Abstract—The effects of anisotropy on the frequency-dependent properties of multi-coupled asymmetric fin-lines due to uniaxial and biaxial substrates are investigated in this paper. The asymmetric multi-port fin-lines characteristics are obtained by using the full wave modal analysis procedure in conjunction with the Galerkin's method. Numerical results include effect of dielectric anisotropy on propagation constants and modal characteristic impedances of two and three fin-lines on uniaxial and biaxial substrates at different frequencies.

Index Terms—Anisotropic substrate, uniaxial and biaxial substrate, fin-line and multi-conductor transmission lines.

I. INTRODUCTION

FINLINES are widely used as a millimeter wave component due to its various advantages such as reducing size, weight, and cost and in addition because it interfaces easily with other millimeter wave circuits. Unilateral and bilateral structures have been analyzed for isotropic substrates by several authors [1]-[3]. In recent years there have been several investigations published on the incorporation of either uniaxial or biaxial anisotropic media on single unilateral or bilateral finline technology [3]-[4], but very limited or no study has been presented on the effect of anisotropy on two or three edge coupled fin-lines. The main objective of this paper is to present the effect of dielectric anisotropy on edge coupled two and three asymmetric fin-lines. The paper discusses the effect of anisotropy on effective dielectric constant and impedances by applying the anisotropy one by one in all three directions. The effective dielectric constant and characteristic impedances for three asymmetric coupled fin-lines have also been provided on biaxial anisotropic substrate. The numerical procedure used for calculation of effective dielectric constant and impedance is full modal analysis which has been discussed in [2].

II. ANALYTICAL AND THEORETICAL FORMULATION

The geometry of multiple asymmetric unilateral finlines on bi-anisotropic substrate is shown in Fig.1. The cross-section of the structure is assumed to be uniform in z-direction. The complete cross-section has been divided into three regions having the region 1 and 3 as the air and region-2 as anisotropic substrate. A full wave analysis of the above structure corresponding to each region has been discussed as follows:

A. Anisotropic region

The substrate shown in Fig.1 is modeled by utilizing bi-anisotropic tensor properties, which are expressed as

\[
\begin{bmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{bmatrix}
\begin{bmatrix}
\mu_x & 0 & 0 \\
0 & \mu_y & 0 \\
0 & 0 & \mu_z
\end{bmatrix}
\]

With the permittivity in tensor form, the wave equations in the substrate can be derived from Maxwell’s equations. In general the wave equations are coupled partial differential equations given by [3, 4].

\[
\frac{\partial^2 H}{\partial x^2} + a H_z + b \frac{\partial E_z}{\partial x} = 0
\]

\[
\frac{\partial^2 E_z}{\partial x^2} + c E_z + d \frac{\partial H_z}{\partial x} = 0
\]
where $D_n = nS/b$. For $n \neq 0$, elimination of either $E_z$ or $H_z$, form the coupled set (1) and (2) makes either field satisfy

$$\frac{\partial^2 F}{\partial x^2} + \left(a + c - bd\right) F + ac F = 0 \quad (3)$$

where $F = E_z$ or $H_z$. Suppose we choose $F = E_z$, then $H_z$ can be calculated in terms of $E_z$ from (2), and rest all the fields components can be calculated in terms of $E_z$ and $H_z$.

B. Region1 and Region3-Isotropic region

For isotropic region all the field components are constructed in terms of the component of electric and magnetic fields in each isotropic region, which are expanded in terms of modal fields with unknown coefficients [2].

The propagation constants are evaluated by applying the Galerkin’s method to the transformed Green’s function matrix relating the voltage and electric fields at various boundaries of the structure and solving for the roots of the determinant of the equation (4) in [2].

$$\sum_{k=1}^{n} c_k \sum_{i=0}^{m} p_k G_{1i} L_{i2} L_{i2} + \sum_{k=1}^{n} d_k \sum_{i=0}^{m} q_k G_{i2} L_{i1} L_{i1} = 0$$

$$\sum_{k=1}^{n} c_k \sum_{i=0}^{m} p_k G_{2i} L_{i1} L_{i1} + \sum_{k=1}^{n} d_k \sum_{i=0}^{m} q_k G_{2i} L_{i2} L_{i1} = 0 \quad (4)$$

The set of basis functions used in this analysis are sinusoidal and expressed as follows:

$$V_z(y) = \cos \left(2(n-1)\pi \frac{(y-y_i)}{w_i}\right) \quad V_y(y) = \sin \left(2n\pi \frac{(y-y_i)}{w_i}\right)$$

where $w_i$ being the width of the $i$th fin, $y_i$ is the distance from origin to the center of $i$th fin.

C. Characteristic impedances

Mode characteristics impedance of the coupled unilateral fin-lines lines on bi-anisotropic substrates are evaluated for all hybrid modes in a straightforward manner by calculating the power associated with a given fin-line for a given mode and the corresponding fin-line voltage as shown in [2]. The fin-line mode impedance is given by

$$Z_{ln} = \frac{V_{ln}^2}{P_{ln}} \quad (5)$$

where $V_{ln}$ is the modal voltage of the $l$th slot given by the integral of the electric field across the slot and $P_{ln}$ is the partial modal power associated with the same slot when the $n$th normal mode is excited.

D. Measure of Anisotropy Effect

The effect of dielectric anisotropy has been provided by the measure of anisotropy effect for effective dielectric constant (MAEE) given in equation (6) between effective dielectric constant for isotropic substrate and anisotropic substrate in which anisotropy has been introduced in either of the three directions.

$$MAEE \% = \frac{\varepsilon_{ef}^{an} - \varepsilon_{ef}^{iso}}{\varepsilon_{ef}^{iso}} \times 100 \quad (6)$$

Similarly the measure of anisotropy effect on impedance (MAEI) is given in equation (7)

$$MAEI \% = \frac{Z_{ef}^{an} - Z_{ef}^{iso}}{Z_{ef}^{iso}} \times 100 \quad (7)$$

III. NUMERICAL RESULTS

A computer program has been developed based on the procedure described above to compute the normal mode parameters of asymmetric multiple coupled unilateral fin-lines on uniaxial and biaxial substrates. As the main objective of this paper is to present the results on effect of dielectric anisotropy on fin-line structure, the effect of anisotropy on propagation characteristic and impedance for two and three asymmetric coupled fin-lines have been investigated here for $e_r = 2.22$ and then by changing the dielectric constant in $x, y$ and $z$ direction to 2.7 respectively.

Fig. 2: Effect dielectric constant of two asymmetric edge couple unilateral fin-lines (w1 = 0.1mm, w2 = 0.2mm, s1=1.628 mm, s2 = 0.1mm, s3= 1.528 mm, D = 0.25mm, h = 3.565mm, a = 7.112 mm, h1 = 3.431mm, h2 =3.431mm, $e_r = e_o = 1.0$, Isotropic: $e_r=2.22$; Anisotropic: Case x: $e_r=2.7$, $e_r = e_o = 2.22$; Case y: $e_r=2.7$, $e_r = e_o = 2.7$; Case z: $e_r=2.7$, $e_r = e_o = 2.22$)
Fig. 2 shows the effective dielectric constant for even and odd mode of two coupled fin-lines for isotropic case and then change in effective dielectric constant by applying the anisotropy in x, y and z-direction. It is interesting to observe that due to TE mode excitation fed to fin-line in the propagation direction, the strength of $E_z$ is much smaller than that of $E_y$ and $E_x$. As results, the effect of anisotropy in z-direction is almost negligible. Also, the thickness of substrate (D in Fig.1) in x-direction is very small compared with length of waveguide broad wall in y-direction (b in Fig.1); therefore anisotropic effect in x-direction is smaller than that in y-direction.

Fig. 3(a) shows the anisotropy effect in term of MAEE as defined in equation (6) for two coupled fin-lines having two modes: even and odd. MAEE is more in y-direction due to TE mode excitation to fin-line in the propagation direction. As a result, the effect of anisotropy in z-direction is almost negligible. Also, the thickness of substrate (D in Fig.1) in x-direction is very small compared with length of waveguide broad wall in y-direction (b in Fig.1); therefore anisotropic effect in x-direction is smaller than that in y-direction.

Fig. 3(b) shows the effect of anisotropy on characteristic impedance of two coupled fin-lines having two modes: even and odd. MAEI is more in y-direction due to TE mode excitation to fin-line in the propagation direction. As a result, the effect of anisotropy in z-direction is almost negligible. Also, the thickness of substrate (D in Fig.1) in x-direction is very small compared with length of waveguide broad wall in y-direction (b in Fig.1); therefore anisotropic effect in x-direction is smaller than that in y-direction.

Fig. 4 shows the effect of anisotropy on effective dielectric constant of three asymmetric coupled fin-lines. The effective dielectric constant is almost dispersionless over the frequency band for mode-a and mode-c, but with small variation in mode-b. MAEI is more in y-direction due to TE mode excitation to fin-line in the propagation direction. As a result, the effect of anisotropy in z-direction is almost negligible. Also, the thickness of substrate (D in Fig.1) in x-direction is very small compared with length of waveguide broad wall in y-direction (b in Fig.1); therefore anisotropic effect in x-direction is smaller than that in y-direction.
TABLE-I: EFFECT OF ANISOTROPY ON THREE ASYMMETRIC EDGE COUPLE FINLINES

<table>
<thead>
<tr>
<th>Freq (GHz)</th>
<th>Characteristic Impedances</th>
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<tbody>
<tr>
<td></td>
<td>ZA1</td>
</tr>
<tr>
<td>26</td>
<td></td>
</tr>
<tr>
<td>Isotropic</td>
<td>134.87</td>
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<td>Anisotropic Case x</td>
<td>131.27</td>
</tr>
<tr>
<td>MAE</td>
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<tr>
<td>Case y</td>
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<td>Case z</td>
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<tr>
<td>MAE</td>
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<td>30</td>
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<td>Isotropic</td>
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</tr>
<tr>
<td>MAE</td>
<td>0.133</td>
</tr>
</tbody>
</table>

Fig.5: (a) Effective dielectric constant and (b) Characteristic impedance for three asymmetric couple unilateral fin-lines \([w_1 = 0.1\text{mm}, w_2 = 0.2\text{mm}, w_3 = 0.3\text{mm}, s_1 = 1.498 \text{mm}, s_2 = 0.08 \text{mm}, s_3 = 0.12 \text{mm}, s_4 = 1.258 \text{mm}, D = 0.25\text{mm}, b = 3.556\text{mm}, a = 7.112 \text{mm}, h_1 = 3.431\text{mm}, h_2 = 3.431\text{mm}, \varepsilon_{x} = \varepsilon_{y} = 1.0, \varepsilon_{z} = 2.22; \text{Anisotropic: Case}_x: \varepsilon_{x} = 2.7, \varepsilon_{y} = \varepsilon_{z} = 2.22; \text{Case}_y: \varepsilon_{x} = 2.7, \varepsilon_{y} = \varepsilon_{z} = 2.22; \text{Case}_z: \varepsilon_{x} = 2.7, \varepsilon_{y} = \varepsilon_{z} = 2.22].

V. CONCLUSION

A general procedure to analyze asymmetric and multiple coupled unilateral fin-line structure on uniaxial and biaxial anisotropic substrate is presented which takes into account dielectric anisotropy, which is very important in millimeter wave applications. The effects of anisotropy on effective dielectric constant and characteristic impedances have been provided on multi-fin-lines by introducing the dielectric anisotropy in either of three directions.

REFERENCES