Comments on: Progressive Censoring Methodology: An apprisal by N. Balakrishnan

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Professor Balakrishnan should be congratulated for his excellent survey article on the recent advancement on progressive censoring. Although progressive censoring was introduced in 1965 by Herd, but I must say it became very popular since the appearance of the book by Balakrishnan and Aggarwala in 2000. Extensive work has been done since then and Professor Balakrishnan has nicely reviewed different aspects of progressive censoring in this article. I have two general comments on progressive censoring and I will briefly mention those here.

Parametric Inference: Extensive work has been done on the parametric inferences for different lifetime distributions, in case of progressive censoring. Interestingly, most of the work are from the frequentest view point. Not much attention has been paid on the Bayesian inferences of the unknown parameters. I would like to highlight some of the positive aspects of the Bayesian inferences in case of progressive censoring.

In case of scaled exponential distribution, if the probability density function (PDF) has the following form;

$$f(x;\lambda) = \lambda e^{-\lambda x}; \qquad x > 0, \tag{1}$$

then for fixed $(R_1, \ldots, R_m)$, based on the progressively censored sample $x_{1:m:n}, \ldots, x_{m:m:n}$, the maximum likelihood estimator (MLE) of λ is

$$\widehat{\lambda} = \frac{m}{\sum_{i=1}^{m}(R_i+1)x_{i:m:n}}. \tag{2}$$

In this case the distribution of $\widehat{\lambda}$ can be obtained using the fact that $\sum_{i=1}^{m}(R_i+1)X_{i:m:n}$ has a gamma distribution. Therefore, $100(1-\alpha)\%$ confidence interval becomes;

$$\left(\frac{\chi^2_{2m,\alpha/2}}{2\sum_{i=1}^{m}(R_i+1)x_{i:m:n}}, \quad \frac{\chi^2_{2m,1-\alpha/2}}{2\sum_{i=1}^{m}(R_i+1)x_{i:m:n}}\right). \tag{3}$$

Now for Bayesian inferences it is natural to assume the conjugate gamma prior on λ. If it is assumed that the prior $\pi(\lambda)$ has the following form;

$$\pi(\lambda) \propto \lambda^{a-1}e^{-b\lambda}; \qquad \lambda > 0, \tag{4}$$

then it can be easily shown that the Bayes estimate of λ under squared error loss function is;

$$\widehat{\lambda}_{Bayes} = \frac{a+m}{b+\sum_{i=1}^{m}(R_i+1)x_{i:m:n}}. \tag{5}$$

In this case $100(1-\alpha)\%$ credible interval of λ is

$$\left(\frac{\chi^2_{2(a+m),\alpha/2}}{2(b+\sum_{i=1}^{m}(R_i+1)x_{i:m:n})}, \quad \frac{\chi^2_{2(a+m),1-\alpha/2}}{2(b+\sum_{i=1}^{m}(R_i+1)x_{i:m:n})}\right). \tag{6}$$

Interestingly, as expected under the non-informative priors, *i.e.* when $a = b = 0$, the MLE and Bayes estimate are equal. Moreover, the $100(1-\alpha)\%$ confidence and credible intervals also become equal in this case.

But Bayesian approach has its advantages in the following cases. For example, let us consider (i) hybrid censored type-II progressive censoring, see for example Childs, Chandrasekar and Balakrishnan (2007) or Kundu and Joarder (2006), (ii) Type-II progressively censored competing risks model, see for example Kundu, Kannan and Balakrishnan (2004), or (iii) Type-I progressive censoring. In all the above cases, even when the lifetime distribution of the item are exponential, the exact distributions of the corresponding MLEs are quite complicated. The exact confidence intervals can not be obtained in explicit forms. On the other hand the Bayes estimates (with respect to squared error loss function) and $100(1-\alpha)\%$ credible intervals can be obtained explicitly in all the above cases. Similar gains are expected for other distributions also. More work is needed in this direction.

Optimal Censoring Plan: As Professor Balakrishnan has indicated in this article that finding the optimal censoring plan, *i.e.* finding the optimal $\{R_1, \ldots, R_m\}$ is an important practical problem. For one-parameter family, the problem is not difficult. Optimal $\{R_1, \ldots, R_m\}$ can be obtained by maximizing the Fisher information of the unknown parameter with respect to different choices of $\{R_1, \ldots, R_m\}$. In case of two-parameter family, the problem is not that trivial. For example, in case of two-parameter Weibull distribution, some of the choices are; (i) maximize the determinant of the Fisher information matrix, or (ii) maximize the trace of the Fisher information matrix. But in both the cases the optimal solution is not scale invariant. In case of Weibull distribution, if all the observations are multiplied by a known constant, the optimal choice of $\{R_1, \ldots, R_m\}$ might change, which may not be very desirable. Due to this, I feel minimizing the variance of the p-th percentile estimator or some of its variants may be more appropriate, see for example Zhang and Meeker (2005) or Kundu (2007) in this connection. More attention is needed in this issue.

References

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