A Via-Free Left-Handed Transmission Line with Radial Stubs

G. Naga Satish¹, K. V. Srivastava², A. Biswas³, D. Kettle⁴

¹Department of Electrical Engineering, Indian Institute of Technology Kanpur
Uttar Pradesh, India-208016.
²gnsatish@iitk.ac.in
³kvs@iitk.ac.in
⁴abiswas@iitk.ac.in

 Abstract — A novel via-free microstrip Left Handed (LH) Transmission Line (TL) utilizing Radial Stubs (RS) is proposed. The microstrip RS is approximated by a series combination of inductance and capacitance. The electrical equivalent circuit of LH TL unit cell including the equivalent inductance and capacitance of radial stub is discussed. The balanced and unbalanced LH TL is implemented in microstrip technology and backward waves are confirmed by both full wave simulations and experiments. Additionally an open-circuited Zeroth Order Resonators (ZOR) with their resonating properties in balanced and unbalanced left handed transmission lines are also verified by full wave simulations and experiments.

Index Terms — Metamaterials, Left Handed materials, Backward waves, Radial Stubs, Zeroth order Resonator.

I. INTRODUCTION

In Left Handed (LH) materials the electric field vector, magnetic field vector and wave vector forms a left handed triad resulting in electromagnetic waves with anti parallel phase and group velocities. This type of materials with their exotic properties was first theoretically proposed by Veselago in 1968[1]. These artificial materials were experimentally verified by combining Split Ring Resonators and thin wires by D. R. Smith [2]. Since then there has been a growing interest in the development of left handed materials. The transmission line (TL) approach of LH materials with cascaded connection of series capacitance and shunt inductance, under the homogeneous limit, were shown to support electromagnetic waves with anti parallel phase and group velocities producing backward waves. In microstrip technology the quasi lumped element implementation of series capacitance and shunt inductance is obtained by inter digital capacitor (IDC) and a shorted TL to ground plane through via respectively. The quasi lumped element implementation of LH TL produces unavoidable parasitic series inductance and shunt capacitance forming a composite right/left Handed (CRLH) TL. Based on the unique properties of CRLH TLs novel microwave devices with varied advantages were designed [3].

Direct implementation of microstrip CRLH TL requires complex vias penetrating from top plane to ground plane. This is not suitable for high volume and cost effective production of LH TLs. Another implementation can be a coplanar waveguide configuration [4] which can accommodate shunt inductances without having vias because the ground conductors are on the same plane as that in the signal line. However the single metal layer structure is not suitable for some applications such as 2D texture surfaces.

In this paper, a novel via free microstrip implementation of LH TL utilizing radial stubs (RS) is proposed. This facilitates easy and cost effective solution for the fabrication of LH TLs. The input impedance of microstrip RS is approximated by a series combination of inductance and capacitance. The electrical equivalent circuit of the proposed LH TL unit cell is obtained by modifying the shunt inductive arm of the CRLH TL equivalent circuit with effective inductance of meander line and RS in series with capacitance of RS connected to ground. In order to confirm backward waves a LH TL in both balanced and unbalanced configuration is implemented in microstrip technology and dispersion diagram of unit cell is obtained by full wave simulations. Further the open circuited zeroth-order resonance (ZOR) properties of the proposed LH TL are verified for different lengths of LH transmission line by full wave simulations and experiments.

II. RADIAL STUB LH TL THEORY

The layout of proposed LH TL unit cell and its electrical equivalent circuit is shown in Fig. 1(a) and (b) respectively.

![Fig. 1. LH TL with radial stubs. (a) Microstrip implementation. (b) Equivalent circuit of the unit cell.](image)

In the equivalent circuit of Fig. 1(b) C_L represents capacitance due to gaps in inter digital capacitor (IDC); L_R and C_R represents parasitic inductance due to current flowing in the
fingers of the IDC. The shunt inductance $L_L$ is the combination of inductance of meander line and radial stub. $C_n$ is capacitance due to radial stub alone and $C_R$ is the parasitic capacitance between fingers of IDC and ground plane. The equivalent circuit in Fig. 1(b) is the modified version of CRLH TL symmetrical T-equivalent circuit [10] where the shunt inductive arm is replaced by series combination of inductance and capacitance of the RS terminated meander line. The dispersion characteristics of the equivalent circuit in Fig. 1(b) is studied in [5] and shown that for good LH bandwidth the series impedance of $L_L$ and $C_n$ should be dominantly inductive. In the layout of Fig. 1(a) the series combination of meander line and radial stub provide dominant inductance to support backward waves over a wide band.

A. Radial Stub.

RS has been widely used in many microwave circuits such as matching networks, bias lines; low pass filters etc to ground wide band RF signal. The RS will realize a short circuit at the point where it placed, making the metal via hole in the planar wide band RF signal. The RS will realize a short circuit at the point where it placed, making the metal via hole in the planar circuit unnecessary [6]. The inset of Fig. 2 shows the geometrical parameters and approximate equivalent circuit of a microstrip radial stub.

$$J_0(x) = 1, J_1(x) = \frac{x}{2}$$
$$Y_0(x) = \frac{2}{\pi} \left[ \ln \left( \frac{x}{2} \right) + 0.5772 \right]$$
$$Y_1(x) = \frac{x}{2} \left[ \ln \left( \frac{x}{2} \right) + 0.5772 \right] - \frac{x}{4} - \frac{1}{x}$$

and keep up to the first order terms, resulting in

$$Z_{in} \approx -\frac{j 120 \pi h}{\sqrt{\varepsilon_{eff}}} \left[ \frac{2}{\varepsilon_{ds}} - \left( \ln \left( \frac{r_o}{r_i} \right) - \frac{1}{2} \right) \right]$$

which is equivalent to a series combination of an inductor $L_{rs}$ and capacitor $C_{rs}$ and is given by

$$L_{rs} \approx \frac{120 \pi h}{\varepsilon_{ds}} \left[ \ln \left( \frac{r_o}{r_i} \right) - \frac{1}{2} \right] H, C_{rs} \approx \frac{0.05 \pi \varepsilon_{eff}}{240 \pi h} F$$

From the above equations it can be observed that the variation of $L_{rs}$ and $C_{rs}$ depends on the dielectric substrate and mainly on $r_o$ and $\theta$. Fig. 2 shows the comparison of variation of input reactance of the microstrip RS over a wide frequency range using S. L March’s formula and the approximation in (5). The approximated input impedance is very closely matching with S. L March formula.

B. Dispersion Characteristics.

The dispersion diagram of the periodic LH TL with unit cell shown in Fig. 1 can be obtained by applying periodic boundary conditions to the unit cell. By expressing unit cell in terms of the [ABCD] matrix and applying Bloch-Fluent theorem [10], one can obtain the dispersion relation of the periodic TL as

$$\beta = \frac{1}{\rho} \cos^{-1} \left[ 1 + Z(\omega)Y(\omega) \right]$$

where $\rho$ is the period of the unit cell, and $Z(\omega)$ and $Y(\omega)$ are the impedance and admittance of the series and shunt elements in the unit cell, respectively, given by

$$Z(\omega) = \frac{1}{2} \left( \frac{1}{j \omega C_L + j \omega L_R} \right)$$
$$Y(\omega) = \frac{1}{\left( j \omega L_L + \frac{1}{j \omega C_R} \right) + j \omega C_R}$$

When $\omega L_L >> 1/\omega C_R$, $Y(\omega)$ approaches the admittance of the $L_L C_R$ tank resonator and, therefore, $C_R$ has less influence on propagation characteristics. A detailed analysis of the equivalent circuit in Fig. 1(b) is presented in [5] and it is shown that good LH bandwidth can be obtained by efficient grounding of the RF signal through the capacitance $C_n$ in the
shunt arm. Fig. 3 shows the theoretical dispersion characteristics with considerable LH bandwidth for the LH TL unit cell with typical values of inductances and capacitances obtained with RS. By appending RS to the meander line an additional inductance \( L_{rs} \) is added to the inductance of meander line and the capacitance \( C_{rs} \) which provides very low impedance path to ground the RF signals. So, by including RS in the shunt inductive network of the standard CRLH transmission line, an LH TL with considerable bandwidth can be obtained. The criterion for LH mode and the condition for balanced LH TL are same as (7) and (8) of ref. [5].

**C. Zeroth-Order Resonator.**

When a LH TL is open-ended or short ended, it produces standing waves due to open/short boundary conditions and becomes a resonator. The LH TL resonator exhibits some unusual characteristics like allowing negative and ZORs apart from positive resonances as in normal RH TLs. The ZOR is particularly useful because at these frequencies the LH TL resonates with infinite wavelength independent of the TL length [9]. It was shown that the ZOR frequency of an unbalanced open ended LH TL is equal to the resonance frequency of shunt arm and in a balanced LH TL and it is equal to the resonance frequency of series arm, in the equivalent circuit, respectively [10]. A zeroth-order resonators with different number of unit cells is designed using the proposed LH TL unit cell.

### IV. SIMULATIONS AND EXPERIMENTS

The LH TL is designed on a RT Duroid 6010 dielectric substrate with a thickness of 1.27 mm and 1 oz thick copper coating on both sides. The dispersion characteristics of both balanced and unbalanced LH unit cells are obtained by MOM based full wave simulation. It can be seen from the Fig. 4(b) that the LH band exists with a negative slope in positive \( \beta \) region, that is, anti parallel phase and group velocities and, therefore, the backward wave propagation has been confirmed numerically. The LH band in balanced and unbalanced LH TLs is from 3.5 to 4.5 GHz and 3.2 to 4.1 GHz respectively. The RH mode in unbalanced line starts from 5.2 GHz. The shunt resonant frequency for the unbalanced LH TL unit cell with the equivalent circuit in Fig. 1(b) is obtained as 4.2 GHz after calculating the series resonant frequency using parameter extraction method described in [10].

![Fig. 3. Dispersion characteristics of unit cell with \( C_L = 0.6 \) pF, \( L_R = 1.5 \) nH, \( L_L = 5 \) nH, \( C_R = 0.6 \) pF and \( C_{rs} = 0.5 \) pF.](image)

![Fig. 4. Full wave simulated dispersion characteristics of balanced and un-balanced LH TLs. (a) Geometrical parameters of unit cell. (b) Dispersion characteristics by full wave simulation.](image)
VI. CONCLUSION

A via free LH TL was proposed by using RS. The RS is approximated by a series connection of inductance and capacitance and its input impedance is compared with the traditional formula. The equivalent circuit of the LH TL unit cell is studied including the approximated inductance and capacitance of radial stub. A microstrip balanced and unbalanced LH TL was implemented and backward wave frequency region is confirmed by both MOM full wave simulations and experiments. The experimental results agree well with the simulations.

Fig. 6. Measured ZOR frequencies for the open circuited balanced and unbalanced LH TLs with unit cell dimensions as in Fig. 4 and w=g = 0.2 mm

ACKNOWLEDGEMENT

The authors wish to acknowledge the assistance provided by S. K. Kole in the fabrication of PCBs.

REFERENCES