Experimental Error

Debasis Kundu
Department of Mathematics and Statistics
Indian Institute of Technology Kanpur

Outline of the Talk

Outline of the Talk

- Experiment

Outline of the Talk

- Experiment
- Data

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- Data
- Information

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- Data
- Information
- Different Types of Models

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- Different Types of Models
- Different Forms of Errors

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- Different Types of Models
- Different Forms of Errors
- Estimating Models/ Model Parameters
- Estimating Errors
- Conclusions

Experiment:

Experiment:

What is an experiment?

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Experiment is a process which produces outcomes/ data.

Experiment:

We divide the experiments into two broad classes

EXPERIMENT:

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- Random Experiment

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- Random Experiment
- Non-Random Experiment

EXPERIMENT:

What is a Random Experiment?

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Random experiment is an experiment which produces random/ non-predictable outcomes. That means even if we repeat the experiment under the same environmental conditions, the outcomes may not be the same, whatever be the reason.

Experiment:

Examples:

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- Toss a coin 10 times.

EXPERIMENT:

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- Rumford Cooling experiment (1798).

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- ECG data of a particular person.

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- Toss a coin 10 times.
- Rumford Cooling experiment (1798).
- ECG data of a particular person.
- Stock price of a particular brand.

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Most of the experiments in nature are random experiments.

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AIM: Trying to extract maximum information from the observed DATA

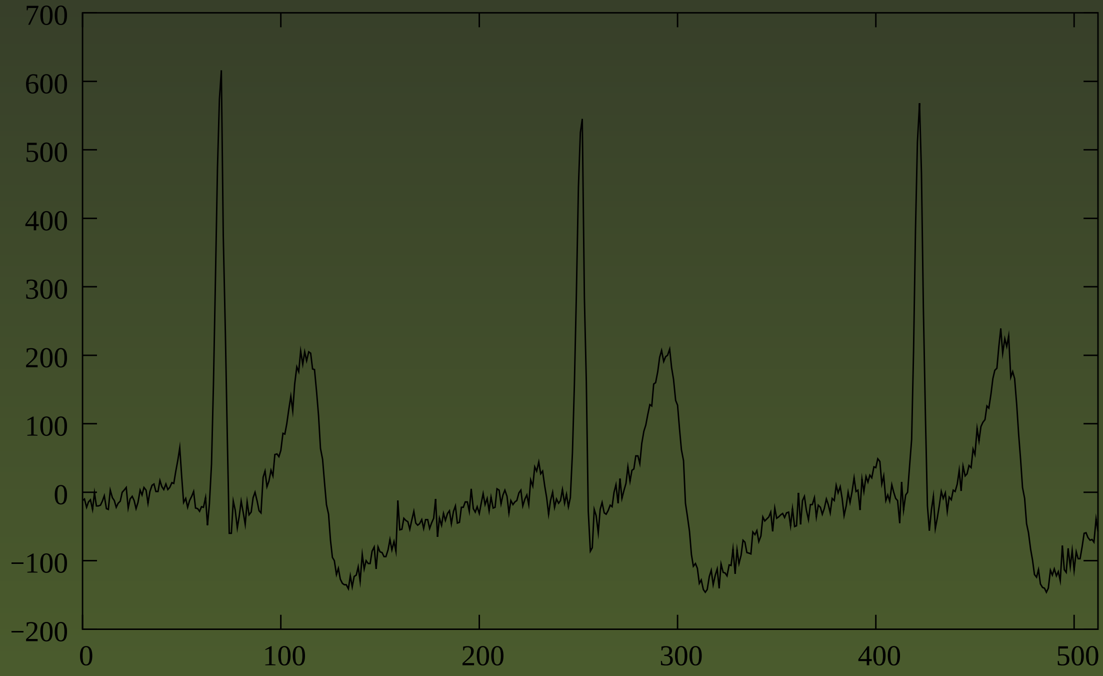
DATA

Example: Rumford Cooling Experiment
(Amount of heat generated by friction)

Time (min)	Temperature (^0F)	Time (min)	Temperature (^0F)
4	126	24	115
5	125	28	124
7	123	31	113
12	120	34	112
14	119	37.5	111
16	118	41	110
20	116		

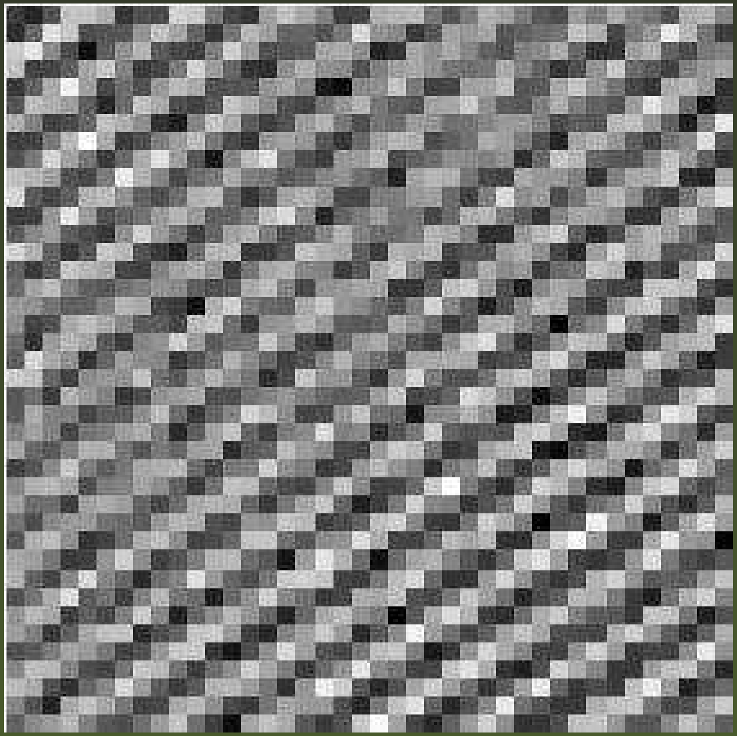
DATA

Example (ECG Data)



DATA

Example (Texture Data)



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Here, Information/ Signal is a very generic term. It may be completely deterministic or completely random or mixed. Our aim is to extract the Information/ Signal from the (noisy) Data.

INFORMATION

Why is it necessary?

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- To understand the process itself.

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Why is it necessary?

- To understand the process itself.
- Prediction purposes.
- Storing purposes.
- Designing the experiments.

INFORMATION

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Intuitively we feel that if the Information/ Signal is much more than the noise/ error, we should be able to extract the Information/ Signal from the noisy data. Otherwise it will be almost impossible.

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Intuitively we feel that if the Information/ Signal is much more than the noise/ error, we should be able to extract the Information/ Signal from the noisy data. Otherwise it will be almost impossible.

Therefore, it is clear that understanding the noise/ error is very important in any $scientific$ experiment.

Information

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$$\text{Information} \approx \text{Model}$$

MODELS

Models

Data = Model + Error

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Error has several components

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Data = Model + Error

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- Modeling Error
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- Data Collection Error
- Calibration Error

MODELS

Different Types of Model

MODELS

Different Types of Model

- Mechanistic Model

Models

Different Types of Model

- Mechanistic Model
- Functional Model

Models

Mechanistic Model:

MODELS

Mechanistic Model:

A mechanistic model is a model based on the exact mechanism of an experiment. It is usually quite complex but most of the time it is quite effective.

Models

Functional Model:

MODELS

Functional Model:

It is purely a mathematical model. One does not need any experimental knowledge to construct a functional model.

MODELS

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Aim: From $\{X_1, Y_1\}, \ldots, \{X_n, Y_n\}$ we want to estimate $f(\cdot)$.

EXAMPLES

Rumford Cooling Data

- Mechanistic Model:
- Based on Newton's law of cooling the following model has been used:

$$Y(t) = 60 + 70e^{-\theta t} + \epsilon(t)$$

Examples

ECG Data

- Functional Model:

$$Y(t) = \sum_{k=1}^{p} \{A_k \cos(\omega_k t) + B_k \sin(\omega_k t)\} + \epsilon(t)$$

Examples

Texture Data

- Functional Model:

$$Y(m,n) = \sum_{k=1}^{p} \{A_k \cos(\omega_k m + \lambda_k n) + B_k \sin(\omega_k m + \lambda_k n)\} + \epsilon(m,n)$$

Models: Estimation

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Possible Approaches:

- From Y and X estimate $f(\cdot)$. (Non-Parametric Approach)
- Assume $f(X) = f(X, \theta)$, here the form of $f(\cdot)$ is known but the parameter θ is unknown. Therefore, estimate θ assuming $f(\cdot)$ is known. Once θ has been estimated and since $f(\cdot)$ is known, therefore $f(X, \theta)$ becomes known. (Parametric Approach)

Errors

Different forms of Errors:

ERRORS

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- Random Errors

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- Systematic Errors

Errors

Different Assumptions on Errors:

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- Spurious errors.
- Errors have some particular probability distribution.

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Why are the assumptions necessary?

- Choosing the correct model.
- Effective estimation of the unknown parameters.
- Effective prediction of the future observations if necessary.
- Constructing error bars of the estimates.
- Deriving properties of the estimates.

ERRORS

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- If the error assumptions are true, then be HAPPY otherwise
- Change the model, change the error assumptions and continue the process.

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Least Squares Estimators (Most Popular One)

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- It works very well if the errors are independent with mean zero and same variances.
- If the error assumptions are not met then the estimates can be quite bad.

Estimating Model Parameters

Generalized Least Squares Estimators

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- It works much better than the least squares estimators.

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- The estimates are quite robust as compared to the least squares estimators.

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Least Absolute Deviations

$$\min_{\theta} \sum_{i=1}^{n} |Y_i - f(X_i, \theta)|$$

- It works very well if the errors are spurious in nature.
- The estimates are quite robust as compared to the least squares estimators.
- Computationally it is very demanding.

Conclusions

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All experimental data will have errors

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To extract maximum information from the data

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- Proper Error Assumptions/ Error Analysis Are Required

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Professional Statistician’s Help is Needed

Thank You