Time Truncated Acceptance Sampling Plans for Generalized Exponential Distribution

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Abstract

Acceptance sampling plans for generalized exponential distribution when the lifetime experiment is truncated at a pre-determined time, are provided in this manuscript. The tables are provided for the minimum sample size required to ensure a certain median life of the experimental unit when the shape parameter is two. The operating characteristic function values of the sampling plans and the associated producer's risks are also presented. It is shown that the tables presented here can be used if instead of median life, other percentile life is chosen as the criterion or if the shape parameter is not two. Examples are provided for illustrative purposes.

Keywords: Acceptance sampling plan; Operating characteristic function value; Median and percentile points; Consumer and Producer's risks.

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1 Introduction

Acceptance sampling plan is an essential tool in the Statistical Quality Control. In most of the statistical quality control experiment, it is not possible to perform hundred percent inspection, due to various reasons. The acceptance sampling plan was first applied in the US Military for testing the bullets during World War II. For example, if every bullet was tested in advance, no bullets were available for shipment, and on the other hand if no bullets were tested, then disaster might occur in the battle field at the crucial time. Acceptance sampling plan is a 'middle path' between hundred percent inspection and no inspection at all.

In the acceptance sampling plan, a consumer decides to accept or reject the lot based on a random sample collected from the lot. The problem can be formulated as follows. Suppose, n units are placed in a life test and the experiment is stopped at a predetermined time T. The number of failures till the time point T is observed, and suppose it is m. The lot is accepted if m is less than or equal to the acceptance number, say c, otherwise it is rejected. Therefore, any acceptance sampling plan provides n, the number of units on test, and the acceptance number c. For a given acceptance sampling plan, the consumer's and producer's risk are the probabilities that a bad lot is accepted and a good lot is rejected, respectively. Usually, with every acceptance sampling plan, the associated consumer's and producer's risks are also presented.

The standard approach to handle this problem is to assume a parametric model for the lifetime distribution and then find the minimum sample size needed to ensure a certain mean/ median life of the lifetime distribution of the items in the lot, when the experiment is stopped at a pre-determined time, say T. Therefore, in any time truncated acceptance sampling plan, other than n, c, T, there will be another component, say θ_m, where θ_m is the specified mean/ median life of the distribution and it acts as a *quality* parameter for the lifetime distribution under consideration.

Extensive work has been done on the acceptance sampling plan since its inception. Different parametric forms have been assumed and extensive tables are available for different parametric values and for different sample sizes. Acceptance sampling plans based on truncated life tests for exponential distribution was first discussed by Epstein [5], see also Sobel and Tischendrof [15]. The results were extended for the Weibull distribution by Goode and Kao [6]. Gupta and Groll [7] and Gupta [8] provided extensive tables on acceptance sampling plans for gamma, normal and log-normal distributions. Kantam and Rosaiah [11], Kantam *et al.* [12], Rosaiah and Kantam [14], and Balakrishnan *et al.* [4] provide the time truncated acceptance plans for half-logistics, log-logistics, Rayleigh and generalized Birnbaum-Saunders distributions respectively.

Recently, it is observed that the generalized exponential distribution has been used quite effectively to analyze lifetime data. In many cases it is observed that it provides a better fit than the Weibull, gamma, log-normal or generalized Rayleigh distributions. The main aim of this paper is to develop the time truncated acceptance sampling plans for the generalized exponential distribution and compare the results with the existing ones. It is known that for the generalized exponential distribution mean is not in a compact form, but the median is in a compact form. Moreover, it is suggested by Gupta [8] that for a skewed distribution the median represents a better *quality* parameter than the mean. On the other hand, for a symmetric distribution, mean is preferable to use as a *quality* parameter. Since generalized exponential distribution is a skewed distribution we prefer to use the median as the quality parameter, and it will be denoted by θ_m. In this manuscript, based on the assumptions that the lifetime follows generalized exponential distribution, we present a methodology to find the minimum sample size required to ensure a specified median life of the items under study. It is further assumed that the life testing experiment will be stopped at a pre-determined time T, if more than c failures does not occur before that stipulated time. Otherwise the experiment is stopped as soon as $(c+1)$-th failure occurs.

The lot is accepted if the specified median is greater than a specified quantity (to ensure a certain *quality* of the product) with a pre-fixed probability $1-P^*$, specified by the consumer and it is known as the consumer's risk. For a given acceptance sampling plan, a good lot might be rejected with a non-zero probability and that is known as the producer's risk. For different acceptance plans, we present the associated producer's risk also, based on the operating characteristic function values. In practice, instead of median life the consumer may prefer to characterize the *quality* based on some other percentile point (may be 75-th percentile point). We discuss how to use the present tables (based on medians) for other percentile points also. Two examples have been discussed for illustrative purposes.

Rest of the paper is organized as follows. In section 2, we give a brief description of the generalized exponential (GE) distribution. Acceptance sampling plans based on the median are provided in section 3. How these tables can be used for other percentile points also are discussed in section 4. An approximation of the minimum sample size is provided in section 5. Descriptions of the tables and illustrative examples are provided in section 6 and finally we conclude the paper in section 7.

2 Generalized Exponential Distribution

The two-parameter generalized exponential distribution has the following probability density function (PDF);

$$f(x;\alpha,\lambda) = \frac{\alpha}{\lambda} e^{-\frac{x}{\lambda}} \left(1 - e^{-\frac{x}{\lambda}}\right)^{\alpha-1}; \qquad x > 0. \tag{1}$$

Here $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters respectively. From now on a generalized exponential random variable with the PDF (1) will be denoted by GE(α,λ).

As the name suggests, this is an extension of the exponential distribution, similarly as the Weibull and gamma distribution, but in different ways.

The two-parameter generalized exponential distribution was originally introduced by Gupta and Kundu [9] as a possible alternative to the well known Weibull and Gamma distributions. Since then extensive work has been done on this distribution. It is further observed that the generalized exponential distribution can be used quite effectively in many circumstances, in place of log-normal or generalized Rayleigh distribution also. Statistical inferences, order statistics, closeness properties with other distributions have been discussed by several authors. The readers are referred to the recent review article by Gupta and Kundu [10] for a current account on the generalized exponential distribution.

It is observed that the shape of the PDF and hazard functions (HF) of the generalized exponential distribution depend on the shape parameter α. The PDF is a decreasing function or an unimodal function if $0 < \alpha \leq 1$ or $\alpha > 1$ respectively. The HF of the generalized exponential distribution is a decreasing function if $\alpha < 1$ and for $\alpha > 1$ it is an increasing function. The PDFs and HFs of the generalized exponential distribution are very similar to those of Weibull and gamma distributions. It is also observed in different studies that generalized exponential distribution might fit better than Weibull or gamma distribution in some cases. In different studies it has been shown that for certain ranges of the parameter values, it is extremely difficult to distinguish between GE and Weibull, gamma, log-normal, generalized Rayleigh distributions.

The cumulative distribution function (CDF) of GE(α, λ) is given by

$$F_{GE}(x; \alpha, \lambda) = \left(1 - e^{-x/\lambda}\right)^{\alpha}. \tag{2}$$

If $X \sim$ GE(α, λ),then the mean and variance of X can be expressed as

$$E(X) = \lambda \left[\psi(\alpha + 1) - \psi(1)\right], \quad V(X) = \lambda^2 \left[\psi'(1) - \psi'(\alpha + 1)\right]. \tag{3}$$

Here $\psi(\cdot)$ and $\psi'(\cdot)$ are the digamma and polygamma functions respectively, *i.e.*

$$\psi(u) = \frac{d}{du}\,\Gamma(u), \quad \psi'(u) = \frac{d}{du}\,\psi(u), \quad \text{where} \quad \Gamma(u) = \int_0^\infty x^{u-1}e^{-x}dx.$$

It is clear that both the mean and variance are increasing functions of λ. The p-th percentile point of GE(α, λ), say $\theta_p = F_{GE}^{-1}(p; \alpha, \lambda)$ is given by

$$\theta_p = -\lambda \ln\left(1 - p^{\frac{1}{\alpha}}\right). \tag{4}$$

Therefore, the median of GE(α, λ) becomes;

$$\theta_m = -\lambda \ln\left(1 - \left(\frac{1}{2}\right)^{\frac{1}{\alpha}}\right). \tag{5}$$

From now on unless **otherwise** mentioned, we treat θ_m as the *quality* parameter. From (5) it is clear that for fixed $\alpha = \alpha_0$, $\theta_m \geq \theta_m^0 \Leftrightarrow \lambda \geq \lambda_m^0$, where

$$\lambda_m^0 = \frac{\theta_m^0}{-\ln\left(1 - \left(\frac{1}{2}\right)^{\frac{1}{\alpha_0}}\right)}. \tag{6}$$

Note that λ_m^0 also depends on α_0, for brevity we do not make it explicit. Now we develop the acceptance sampling plans for the generalized exponential distribution to ensure that the median lifetime of the items under study exceeds a pre-determined *quality* provided by the consumer say θ_m^0, equivalently λ exceeds λ_m^0, with a minimum probability P^*.

3 Acceptance Sampling Plans

In this section, we provide the acceptance sampling plans under the assumptions that lifetime distribution follows a two-parameter GE(α, λ). It is further assumed that the shape parameter α is known. In acceptance sampling plans, usually the test terminates at a pre-specified time T and the number of failures (not the actual failure times) during this time point **are** noted. Based on the number of failed items, a confidence limit (lower) on the median (in this case) is formed. Alternatively, based on the number of failures, it is then desired to establish a specified median life with a given probability of at least P^*, specified by the consumer. In this proposed acceptance sampling plans, the decision to accept the specified median takes place, if and only if the number of failures m at the end of the time point T does not exceed c, the acceptance number. Naturally, if more than c failures already occurs before T, there is no point in continuing the test. In this case as soon as $(c+1)$-th failure takes place before time point T, the test terminates with the decision not to accept the lot.

Under these circumstances, one wants to find out the smallest sample size necessary to achieve these objectives. Therefore, as mentioned earlier an acceptance sampling plan consists of (a) the number of units n to be used for testing purposes, (b) the acceptance number c, (c) the ratio $\frac{T}{\lambda_m^0}$, where λ_m^0 is same as defined in (6), corresponds to θ_m^0, the specified median life of the given population GE(α, λ) and T is the maximum testing time. The shape parameter α_0 and the prescribed (bare) median life are provided before hand. The choice of c, T and n will be made in general from the producer's risk, which is the probability of rejecting a good lot, *i.e.*, a lot for which the true median life is greater than or equal to the specified median life. On the other hand the consumer's risk is fixed in this formulation and can not exceed 1 - P^*. Therefore, it can be seen that P^* is the confidence level in the sense that the chance of rejecting a lot having median $\theta \leq \theta_m^0$ is at least P^*.

Finally it should be pointed out clearly that whenever we are talking about a lot, it means a lot of very large size, so that binomial distribution can be used. Moreover, the acceptance and rejection of the lot are equivalent to the acceptance or rejection of the hypothesis on the *quality* parameter, namely $\theta \geq \theta_m^0$. The problem can be described mathematically as follows; given a number $0 < P^* < 1$, an experimental (maximum) time point T, the median value θ_m^0 and an acceptance number c, we want to find the smallest positive integer n, so that if the observed number of failures m does not exceed c, it is ensured that $\theta_m \geq \theta_m^0$ with a minimum probability P^*.

In this case for given c, P^*, T, α_0 and θ_m^0, we need to find n, the smallest positive integer, which satisfies the inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - P^*, \tag{7}$$

where

$$p = F_{GE}(T; \alpha, \lambda) = \left(1 - e^{-\frac{T}{\lambda_m}}\right)^{\alpha}. \tag{8}$$

It is clear that p depends only on the ratio $\frac{T}{\lambda_m^0}$. It is a monotonically increasing function of $\frac{T}{\lambda_m^0}$ and it is a decreasing function of λ_m^0. Because of the monotonicity, it may be observed that in (7) we can establish with probability P^* that $F_{GE}\left(\frac{T}{\lambda}\right) \leq F_{GE}\left(\frac{T}{\lambda_m^0}\right)$, which implies $\lambda \geq \lambda_m^0$. Therefore, if n is the smallest integer which satisfies (7), then for the same n, replacing p with $F_{GE}(T; \alpha_0, \lambda)$, (7) will satisfy for all $\lambda \geq \lambda_m^0$. Note that p as defined in (8) depends only on the ratio $\frac{T}{\lambda_m^0}$ for fixed $\alpha = \alpha_0$.

In Table 1, we present the minimum values of n, satisfying (7) for $P^* = 0.75$, 0.90, 0.95, 0.99 and for $\frac{T}{\lambda_m^0} = 0.628$, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, keeping α_0 fixed. We mainly choose these P^* and $\frac{T}{\lambda_m}$ values so that we can compare our results with those obtained by Gupta and Groll [7], Gupta [8], Kantam *et al.* [12], Baklizi and El Masri [2] and Balakrishnan *et al.* [4].

3.1 Operating Characteristic Functions of the Sampling Plans $\left(n, c, \frac{T}{\lambda_m^0}\right)$

The operating characteristic (OC) function of the sampling plan $\left(n, c, \frac{T}{\lambda_m^0}\right)$ provides the probability of accepting the lot. For the above acceptance sampling plan this probability is given by

$$OC(p) = P\{\text{Accepting a lot}\} = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} = 1 - B_p(c+1, n-c), \tag{9}$$

here $B_p(c+1, n-c)$ is the incomplete beta function and here p is as defined in (8). $B_p(c+1, n-c)$ is an increasing function of p, and therefore, $OC(p)$ is a decreasing function of p. Moreover, for fixed T, p is a decreasing function of $\lambda \geq \lambda_m^0$. Based on (9), for fixed $\alpha = \alpha_0$, and c, the operating characteristic function values as a function of $\dfrac{\lambda}{\lambda_m^0}$ are presented in Table 2, for different values of P^* and for the given acceptance sampling plans.

3.2 Producer's Risk

The producer's risk is the probability of rejection of the lot, when $\theta_m \geq \theta_m^0$, or equivalently $\lambda \geq \lambda_m^0$. It can be computed as follows;

$$\begin{aligned} PR(p) &= P\{\text{Rejecting a lot}\} = 1 - P\{\text{Accepting the Lot} | \lambda > \lambda_m^0\} \\ &= \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} = B_p(c+1, n-c). \end{aligned}$$

For the given sampling plan, and for a given value of the producer's risk, say γ, one may be interested in knowing the minimum value of $\dfrac{\lambda}{\lambda_m^0}$, that will ensure the producer's risk to be at most γ. The $\dfrac{\lambda}{\lambda_m^0}$ is the smallest quantity for which $p = \left(1 - e^{-\frac{T}{\lambda}}\right)^{\alpha_0} = \left(1 - e^{-\frac{T}{\lambda_m^0} \times \frac{\lambda_m^0}{\lambda}}\right)^{\alpha_0}$ satisfies the inequality

$$PR(p) = \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \leq \gamma. \tag{10}$$

For a given acceptance sampling plan $\left(n, c, \dfrac{T}{\lambda_m^0}\right)$, and for a given P^*, the minimum value of $\dfrac{\lambda}{\lambda_m^0}$, satisfying (10) are computed and presented in Table 3.

4 Extensions and Approximations

4.1 Sampling Plans for Other Percentile Points

So far we have discussed the acceptance sampling plans for a given median life. Now in this section we want to describe how these tables can be used for other percentile points also. Suppose, it is desired to obtain the acceptance sampling plans for the given p-th percentile point of GE(α, λ) given by

$$\theta_p = -\lambda \ln\left(1 - p^{\frac{1}{\alpha}}\right). \tag{11}$$

In this case, we are treating θ_p as the *quality* parameter and it is desired that given $\alpha = \alpha_0$, we want an acceptance sampling plans such that $\theta_p \geq \theta_p^0$, equivalently $\lambda_p \geq \lambda_p^0$, where

$$\lambda_p = \frac{\theta_p}{-\ln\left(1 - p^{\frac{1}{\alpha_0}}\right)}, \quad \text{and} \quad \lambda_p^0 = \frac{\theta_p^0}{-\ln\left(1 - p^{\frac{1}{\alpha_0}}\right)}.. \tag{12}$$

Let us denote the time truncation parameter as $\widetilde{T}$ which may be different than T. Therefore, here for given c, P^*, $\widetilde{T}$, α_0, and θ_p^0, we want to find n, the smallest positive integer n, which satisfies

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - P^*, \tag{13}$$

where

$$p = \left(1 - e^{-\frac{\widetilde{T}}{\lambda_p^0}}\right)^{\alpha_0}. \tag{14}$$

Therefore, Table 1 (based on median) can be used for other percentiles also if $\dfrac{\widetilde{T}}{\lambda_p^0} = \dfrac{T}{\lambda_m^0}$.

4.2 Sampling Plans for Other Shape Parameters

In Table 1 we have presented the sampling plans when $\alpha_0 = 2$. But the natural question is how to use this table for other shape parameters also. Let α_0 denote the tabulated value and α denote the true value. In this case also let us denote by $\widetilde{T}$, the time truncation parameter associated with the new α value, which may be different than T. In this subsection only let us denote

$$\lambda_m = -\frac{\theta_m}{\ln\left(1-\frac{1}{2}\right)^{\frac{1}{\alpha}}} \tag{15}$$

and λ_m^0 is same as defined in (6). Therefore, when the shape parameter is α,

$$P\{\text{Accepting the Lot}\} = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}, \tag{16}$$

here $p = \left(1 - e^{-\frac{\widetilde{T}}{\lambda_m}}\right)^{\alpha}$. Now equating

$$p = \left(1 - e^{-\frac{\widetilde{T}}{\lambda_m}}\right)^{\alpha} = \left(1 - e^{-\frac{T}{\lambda_m^0}}\right)^{\alpha_0} \tag{17}$$

we obtain

$$\frac{\widetilde{T}}{\lambda_m} = -\ln\left[1 - \left(1 - e^{-\frac{T}{\lambda_m^0}}\right)^{\frac{\alpha_0}{\alpha}}\right]. \tag{18}$$

Therefore, the same table can be used for other α values also, using $\frac{\widetilde{T}}{\lambda_m}$ as given in (18), instead of $\frac{T}{\lambda_m^0}$

4.3 Approximations

As it has been mentioned earlier that Tables 1 and 4 have been obtained using a trial and error method on n, and using the monotonicity property of n with respect to p. In all these calculations, it has been assumed that the lot is very large and p is not very small, so that the binomial approximation can be used. If p is very small and n is large, then binomial distribution is approximated by the Poisson distribution with mean $\beta = np$. Therefore, (7) can be written as

$$\sum_{i=0}^{c} \frac{e^{-\beta}\beta^i}{i!} \le 1 - P^*, \tag{19}$$

where $\beta = n\left(1 - e^{-\frac{T}{\lambda_m^0}}\right)^{\alpha_0}$. We have

$$\sum_{i=0}^{c} \frac{e^{-\beta}\beta^i}{i!} = 1 - G_{c+1}(\beta, 1), \tag{20}$$

where $G_k(x,\delta)$ denotes the cumulative distribution function of a gamma distribution with the shape and scale parameters as k and δ respectively, *i.e.*

$$G_k(x;\delta) = \frac{\delta^k}{\Gamma(k)} \int_0^x t^{k-1} e^{-\delta t} dt. \tag{21}$$

Therefore, if γ_{c+1,P^*} denotes the P^* percentage point of a standardized (scale parameter one) gamma variable with the shape parameter $c+1$, then

$$n \approx \left[\frac{\gamma_{c+1,P^*}}{\left(1 - e^{-\frac{T}{\lambda_m^0}}\right)^{\alpha_0}} \right] + 1, \tag{22}$$

here $[x]$ represents the largest integer less than or equal to x. Now using the relation between the gamma and χ^2 random variables, we immediately obtain

$$n \approx \left[\frac{\chi^2_{2c+2,P^*}}{2\left(1 - e^{-\frac{T}{\lambda_m^0}}\right)^{\alpha_0}} \right] + 1, \tag{23}$$

here χ^2_{2c+2,P^*} denotes the P^* percentage point of a χ^2 variable with degrees of freedom $2c+2$.

4.4 Descriptions of Tables and Examples

In Table 1 we provide the minimum sample size required to ascertain that the median life exceeds θ_m^0 with probability at least P^*, the corresponding acceptance number c and when $\alpha_0 = 2$. It has been prepared by using (a) trial and error method on n, (b) monotonicity property of n with respect to p, and (c) binomial probabilities. For example in Table 1, when $P^* = 0.90$, $\frac{T}{\lambda_m^0} = 1.571$, $c = 2$, the corresponding table value is 6. It implies that out of 6 items, if 2 items fail before time point T, then a 90% upper confidence interval of λ will be $(\frac{T}{1.571}, \infty)$. In other words, if out of 9 items, less than or equal to 2 items fail before time point T, then we can accept the lot with probability 0.90 with the assurance that

$$\lambda \geq \frac{T}{1.571} \quad \Leftrightarrow \quad \theta_m \geq \frac{T}{1.571} \times \left(-\ln\left(1 - \sqrt{\frac{1}{2}}\right)\right) = T \times 0.782.$$

Table 2 represents operating characteristic function values for the time truncated acceptance sampling plan obtained from Table 1, for different values of P^* and for different values of $\frac{\lambda}{\lambda_m^0}$, when $c = 2$. For example, when $P^* = 0.90$, $\frac{T}{\lambda_m^0} = 1.571$, $c = 2$, the table value is 0.9556 when $\frac{\lambda}{\lambda_m^0} = 4$. It implies, if one accepts the above time truncated acceptance sampling plan, *i.e.* the lot is accepted if out of 6 items, less than or equal to 2 items fail before time point T, then if $\lambda \geq 4 \times \frac{T}{1.571}$ or $\theta_m \geq T \times 4 \times 0.782$, then the lot will be accepted with probability at least 0.9556.

Table 3 represents the minimum ratio of the true median life to the specified median life for the acceptance of a lot with the producer's risk 0.05 and when $\alpha_0 = 2$. In this case for example, when the consumer's risk is 10%, *i.e.* $P^* = 0.90$, $c = 2$, $\frac{T}{\lambda_m^0} = 1.571$, the table value $\frac{\lambda}{\lambda_m^0} = \frac{\theta_m}{\theta_m^0} = 3.9$. It implies if $\theta_m \geq T \times 0.782 \times 3.9$, then with $n = 6$ (obtained from Table 1) and $c = 2$, the lot will be rejected with probability less than or equal to 0.05.

Example 1: Suppose it is assumed that the lifetime distribution of the product under study follows a generalized exponential distribution with $\alpha_0 = 2$. An experimenter wants to know the minimum sample size to be considered to make a decision (accepting or rejecting the lot), when he/ she wants the true median life should be at least $\theta_m^0 = 1000$ units with the probability of accepting a bad lot less than or equal to 0.01 or $P^* = 0.99$. It is also assumed that the maximum affordable time is 767 units and the maximum affordable number of failures is 2. Since

$$\lambda_m^0 = \frac{1000}{-\ln\left(1 - \left(\frac{1}{2}\right)^{\frac{1}{2}}\right)} = 814.37, \quad \text{and} \quad \frac{T}{\lambda_m^0} = 0.942, \tag{24}$$

from Table 1 we obtain $n = 15$. Therefore, out of 15 items if not more than 2 items fail before $T = 767$ units of time, the lot can be accepted with the assurance that the true median life is at least 1000 with the probability 0.99.

Example 2: Continuing with the same example, suppose instead of median life the practitioner wants that the 75-th percentile life should exceed 1275 units, where the affordable time is 1000 units and the affordable number of failures is 5. In this case

$$\lambda_p^0 = \frac{1275}{-\ln(1-\sqrt{0.75})} = 634.2 \quad \Rightarrow \quad \frac{T}{\lambda_p^0} = 1.57. \tag{25}$$

If we want $P^* = 0.95$, and $c = 5$, then using Table 1 we get $n = 14$, using the column corresponds $\frac{T}{\lambda_m^0} = 1.571$.

Example 3: Now we consider a data set which was already considered by Wood [17], Rosaiah and Kantam [14] and recently by Balakrishnan *et al.* [4]. The data set represents the failure time in hours of a software, which represents the lifetimes from the starting of the execution of the software until which the failure of the software is experienced. We have the following observations; 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, 5218. The problem can be stated as follows. If the assured median life is 1000 hours, then with $P^* = 0.90$, whether the lot can be accepted or not?

First we check whether the generalized exponential distribution can be used or not. The MLEs of α and λ are 2.6531 and 0.6547 respectively. The Kolmogorov-Smirnov distance between the observed and fitted distribution functions is 0.125 with the associated p value 0.99. Therefore, generalized exponential distribution provides a very good fit. Now in our study we have assumed α to be known as 2.65 and $T = 1070$ hours. Based on that we obtain

$$\lambda_m^0 = \frac{1000}{-\ln\left(1-\left(\frac{1}{2}\right)^{\frac{1}{2.65}}\right)} = 680.73, \quad \text{and} \quad \frac{T}{\lambda_m^0} = 1.572. \tag{26}$$

From Table 1, corresponds to the column $\frac{T}{\lambda_m^0} = 1.571$, and for $P^* = 0.90$ we obtain $n = 9$ when $c = 4$. Therefore, if the number of failures before $T = 1070$ hours, is less than or equal to 4, we can accept the lot with the assured median level 1000 hours, with probability 0.90. Since the number of failures before $T = 1070$ hours is only 2, therefore we can accept the lot with the above specifications.

5 Conclusions

In this paper we have considered the time truncated acceptance sampling plan for the generalized exponential distribution. It is assumed that the shape parameter is known and we have presented the table for the minimum sample size required to guarantee a certain median life of the experimental units. We have also presented the operating characteristic function values and the associated producer's risks. Although we have provided the Tables when the shape parameter is 2, and for the medians only, but the tables can be used for other shape parameters and other percentiles also. We have provided several examples to illustrate the tables. It may be pointed out that Aslam and Shabaz [1] also considered the economic reliability test plans using the generalized exponential distribution. The problem of interests and the approaches are quite different than the present manuscript.

Finally it should be mentioned that our results can be used for other distributions also which can be converted to generalized exponential distribution. For example the generalized Rayleigh (scaled Burr Type X) distribution, introduced by Surles and Padgett [16], see also Kundu and Raqab [13], can be easily converted to generalized exponential distribution. Therefore, our tables can be used to develop the acceptance sampling plan for generalized Rayleigh distribution.

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Table 1: Minimum sample size necessary to assure that the median life exceeds a given value θ_m^0, with probability P^* and the corresponding acceptance number c when the shape parameter $\alpha_0 = 2$.

		T/λ_m^0							
P^*	$c \downarrow$	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	5	3	2	2	1	1	1	1
	1	9	5	4	3	2	2	2	2
	2	13	8	6	5	4	3	3	3
	3	17	10	8	6	5	4	4	4
	4	21	13	9	8	6	5	5	5
	5	25	15	11	9	7	6	6	6
	6	29	17	13	11	8	8	7	7
	7	33	20	15	12	10	9	8	8
	8	36	22	16	14	11	10	9	9
	9	40	24	18	15	12	11	10	10
	10	44	27	20	16	13	12	11	11
0.90	0	7	3	2	2	2	1	1	1
	1	12	7	5	4	3	2	2	2
	2	17	10	7	6	4	4	3	3
	3	22	13	9	7	5	5	4	4
	4	26	15	11	9	7	6	5	5
	5	30	18	13	11	8	7	6	6
	6	35	20	15	12	9	8	8	7
	7	39	23	17	14	10	9	9	8
	8	43	25	19	15	12	10	10	9
	9	47	28	20	17	13	11	11	10
	10	51	30	22	18	14	12	12	11
0.95	0	9	5	4	3	2	1	1	1
	1	15	9	6	5	4	3	3	2
	2	20	12	8	6	5	4	3	3
	3	25	14	10	8	6	5	5	4
	4	29	17	12	10	7	6	6	5
	5	34	20	14	11	8	7	7	6
	6	38	22	16	13	10	8	8	7
	7	43	25	18	15	11	10	9	8
	8	47	28	20	16	12	11	10	10
	9	51	30	22	18	13	12	11	11
	10	55	33	24	19	15	13	12	12
0.99	0	14	8	5	4	3	2	2	1
	1	21	12	8	6	4	3	3	3
	2	26	15	10	8	6	4	4	4
	3	32	18	13	10	7	6	5	5
	4	37	21	15	12	8	7	6	6
	5	42	24	17	13	10	8	7	7
	6	46	27	19	15	11	9	8	8
	7	51	30	21	17	12	10	9	9
	8	56	32	23	18	13	11	11	10
	9	60	35	25	20	15	13	12	11
	10	65	38	27	22	16	14	13	12

Table 2: OC values for the time truncated acceptance sampling plan $(n, c, T/\lambda_m^0)$ for a given P^*, when $c = 2$ and $\alpha_0 = 2$.

			α/α^0					
P^*	n	T/λ_m	2	4	6	8	10	12
0.75	13	0.628	0.8590	0.9934	0.9992	0.9998	1	1
	8	0.942	0.8124	0.9890	0.9986	0.9997	0.9999	1
	6	1.257	0.7640	0.9830	0.9976	0.9995	0.9998	0.9999
	5	1.571	0.7115	0.9751	0.9962	0.9991	0.9997	0.9999
	4	2.356	0.5512	0.9403	0.9888	0.9971	0.9990	0.9996
	3	3.141	0.6103	0.9439	0.9886	0.9969	0.9989	0.9996
	3	3.972	0.4311	0.8819	0.9716	0.9914	0.9969	0.9987
	3	4.712	0.2897	0.8002	0.9439	0.9814	0.9928	0.9969
0.90	17	0.628	0.7507	0.9857	0.9982	0.9996	0.9999	1
	10	0.942	0.6992	0.9785	0.9970	0.9994	0.9998	0.9999
	7	1.257	0.6714	0.9726	0.9959	0.9991	0.9997	0.9999
	6	1.571	0.5796	0.9556	0.9927	0.9983	0.9995	0.9998
	4	2.356	0.5512	0.9403	0.9888	0.9971	0.9990	0.9996
	4	3.141	0.2952	0.8400	0.9622	0.9888	0.9960	0.9984
	3	3.972	0.4311	0.8819	0.9716	0.9914	0.9969	0.9987
	3	4.712	0.2897	0.8002	0.9439	0.9814	0.9928	0.9969
0.95	20	0.628	0.6634	0.9776	0.9971	0.9994	0.9998	0.9999
	12	0.942	0.5837	0.9641	0.9948	0.9988	0.9997	0.9999
	8	1.257	0.5792	0.9594	0.9937	0.9985	0.9996	0.9998
	6	1.571	0.5697	0.9556	0.9927	0.9983	0.9995	0.9998
	5	2.356	0.3443	0.8800	0.9751	0.9932	0.9977	0.9991
	4	3.141	0.2952	0.8400	0.9622	0.9888	0.9960	0.9984
	3	3.972	0.4311	0.8819	0.9716	0.9914	0.9969	0.9987
	3	4.712	0.2897	0.8002	0.9439	0.9814	0.9928	0.9969
0.99	26	0.628	0.4943	0.9554	0.9938	0.9986	0.9996	0.9999
	15	0.942	0.4252	0.9355	0.9900	0.9977	0.9993	0.9997
	10	1.257	0.4120	0.9254	0.9876	0.9970	0.9991	0.9997
	8	1.571	0.3520	0.9007	0.9819	0.9954	0.9985	0.9994
	6	2.356	0.2011	0.8061	0.9556	0.9874	0.9957	0.9983
	4	3.141	0.2952	0.8400	0.9622	0.9888	0.9960	0.9984
	4	3.972	0.1386	0.7014	0.9124	0.9709	0.9888	0.9952
	4	4.712	0.0601	0.5512	0.8400	0.9403	0.9754	0.9888

Table 3: Minimum ratio of true median life to specified median life for the acceptance of a lot with producer's risk of 0.05, when $\alpha_0 = 2$.

		T/λ_m^0							
P^*	$c \downarrow$	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	7.30	8.30	9.00	11.20	11.50	15.30	19.10	23.00
	1	3.41	3.59	4.12	4.21	4.52	6.03	7.55	9.04
	2	2.60	2.86	3.11	3.40	4.20	4.14	5.20	6.20
	3	2.25	2.37	2.67	2.63	3.30	3.32	4.15	5.00
	4	2.05	2.22	2.22	2.50	2.79	2.86	3.57	4.30
	5	1.92	2.02	2.10	2.20	2.46	2.56	3.20	3.84
	6	1.82	1.88	2.02	2.16	2.23	2.97	2.93	3.52
	7	1.75	1.85	1.95	1.97	2.40	2.74	2.73	3.28
	8	1.67	1.76	1.80	1.97	2.23	2.56	2.58	3.10
	9	1.63	1.69	1.77	1.84	2.09	2.42	2.45	2.94
	10	1.60	1.68	1.74	1.731	1.98	2.30	2.34	2.81
0.90	0	8.70	8.30	9.00	11.20	16.70	15.03	19.10	23.00
	1	4.02	4.41	4.80	5.15	6.31	6.03	7.53	9.04
	2	3.05	3.31	3.50	3.90	4.20	5.60	5.17	6.20
	3	2.63	2.83	2.92	3.00	3.30	4.40	4.16	5.00
	4	2.34	2.45	2.61	2.78	3.30	3.71	3.57	4.30
	5	2.15	2.30	2.41	2.63	2.90	3.28	3.20	3.84
	6	2.06	2.12	2.27	2.34	2.61	2.97	3.71	3.52
	7	1.96	2.05	2.17	2.29	2.39	2.74	3.43	3.30
	8	1.88	1.94	2.09	2.11	2.49	2.56	3.21	3.10
	9	1.82	1.90	1.94	2.09	2.34	2.42	3.03	2.94
	10	1.76	1.83	1.90	1.97	2.21	2.30	2.88	2.81
0.95	0	9.84	10.90	13.00	14.00	16.80	15.30	19.01	23.01
	1	4.56	5.11	5.35	6.03	7.73	8.41	10.16	9.04
	2	3.36	3.71	3.81	3.89	5.10	5.59	5.20	6.20
	3	2.84	2.97	3.15	3.34	3.94	4.40	5.50	4.98
	4	2.50	2.67	2.79	3.03	3.30	3.71	4.64	4.30
	5	2.33	2.48	2.56	2.63	2.90	3.28	4.10	3.84
	6	2.17	2.27	2.39	2.52	2.94	2.97	3.71	3.52
	7	2.08	2.18	2.27	2.44	2.69	3.19	3.43	3.28
	8	1.99	2.11	2.18	2.24	2.49	2.97	3.21	3.85
	9	1.91	2.00	2.10	2.21	2.34	2.79	3.03	3.63
	10	1.85	1.96	2.04	2.07	2.41	2.64	2.88	3.45
0.99	0	12.40	14.40	14.16	16.10	21.05	22.31	28.44	23.50
	1	5.49	6.03	6.40	6.70	7.73	8.41	10.61	12.71
	2	3.90	4.25	4.41	4.77	5.83	5.60	7.14	8.40
	3	3.30	3.50	3.78	3.94	4.50	5.25	5.50	6.60
	4	2.90	3.07	3.27	3.50	3.75	4.40	4.64	5.57
	5	2.65	2.80	2.89	3.01	3.62	3.86	4.10	4.91
	6	2.44	2.61	2.72	2.84	3.24	3.47	3.71	4.45
	7	2.32	2.47	2.56	2.71	2.95	3.19	3.43	4.11
	8	2.22	2.31	2.43	2.49	2.73	2.97	3.71	3.84
	9	2.12	2.23	2.33	2.42	2.76	3.11	3.49	3.63
	10	2.06	2.17	2.24	2.37	2.60	2.94	3.30	3.45