Discussions on "Hierarchical Bayesian auto-regressive models for large space time data with applications to ozone concentration modelling" by S.K. Sahu and K.S. Bakar

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First I would like to congratulate Sujit K. Sahu and Khandoker Shuvo Bakar for a valuable and timely contribution to an important topic on Bayesian spatio-temporal modelling. As the authors have correctly mentioned, in recent time a large volume of space-time data is collected everyday using the global geographic information systems. Adequate modelling techniques are needed to extract useful information out of these large data sets. Frequentist approaches may be very difficult to implement, hence Bayesian inference is the most natural choice. The authors proposed spatial auto-regressive models and an approximation has been carried out using Gaussian predictive process (GPP) approach, which is an extension of the method suggested by Banerjee et al. [1]. Bayesian analysis has been performed under the assumptions of appropriate priors on the unknown parameters, and using MCMC techniques. Finally, the authors analyzed a very large dataset on ozone concentrations, consisting of over one million observations. The proposed model has a very good predictive power.

Having said this, I must add that the paper raises as many questions as it has answered, just as any good article usually does. My main questions are along the line of how the proposed model can be made more flexible, and how difficult it will be to implement in practice. The authors used the transformed data (square root), since it leads to symmetry, although it has been mentioned that any other transformation such as logarithm is also possible. In my opinion, this transformation of the data plays an important role in the prediction prob-

lem although not as much in the analysis (estimation) part. Therefore, natural question is how difficult it will be to have a data driven transformation. I would suggest two ways to approach this problem. One way is to take different transformations from the class of Box-Cox type transformations, and choose the best one using some criterion as described in this paper. Alternatively, one might impose a prior on the parameter in the Box-Cox transformation and then perform the usual Bayesian analysis.

Another question concerns the autoregressive parameter ρ. Is there any strong prior evidence that $\rho > 0$. A uniform (0,1) prior has been chosen on ρ, and I wonder how the results will differ if a uniform (-1,1) prior is assumed? Have the authors thought about this? Moreover, three covariates which have been used in this article are strongly correlated, so why the prior variance on $\boldsymbol{\beta}$ has been taken as $10^4\mathbf{I}$. Is it not more meaningful to take some correlation structure on the prior of $\boldsymbol{\beta}$, or the diagonal structure be chosen so that it is easy to implement.

I am also some what surprised by the choice of the hyper parameter values. The choices appear to be restrictive, *viz* (2,1), (1,1), (10,10). I wonder why very small values like (0.0001,0.0001) or very large values like (10000,10000) have not been tried. These type of hyper-parameter values have been suggested by many authors including Berger. In my opinion it can make a difference in the prediction process.

In summary, I would like to reiterate that the present paper will have a lasting effect in the literature on modeling large scale data set with space-time components. In particular, the associated R package spTimer will be really helpful to the practitioner.