	0.3330
		MSE	1.99E−02	1.88E−02	7.88E−03	8.57E−03	2.30E−02	3.12E−02

Table 2
Results for no outlier t_m noise.

Signal parameters	(m)	Methods						
			HUSM	ANSM	RASM	L1SM	LSE	PME
$\omega_1^0 = 0.4$	10	AV	0.4002	0.4003	0.4003	0.4003	0.4002	0.4001
		MSE	1.28E−06	1.32E−06	1.42E−06	1.50E−06	1.23E−06	2.32E−06
	30	AV	0.4001	0.4001	0.4001	0.4001	0.4000	0.4001
		MSE	9.30E−07	9.44E−07	1.41E−06	1.45E−06	1.00E−06	2.29E−06
$\omega_2^0 = 0.6$	10	AV	0.6000	0.6000	0.5999	0.5999	0.5999	0.6001
		MSE	4.66E−06	4.66E−06	5.54E−06	5.84E−06	5.05E−06	5.24E−06
	30	AV	0.5999	0.5999	0.5997	0.5996	0.6000	0.6001
		MSE	3.49E−06	3.38E−06	5.10E−06	5.64E−06	3.81E−06	4.15E−06
$\alpha_1^0 = 1.5$	10	AV	1.4876	1.4860	1.4805	1.4831	1.4873	1.4856
		MSE	3.04E−02	3.10E−02	3.64E−02	3.73E−02	2.61E−02	4.90E−02
	30	AV	1.4937	1.4925	1.4887	1.4870	1.4999	1.4806
		MSE	2.66E−02	2.84E−02	3.12E−02	4.06E−02	2.31E−02	4.43E−02
$\alpha_2^0 = 0.9$	10	AV	0.8870	0.8838	0.8796	0.8769	0.8923	0.8854
		MSE	1.50E−02	1.46E−02	2.23E−02	2.48E−02	1.69E−02	1.96E−02
	30	AV	0.9015	0.9037	0.8997	0.8968	0.8963	0.8880
		MSE	1.36E−02	1.26E−02	2.02E−02	2.25E−02	1.71E−02	1.71E−02
$\beta_1^0 = 1.2$	10	AV	1.1991	1.2023	1.2165	1.2164	1.1935	1.1720
		MSE	4.50E−02	4.73E−02	4.86E−02	5.10E−02	4.35E−02	6.72E−02
	30	AV	1.1795	1.1777	1.1865	1.1831	1.1749	1.1755
		MSE	3.65E−02	3.73E−02	5.17E−02	4.09E−02	3.80E−02	6.21E−02
$\beta_2^0 = 0.3$	10	AV	0.3003	0.3009	0.2924	0.2923	0.2936	0.3029
		MSE	5.27E−02	5.26E−02	5.81E−02	5.95E−02	5.71E−02	5.87E−02
	20	AV	0.2878	0.2868	0.2684	0.2611	0.2909	0.3053
		MSE	3.80E−02	3.72E−02	5.28E−02	5.25E−02	3.80E−02	4.12E−02

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We now consider the function

$$Q_2\left(\underset{\sim}{\theta}\right) = \sum_{t=1}^{n} \rho\left(y_t^{(1)} - \alpha \, Cos\, \omega t - \beta \, Sin\, \omega t\right), \quad (18)$$

Table 3
Results for no outlier mixture normal with varying window width.

Signal parameters	Time window length	Methods	HUSM	ANSM	RASM	L1SM	LSE	PME
$\omega_1^0 = 0.4$	100	AV	0.3993	0.3993	0.3990	0.3990	0.3992	0.3989
		MSE	4.42E−06	4.78E−06	7.53E−06	9.01E−06	4.78E−06	6.26E−06
	500	AV	0.3999	0.3999	0.3998	0.3998	0.3999	0.3997
		MSE	4.50E−08	4.87E−08	8.86E−08	1.10E−07	4.59E−08	5.48E−08
	800	AV	0.4000	0.4000	0.4000	0.4000	0.4000	0.3999
		MSE	7.08E−09	7.47E−09	1.48E−08	1.75E−08	8.80E−09	3.03E−08
$\omega_2^0 = 0.6$	100	AV	0.5997	0.5997	0.5998	0.5997	0.5996	0.6005
		MSE	1.30E−05	1.23E−05	8.57E−06	9.65E−06	1.61E−05	1.63E−05
	500	AV	0.6000	0.6000	0.6000	0.6000	0.6000	0.6000
		MSE	1.47E−07	1.35E−07	3.73E−08	3.77E−08	1.66E−07	1.95E−07
	800	AV	0.6000	0.6000	0.6000	0.6000	0.6000	0.6000
		MSE	3.00E−08	2.82E−08	6.60E−09	6.83E−09	3.36E−08	4.95E−08
$\alpha_1^0 = 1.5$	100	AV	1.5777	1.5798	1.6015	1.6063	1.5790	1.5950
		MSE	2.77E−02	3.00E−02	4.91E−02	5.77E−02	3.08E−02	3.53E−02
	500	AV	1.5331	1.5339	1.5441	1.5475	1.5355	1.7395
		MSE	6.21E−03	6.73E−03	1.29E−02	1.56E−02	6.66E−03	5.98E−02
	800	AV	1.4869	1.4874	1.4836	1.4839	1.4850	1.7928
		MSE	2.73E−03	2.90E−03	5.68E−03	6.76E−03	3.46E−03	8.70E−02
$\alpha_2^0 = 0.9$	100	AV	0.9161	0.9155	0.8948	0.8898	0.9146	0.8999
		MSE	1.21E−02	1.12E−02	6.40E−03	7.01E−03	1.37E−02	1.39E−02
	500	AV	0.8991	0.8992	0.8964	0.8958	0.8992	0.8904
		MSE	3.28E−03	3.09E−03	8.50E−04	8.24E−04	3.45E−03	4.39E−03
	800	AV	0.8963	0.8964	0.8960	0.8954	0.8977	0.8977
		MSE	2.24E−03	2.08E−03	4.77E−04	4.51E−04	2.50E−03	3.67E−03
$\beta_1^0 = 1.2$	100	AV	1.1203	1.1178	1.0834	1.0777	1.1166	1.0890
		MSE	4.26E−02	4.59E−02	7.66E−02	9.06E−02	4.68E−02	5.69E−02
	500	AV	1.1500	1.1483	1.1286	1.1219	1.1494	0.7733
		MSE	9.31E−03	1.00E−02	1.76E−02	2.14E−02	9.34E−03	1.84E−01
	800	AV	1.2000	1.1987	1.1977	1.1975	1.2007	0.4817
		MSE	3.81E−03	4.02E−03	8.56E−03	1.05E−02	4.48E−03	5.17E−01
$\beta_2^0 = 0.3$	100	AV	0.2691	0.2696	0.2716	0.2693	0.2647	0.3023
		MSE	3.74E−02	3.47E−02	2.46E−02	2.86E−02	4.43E−02	4.41E−02
	500	AV	0.2916	0.2910	0.2998	0.2995	0.2900	0.3034
		MSE	1.19E−02	1.10E−02	2.85E−03	2.74E−03	1.32E−02	2.96E−02
	800	AV	0.3120	0.3115	0.3036	0.3025	0.3112	0.3188
		MSE	5.15E−03	4.81E−03	1.23E−03	1.20E−03	6.06E−03	6.01E−03

Table 4
Results for model with integer relation $(2\omega_1^0 = \omega_2^0)$ among frequencies.

Signal parameters	Methods	HUSM	ANSM	RASM	L1SM	LSE	PME
$\omega_1^0 = 0.4$	AV	0.3999	0.3999	0.3998	0.3998	0.4000	0.3996
	MSE	4.20E−07	4.32E−07	9.28E−07	1.13E−06	4.71E−07	1.73E−06
$\omega_2^0 = 0.8$	AV	0.7999	0.7999	0.8000	0.8000	0.7999	0.7999
	MSE	1.87E−06	1.77E−06	7.37E−07	6.96E−07	1.92E−06	3.15E−06
$\alpha_1^0 = 1.5$	AV	1.4931	1.4956	1.4976	1.4863	1.4844	1.5141
	MSE	9.91E−03	1.01E−02	1.99E−02	2.51E−02	1.32E−02	3.44E−02
$\alpha_2^0 = 0.9$	AV	0.8912	0.8917	0.8880	0.8852	0.8922	0.8859
	MSE	8.18E−03	7.71E−03	3.13E−03	3.44E−03	8.95E−03	1.01E−02
$\beta_1^0 = 1.2$	AV	1.1603	1.1588	1.1370	1.1437	1.1687	1.1064
	MSE	1.65E−02	1.74E−02	3.19E−02	3.73E−02	1.80E−02	4.83E−02
$\beta_2^0 = 0.3$	AV	0.2884	0.2883	0.2922	0.2953	0.2923	0.2890
	MSE	2.26E−02	2.15E−02	7.95E−03	7.56E−03	2.38E−02	3.22E−02

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where, $\rho(\cdot)$ is the same function as chosen in Step I. $\left(\hat{\alpha}_2, \hat{\beta}_2, \hat{\omega}_2\right)$ obtained by minimizing (18), with respect to the chosen $\rho(\cdot)$, using the real-coded GA based approach described in Step I are the M-estimates, say, $\left(\hat{\alpha}^0_{2(M)}, \hat{\beta}^0_{2(M)}, \hat{\omega}^0_{2(M)}\right)$, of $(\alpha^0_2, \beta^0_2, \omega^0_2)$.

3.3. Step III: Obtaining estimates of $\alpha^0_3, \beta^0_3, \omega^0_3, \ldots, \alpha^0_p, \beta^0_p, \omega^0_p$

Suppose the number of sinusoids, p, is known, we continue the sequential estimation procedure p times, by adjusting the data with respect to the estimated component, to get the M-estimates of all the signal parameters. In case the number of sinusoids p is unknown, we first estimate p and then apply the sequential procedure estimated number of components times to get sequential M-estimates of the signal parameters.

The flow chart of the proposed sequential M-estimation procedure for a known (or estimated) p is presented in Fig. 1.

4. Simulation studies and real signal analysis

In this section, we present the simulation studies under varied conditions and real signal data analysis to ascertain the performance of the proposed real-coded GA based sequential M-estimation procedure.

4.1. Simulation studies

In the simulation studies, we consider the following simulation model

$$y_t = \sum_{k=1}^{p} \left(\alpha^0_k \, Cos\, \omega^0_k t + \beta^0_k \, Sin\, \omega^0_k t\right) + \varepsilon_t; \quad t = 1, \ldots, n.$$

Table 5
Results for model with integer relation $(\omega^0_1 = 3\omega^0_2)$ among frequencies.

Signal parameters	Methods	HUSM	ANSM	RASM	L1SM	LSE	PME
$\omega^0_1 = 0.9$	AV	0.9004	0.9004	0.9004	0.9003	0.9004	0.9006
	MSE	6.59E−07	7.23E−07	7.78E−07	7.41E−07	7.14E−07	2.54E−06
$\omega^0_2 = 0.3$	AV	0.3001	0.3000	0.3000	0.2999	0.3000	0.3006
	MSE	1.73E−06	1.61E−06	1.08E−06	1.33E−06	1.86E−06	3.08E−06
$\alpha^0_1 = 1.5$	AV	1.4575	1.4493	1.3428	1.3207	1.4652	1.4180
	MSE	1.94E−02	2.17E−02	4.02E−02	4.51E−02	2.08E−02	4.95E−02
$\alpha^0_2 = 0.9$	AV	0.9015	0.9023	0.9028	0.9051	0.9034	0.8788
	MSE	8.12E−03	7.54E−03	4.22E−03	5.02E−03	8.75E−03	1.11E−02
$\beta^0_1 = 1.2$	AV	1.2490	1.2408	1.0716	1.0322	1.2575	1.2802
	MSE	2.00E−02	2.08E−02	3.95E−02	4.85E−02	2.39E−02	6.83E−02
$\beta^0_2 = 0.3$	AV	0.3064	0.3054	0.3032	0.3017	0.3028	0.3467
	MSE	1.74E−02	1.63E−02	9.21E−03	1.32E−02	1.91E−02	2.84E−02

Table 6
Results for model with non-integer relation $(1.5\omega^0_1 = \omega^0_2)$ among frequencies.

Signal parameters	Methods	HUSM	ANSM	RASM	L1SM	LSE	PME
$\omega^0_1 = 0.4$	AV	0.4001	0.4001	0.4001	0.4001	0.4002	0.3999
	MSE	4.20E−07	4.37E−07	9.21E−07	1.10E−06	5.41E−07	2.04E−06
$\omega^0_2 = 0.6$	AV	0.6002	0.6002	0.6001	0.6001	0.6002	0.6004
	MSE	2.04E−06	1.93E−06	8.17E−07	9.03E−07	2.40E−06	3.39E−06
$\alpha^0_1 = 1.5$	AV	1.5090	1.5086	1.5196	1.5226	1.5039	1.5222
	MSE	1.30E−02	1.33E−02	2.70E−02	3.01E−02	1.66E−02	4.48E−02
$\alpha^0_2 = 0.9$	AV	0.8883	0.8887	0.8896	0.8862	0.8911	0.8811
	MSE	6.99E−03	6.59E−03	2.76E−03	3.12E−03	7.62E−03	1.07E−02
$\beta^0_1 = 1.2$	AV	1.1921	1.1926	1.1781	1.1747	1.1969	1.1453
	MSE	1.26E−02	1.31E−02	2.50E−02	2.97E−02	1.50E−02	4.49E−02
$\beta^0_2 = 0.3$	AV	0.3248	0.3248	0.3075	0.3061	0.3255	0.3330
	MSE	1.99E−02	1.88E−02	7.88E−03	8.57E−03	2.30E−02	3.12E−02

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We take $p = 2$ in all the simulation models. n denotes the sample size, i.e. the width of the time window. We assume that the noise sequence $\{\varepsilon_t\}$ is a sequence of i.i.d. random variable with a heavy tail structure. In the simulation studies, we perform simulations to ascertain the performance of the proposed estimators for; (a) different standard heavy tail noise distributions and different levels of noise variance, (b) different lengths of the finite width time window, (c) sinusoidal signals with integer/non-integer relationships among the frequencies and (d) outliers present in the dataset.

Under each of the different scenarios, we compute the signal parameters using the proposed GA based sequential M-estimation technique. Based on the different choices of the $\rho(\cdot)$ function, defined in Section 2, we compute the following four different sequential M-estimates (i) Huber's robust sequential M-estimate (HUSM), (ii) Andrew's robust sequential M-estimate (ANSM), (iii) Ramsey's robust sequential M-estimate (RASM), (iv) L1 sequential M-estimate (L1SM). For each of the sequential M-estimation methods, we report here the average estimates (AV) and the mean square errors (MSE) of the signal

Table 7
Results for outlier contaminated dataset with mixture normal noise.

Signal Parameters	(σ_1, σ_2)	Methods						
			HUSM	ANSM	RASM	L1SM	LSE	PME
$\omega_1^0 = 0.4$	(0.6,0.01)	AV	0.4000	0.4001	0.3999	0.3999	0.3993	0.3990
		MSE	1.81E−07	2.01E−07	5.03E−07	6.65E−07	6.20E−07	1.04E−06
	(1.0,0.1)	AV	0.3999	0.4000	0.3999	0.3998	0.3993	0.3990
		MSE	4.83E−07	5.29E−07	8.77E−07	9.44E−07	9.51E−07	1.11E−06
$\omega_2^0 = 0.6$	(0.6,0.01)	AV	0.6004	0.5999	0.6000	0.6001	0.6045	0.6044
		MSE	9.64E−07	7.64E−07	3.22E−07	3.39E−07	2.19E−05	2.17E−05
	(1.0,0.1)	AV	0.6006	0.6001	0.6002	0.6002	0.6047	0.6049
		MSE	2.59E−06	1.94E−06	6.89E−07	7.62E−07	2.56E−05	2.89E−05
$\alpha_1^0 = 1.5$	(0.6,0.01)	AV	1.5010	1.5033	1.5095	1.5115	1.4994	1.5381
		MSE	5.33E−03	5.71E−03	1.34E−02	1.75E−02	4.91E−03	3.63E−03
	(1.0,0.1)	AV	1.5015	1.5019	1.5135	1.5189	1.4979	1.5230
		MSE	1.02E−02	1.10E−02	2.02E−02	2.30E−02	1.04E−02	9.17E−03
$\alpha_2^0 = 0.9$	(0.6,0.01)	AV	0.8680	0.8984	0.8893	0.8893	0.5087	0.5181
		MSE	4.06E−03	2.37E−03	8.70E−04	1.05E−03	1.68E−01	1.71E−01
	(1.0,0.1)	AV	0.8582	0.8928	0.8790	0.8796	0.5014	0.4792
		MSE	1.37E−02	9.81E−03	3.36E−03	3.70E−03	1.98E−01	2.25E−01
$\beta_1^0 = 1.2$	(0.6,0.01)	AV	1.1645	1.1610	1.1338	1.1310	1.1992	1.1505
		MSE	6.94E−03	7.65E−03	2.23E−02	2.84E−02	4.92E−03	4.60E−03
	(1.0,0.1)	AV	1.1517	1.1483	1.1329	1.1265	1.1858	1.1539
		MSE	1.68E−02	1.88E−02	3.21E−02	3.76E−02	1.38E−02	1.07E−02
$\beta_2^0 = 0.3$	(0.6,0.01)	AV	0.3409	0.2920	0.3049	0.3067	0.7274	0.7105
		MSE	1.19E−02	9.00E−03	4.00E−03	4.33E−03	1.93E−01	1.82E−01
	(1.0,0.1)	AV	0.3638	0.3129	0.3235	0.3173	0.7279	0.7363
		MSE	2.54E−02	1.89E−02	8.01E−03	8.21E−03	2.04E−01	2.10E−01

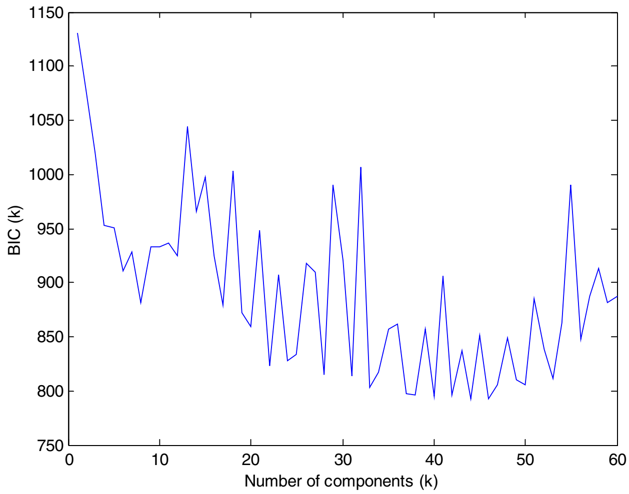

Fig. 2. Plot of the robust BIC function for the AWW signal.

parameter estimates over 100 simulations runs. We also report the corresponding performance of the least squares estimates (LSE) and the periodogram maximize estimates (PME) for comparison.

4.1.1. Performance under two specific heavy tail noise distributions

In this subsection, we observe the performance of the proposed estimators under two specific heavy tail noise distributions. We assume the following two standard heavy tail distributions for the noise sequence $\{\varepsilon_t\}$:

I. A mixture normal distribution: $0.6\,N(0,\sigma_1^2) + 0.4\,N(0,\sigma_2^2)$.
II. A Student's t random variable with m degrees of freedom.

In the simulation model, we take $\alpha_1^0 = 1.5, \beta_1^0 = 1.2, \omega_1^0 = 0.4, \alpha_2^0 = 0.9, \beta_2^0 = 0.3, \omega_2^0 = 0.6$ and the sample size is fixed at $n = 200$. For the mixture normal noise sequence, we consider the 2 combinations of (σ_1, σ_2), namely (0.6,0.01) (low variance case) and (1.0,0.1) (high variance case). For the t distribution, we consider 2 different degrees of freedoms, namely 10 and 30, once again giving 2 different levels of noise variance.

The results for the mixture normal noise are presented in Table 1 and the results for the t noise are presented in Table 2.

From the simulation results, we observe that the proposed robust methods perform quite well for the two examples of heavy tailed noise distributions considered. Among the proposed methods, the best performance is observed for HUSM and ANSM. These methods clearly outperform the traditional LSE and PME methods in terms of giving lower MSE. Performance of all the methods deteriorates as the underlying noise variance increases.

4.1.2. Effect of width of finite width time window on performance

In this subsection, we perform simulations to observe the effect of width of the finite width time window on the performance of the proposed estimators. We consider the simulation model with $\alpha_1^0 = 1.5, \beta_1^0 = 1.2, \omega_1^0 = 0.4, \alpha_2^0 = 0.9, \beta_2^0 = 0.3, \omega_2^0 = 0.6$ and the noise having a mixture normal distribution, $0.6\,N(0,\sigma_1^2) + 0.4\,N(0,\sigma_2^2)$, $\sigma_1 = 1.0$, $\sigma_2 = 0.1$. We consider 3 different widths of the time window, namely, 100, 500 and 800. The results are presented in Table 3. The first row in each of the cells gives the average estimates and the second row gives the mean square errors over the 100 simulation runs.

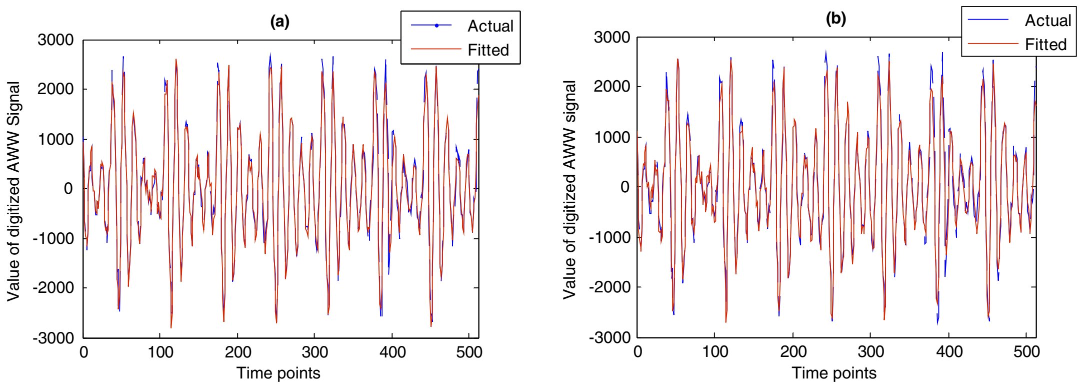

Fig. 3. (a) AWW fit using HUSM, (b) AWW fit using RASM.

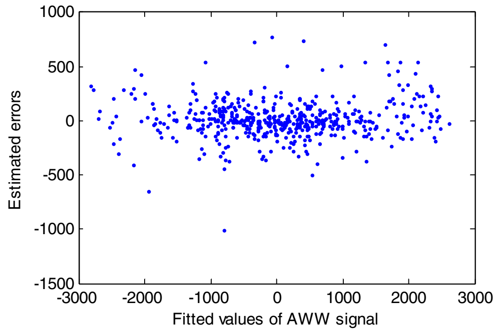

Fig. 4. Plot of the estimated noise against the fitted values for AWW fit.

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We observe that as width of the time window increases, MSEs of all the methods decrease and give very accurate estimates. Significant gain in using the proposed robust methods over PME is observed for the frequency parameters. For almost all the parameters and at all window widths considered, the proposed methods perform better than the LSE and PME.

4.1.3. Performance under integer/non-integer relationship among frequencies

In this subsection, we perform simulations to observe the performance of the proposed estimators when there is integer or non-integer relationship among the frequencies of the model. We consider the following three different scenarios;

Case I: $\alpha_1^0 = 1.5, \beta_1^0 = 1.2, \omega_1^0 = 0.4, \alpha_2^0 = 0.9, \beta_2^0 = 0.3, \omega_2^0 = 0.8$ (Integer relation: $2\omega_1^0 = \omega_2^0$).
Case II: $\alpha_1^0 = 1.5, \beta_1^0 = 1.2, \omega_1^0 = 0.9, \alpha_2^0 = 0.9, \beta_2^0 = 0.3, \omega_2^0 = 0.3$ (Integer relation: $\omega_1^0 = 3\omega_2^0$).
Case III: $\alpha_1^0 = 1.5, \beta_1^0 = 1.2, \omega_1^0 = 0.4, \alpha_2^0 = 0.9, \beta_2^0 = 0.3, \omega_2^0 = 0.6$.

(non-integer relation: $1.5\omega_1^0 = \omega_2^0$).

In each of the above cases, we take $n = 200$ and noise having a mixture normal distribution as in Subsection 4.1.2. The results for the 3 cases are given in Tables 4–6.

From the simulation results of models with integer relationship among the frequencies, we observe that the gain in using the proposed robust methods over traditional PME is quite substantial. The proposed methods perform quite well with integer or non-integer relationships existing among the frequencies and once again clearly outperform the traditional LSE and PME methods.

4.1.4. Performance under outlier contamination

In order to observe the possible effect of outliers present in the data on the performance of the proposed estimators, we perform simulations on outlier contaminated datasets. We contaminate the datasets obtained in 4.1.1 with outliers and

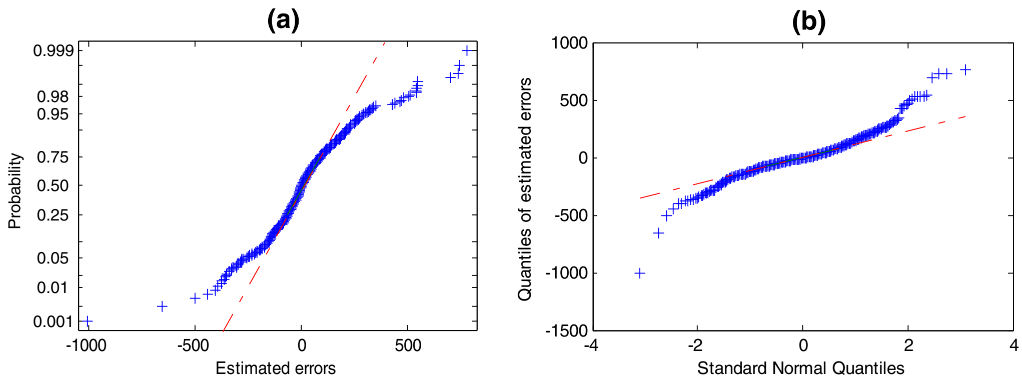

Fig. 5. (a) Normal probability plot and (b) QQ plot of AWW fit errors.

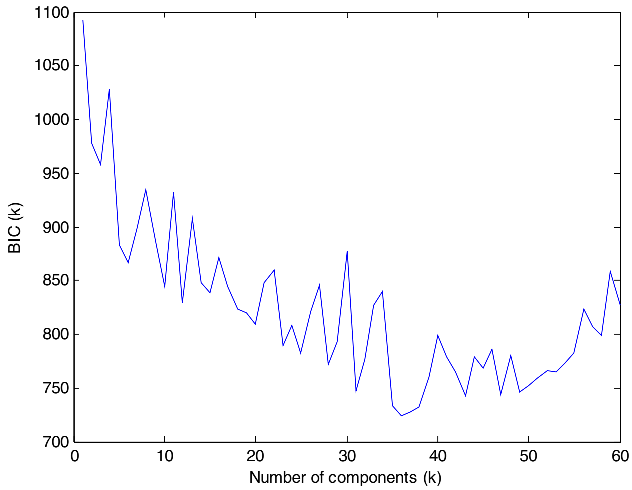

Fig. 6. Plot of the robust BIC function for the AHH signal.

estimate the signal parameters from these outlier contaminated datasets using the proposed estimators and the LSE and PME. Outlier contaminated datasets are obtained by selecting at random observations from a no outlier dataset and adding a predetermined outlier contamination number. In all the outlier contaminated datasets, we take the contamination number as 10. The results for the outlier case simulations are presented in Table 7.

It is observed that the performance of the proposed methods is quite robust with respect to outliers present in the data. As expected the performance of the LSE deteriorates drastically in the presence of nominal percentage of outliers in the data. The proposed methods are reasonably robust with respect to outliers present in the data and perform satisfactorily and much better than the usual LSE and PME with outliers present in the data. In almost all the cases the proposed methods perform better than the traditional LSE and PME methods.

4.2. Real signal data analysis

In this subsection we present some real signal data analysis using the proposed GA based sequential procedure. We consider the digitized speech signals of AWW and AHH sounds of [11]. Since the number of signal components is unknown for these signals, we first estimate the number of signals using robust Bayesian Information Criterion (BIC).

4.2.1. Sequential M-estimation fitting of AWW

For the observed AWW digitized signal, the estimated number of superimposed signals is 44. The plot of the robust BIC function is given in Fig. 2.

We now use the proposed real-coded GA based sequential M-estimation technique for estimation of all the signal parameters corresponding to the 44 components. The fit of the AWW signal using Huber sequential M-estimation (HUSM) approach and the fit using Ramsey sequential M-estimation (RASM) approach is given in Fig. 3. The other sequential M-estimation approaches give similar fits. Plot of the estimated noise against the fitted values is given in Fig. 4. The normal probability plot and the QQ plot of the estimated noise for the HUSM fit are given in Fig. 5. Both the normal probability plot and QQ plot are indicative of heavy tail noise and hence the proposed robust sequential M-estimation technique is more appropriate than usual LSE or PME.

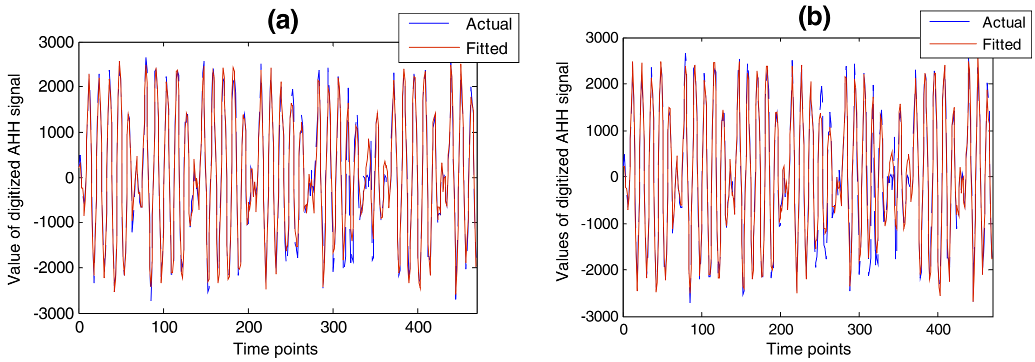

Fig. 7. (a) AHH fit using HUSM, (b) AHH fit using RASM.

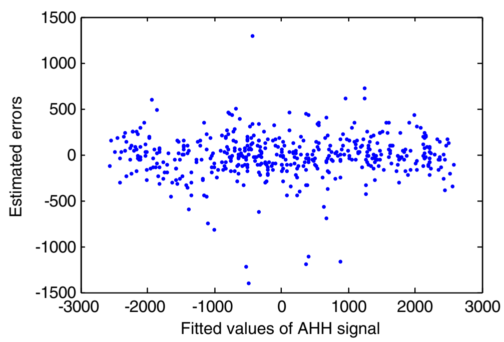

Fig. 8. Plot of the estimated noise against the fitted values for AHH fit.

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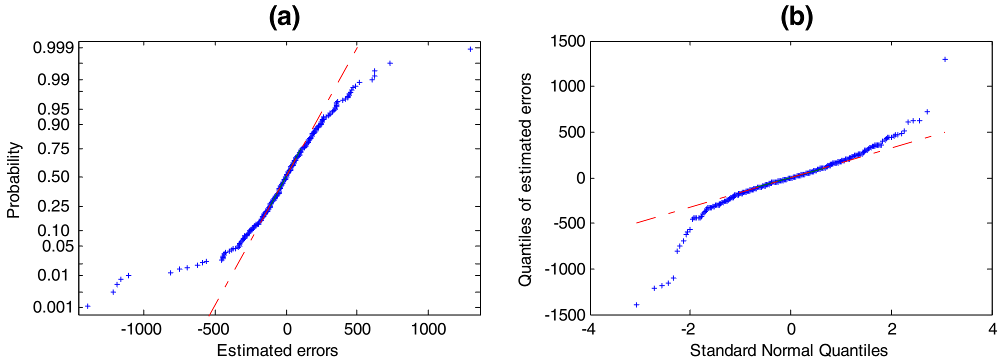

Fig. 9. (a) Normal probability plot and (b) QQ plot of AHH fit errors.

4.2.1. Sequential M-estimation fitting of AHH

We next consider the digitized AHH signal. For the observed signal we estimate the number of component signals as 36. The plot of the robust BIC function is given in Fig. 6. We apply the proposed real-coded GA based M-estimation technique, in a sequential manner, for estimation of all the signal parameters corresponding to the 36 components. Fits of the AHH signal using HUSM and RASM approaches are given in Fig. 7. Plot of the estimated noise against the fitted values is given in Fig. 8. The normal probability plot and the QQ plot of the estimated noise for the HUSM fit are given in Fig. 9. As in the AWW fit, once again both the normal probability plot and QQ plot are indicative of heavy tail noise and hence the proposed robust sequential M-estimation technique is more appropriate.

Fitting of the real life speech signals using the real coded GA based sequential M-estimation technique indicates satisfactory performance of the proposed procedure for fitting signals with large number of components. The proposed GA based sequential robust methods are able to efficiently resolve the large number of signal components sequentially. The error plots further indicate appropriateness and usefulness of the proposed robust estimation technique for the given real life signals.

5. Conclusions

In this paper we propose real-coded GA based sequential robust M-estimation technique for estimation of parameters of nonlinear sinusoidal signal models. The proposed approach of sequential estimation can be applied for estimation of parameters of real valued as well as complex valued sinusoidal models. The proposed estimation technique uses elitist generational GA and robust M-estimation, in a sequential manner, for estimation of the signal parameters. Since the proposed approach estimates the signal parameters in a sequential manner, the method can be easily applied for analyzing real life signals with large number of superimposed sinusoidal components. Furthermore, as the proposed approach is based on M-estimation technique, the estimates obtained are robust to heavy tailed noise and presence of outliers in the data. Extensive simulations and real life signal analysis indicates usefulness and satisfactory performance of the proposed approach.

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