Unified Modelling Theory for Qubit Representation using Quantum Field Graph Models

Vishal Sahni
Quantum-Nano Computing Systems Centre
Dayalbagh Educational Institute (Deemed University)
Dayalbagh, Agra INDIA

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• Linear graph theory, a branch of topology, has been applied to such diverse systems as ranging from electrical networks through real physical systems and “conceptual” socio-economic-environmental systems to creational systems.

• The approach was developed at MSU and pioneered by Prof. Herman E. Koenig, father of Physical System Theory.
The vast success of Physical System Theory as an operational modelling methodology has been attributed to the fact that it is not just based on analogies but invokes the fundamental properties of instrumentation in identifying an appropriate pair of complementary terminal variables (across and through variables) in each discipline of its application.

It is founded on linear graph theory in capturing the structure as a model of interrelationships between parts of the whole, which is the essence of systems thinking.
• Linear graph theory represents one step towards a systems modelling discipline which coordinates various branches of knowledge into onescientific order.  

(Satsangi 2006).
### Example of systems and complementary variables

(Satsangi, 2008)

<table>
<thead>
<tr>
<th>System</th>
<th>Electrical</th>
<th>Mechanical</th>
<th>Hydraulic/ Pneumatic</th>
<th>Transportation</th>
<th>Economic</th>
<th>Ecosystem</th>
<th>Information System</th>
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<tbody>
<tr>
<td>X-across variable (effort variable)</td>
<td>Voltage</td>
<td>Linear velocity (Rotational velocity)</td>
<td>Pressure</td>
<td>Propensity or level-of-service or traffic density</td>
<td>Unit price</td>
<td>Unit energy or Monetary cost</td>
<td>Treatment</td>
<td>Spiritual Potential Difference</td>
</tr>
</tbody>
</table>
This talk presents a quantum field graph model which is facilitated by considering a qubit as a basic building block and representing it through an appropriate linear graph. The system graph is in two separate parts corresponding to a qubit representation ($|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$), consisting of "ket 0" and "ket 1" subsystem graphs.

Unit "Poynting" vectors $|0\rangle$ and $|1\rangle$ behave like quantum across (potential) variables specifying "direction" of energy propagation, as it were, while $\alpha$ and $\beta$ behave like quantum through (flow-rate or "force") variables specifying quantity of energy flow-rate (energy flux or power flow, W/m$^2$). For n independent states, there will be precisely n basis quantum potential vectors (or unit "Poynting" vectors).

The model has been successfully applied for several quantum gates as well as applications such as quantum teleportation and has the potential for successfully modelling systems at the high end of complexity scale.
Single Qubit

This is the simplest case of representation involving a single qubit $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ as shown in Figure 1. A single qubit representation is physically realizable as an “infinitesimal” (nanoscale) element of quantum force-field (e.g. electromagnetic force-field).

Figure 1
\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

where \( |\psi\rangle \) is the “Poynting” energy-flux or power vector (W/m²)

\( |0\rangle \) and \( |1\rangle \) are the Orthogonal Unit Poynting vectors

\( \alpha \) - Scalar Power flow along \( |0\rangle \) unit “Poynting” vector

\( \beta \) - Scalar Power flow along \( |1\rangle \) unit “Poynting” vector

\( \bar{x}_0 \triangleq |0\rangle \quad \bar{x}_1 \triangleq |1\rangle \)

\( y_0 \triangleq \alpha \quad y_1 \triangleq \beta \)

\( Y = \text{Classical through variable vector} \quad Y = \begin{bmatrix} y_0 \\ y_1 \end{bmatrix} \)

\( \bar{X} = \text{Quantum across variable vector} \quad \bar{X} = \begin{bmatrix} \bar{x}_0 \\ \bar{x}_1 \end{bmatrix} \)

\( |\psi\rangle = \text{Power Flow Vector} = Y^T \bar{X} = \begin{bmatrix} y_0 & y_1 \end{bmatrix} \begin{bmatrix} |0\rangle = \bar{x}_0 \\ |1\rangle = \bar{x}_1 \end{bmatrix} \)

\( |\psi\rangle = \langle Y | \bar{X} \rangle = Y^T \bar{X} = \begin{bmatrix} \alpha & \beta \end{bmatrix} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix} = \alpha |0\rangle + \beta |1\rangle \)

We can generalize Poynting vector in W/m² as active and reactive power flow (as in electrical power systems) for completeness, considering \( \alpha \) and \( \beta \) as complex numbers.
Superposition of two independent qubits

\[ |\psi_1 \rangle \equiv (\alpha_1 |0\rangle + \beta_1 |1\rangle); |\psi_2 \rangle \equiv (\alpha_2 |0\rangle + \beta_2 |1\rangle) \]

\[ |\psi \rangle = |\psi_1 \rangle + |\psi_2 \rangle \]

\[ = (\alpha_1 + \alpha_2) |0\rangle + (\beta_1 + \beta_2) |1\rangle \equiv \alpha |0\rangle + \beta |1\rangle \quad \text{where } \alpha = \alpha_1 + \alpha_2 \text{ and } \beta = \beta_1 + \beta_2 \]

\[ (\alpha_1 + \alpha_2) |0\rangle \quad (\beta_1 + \beta_2) |1\rangle \]

\[ \Rightarrow \quad \alpha \equiv \alpha_1 + \alpha_2 \]

\[ \beta \equiv \beta_1 + \beta_2 \]

Figure 2
\[ |\psi_1\rangle = \langle Y_1 | \overline{X_1} \rangle = Y^T \overline{X_1} \]

where \[ Y_1 = \begin{bmatrix} y_{10} \\ y_{11} \end{bmatrix} \quad y_{10} = \alpha_1 \quad y_{11} = \beta_1 \quad \overline{x_{10}} \triangleq |0\rangle \quad \overline{x_{11}} \triangleq |1\rangle \quad \overline{X_1} = \begin{bmatrix} \overline{x_{10}} \\ \overline{x_{11}} \end{bmatrix} \]

\[ Y_1^T \overline{X_1} = \begin{bmatrix} y_{10} & y_{11} \end{bmatrix} \begin{bmatrix} |0\rangle \equiv \overline{x_{10}} \\ |1\rangle \equiv \overline{x_{11}} \end{bmatrix} \]

\[ |\psi_2\rangle = \langle Y_2 | \overline{X_2} \rangle = Y_2^T \overline{X_2} \]

where \[ Y_2 = \begin{bmatrix} y_{20} \\ y_{21} \end{bmatrix} \quad y_{20} = \alpha_2 \quad y_{21} = \beta_2 \quad \overline{x_{20}} \triangleq |0\rangle \quad \overline{x_{21}} \triangleq |1\rangle \quad \overline{X_2} = \begin{bmatrix} \overline{x_{20}} \\ \overline{x_{21}} \end{bmatrix} \]

\[ Y_2^T \overline{X_2} = \begin{bmatrix} y_{20} & y_{21} \end{bmatrix} \begin{bmatrix} |0\rangle \equiv \overline{x_{20}} \\ |1\rangle \equiv \overline{x_{21}} \end{bmatrix} \]

\[ |\psi\rangle = |\psi_1\rangle + |\psi_2\rangle = Y_1^T \overline{X_1} + Y_2^T \overline{X_2} = \begin{bmatrix} Y_1^T & Y_2^T \end{bmatrix} \begin{bmatrix} \overline{X_1} \\ \overline{X_2} \end{bmatrix} \]

\[ = Y^T \overline{X} \quad \text{where} \quad Y \equiv \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad \overline{X} = \begin{bmatrix} \overline{X_1} \\ \overline{X_2} \end{bmatrix} \]
A NOT gate converts an input of $|0\rangle$ to a $|1\rangle$ and vice versa. For the superposition state, $\alpha|0\rangle + \beta|1\rangle$, the output would be $\alpha|1\rangle + \beta|0\rangle$ which is represented using quantum graph in Figure 3.

$$NOT\left(\alpha|0\rangle + \beta|1\rangle\right) = (\beta|0\rangle + \alpha|1\rangle)$$
Hadamard Gate Operation

This example demonstrates a Hadamard gate operation in which qubit $|0\rangle$ is transformed to $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|1\rangle$ to $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$. Figure 4.1 shows the graph representation for both these cases.

![Graph representation of Hadamard gate operation](image)

Figure 4.1
$$|\psi_i\rangle = Y_i^r \overline{X_i} = \begin{bmatrix} \gamma_{i0} & \gamma_{i1} \end{bmatrix} \begin{bmatrix} x_{i0} \\ x_{i1} \end{bmatrix} \triangleq |0\rangle$$

where

$$Y_i = \begin{bmatrix} \gamma_{i0} \\ \gamma_{i1} \end{bmatrix}, \quad \overline{X_i} = \begin{bmatrix} x_{i0} \\ x_{i1} \end{bmatrix}$$

and

$$Y_0 = HY_i, \quad Y_0 = \begin{bmatrix} \gamma_{00} \\ \gamma_{01} \end{bmatrix} \quad \text{and} \quad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

or

$$\begin{bmatrix} \gamma_{\phi0} \\ \gamma_{\phi1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \gamma_{i0} \\ \gamma_{i1} \end{bmatrix}$$

$$\Rightarrow |\psi_0\rangle = Y_0^r \overline{X_\phi}$$

$$= \begin{bmatrix} \gamma_{\phi0} & \gamma_{\phi1} \end{bmatrix} \begin{bmatrix} x_{\phi0} \\ x_{\phi1} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

where

$$\overline{X_\phi} = \begin{bmatrix} x_{\phi0} \\ x_{\phi1} \end{bmatrix}$$
Figure 4.2(a) shows the graph representation for superposition of both cases, i.e. 
\[ \alpha |0\rangle + \beta |1\rangle = \frac{\alpha}{\sqrt{2}} |0\rangle + \frac{\beta}{\sqrt{2}} |1\rangle \] 
and Fig. 4.2(b) shows transfer function representation of Hadamard gate.

**Figure 4.2 (a) : Hadamard Gate Operation**

\[ Y_i \quad Y_o = H Y_i \]

where \[ H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = H^T \]

**Figure 4.2(b) : Transfer function representation**
Given \( |\psi_i\rangle = Y_i^T \overline{X}_i \)

\[
= \begin{bmatrix} \alpha & \beta \end{bmatrix} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix}
\]

\[\Rightarrow |\psi_o\rangle = Y_o^T \overline{X}_o = [HY_i]^T \overline{X}_o = Y_i^T H \overline{X}_o \]

\[
= \begin{bmatrix} Y_{i0} & Y_{i1} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix}
\]

\[
= \begin{bmatrix} \alpha & \beta \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix}
\]

\[
= \frac{1}{\sqrt{2}} (\alpha + \beta) |0\rangle + \frac{1}{\sqrt{2}} (\alpha - \beta) |1\rangle
\]
A Controlled NOT gate has got two qubits, the first one is the control qubit and the second one is the target qubit. If the control qubit is $|0\rangle$, the target qubit is left unchanged. If the control qubit is $|1\rangle$, the target qubit is flipped like a NOT gate. Graph models for both cases are shown in Figure 5(a) and for superposition of states in Figure 5(b).

\[
\begin{align*}
CNOT(\alpha|0\rangle + \beta|1\rangle) &= (\alpha|0\rangle + \beta|1\rangle) & \text{if } C = |0\rangle \\
CNOT(\alpha|0\rangle + \beta|1\rangle) &= (\beta|0\rangle + \alpha|1\rangle) & \text{if } C = |1\rangle
\end{align*}
\]
\[ |a_{i,0}, b_{i,1}⟩ |A⟩ \quad \text{CNOT} \quad |A⟩ |a_{\phi,0}, b_{\phi,1}⟩ \]
\[ |a_{i,0}, b_{i,1}⟩ |B⟩ \quad \varphi \quad |B \oplus A⟩ |a_{\phi,0}, b_{\phi,1}⟩ \]

CNOT gate

\[
\begin{array}{ccc}
\text{CONTROL} & \text{INPUT} & \text{TARGET} \\
C|0⟩ & C|1⟩ & |0⟩ & |1⟩ \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{CONTROL} & \text{TARGET} & \text{OUTPUT} \\
C|0⟩ & C|1⟩ & |0⟩ & |1⟩ \\
\end{array}
\]

\[
\begin{array}{cccc}
\begin{array}{cc}
a₀ & b₀ \\
1 & G
\end{array}
& \begin{array}{cc}
a₁ & b₁ \\
G & G
\end{array}
& \begin{array}{cc}
a₀ & b₀ \\
\alpha & G
\end{array}
& \begin{array}{cc}
a₁ & b₁ \\
\beta & G
\end{array}
\end{array}
\Rightarrow
\begin{array}{cccc}
\begin{array}{cc}
a₀ & b₀ \\
1 & G
\end{array}
& \begin{array}{cc}
a₁ & b₁ \\
G & G
\end{array}
& \begin{array}{cc}
a₀ & b₀ \\
\alpha & G
\end{array}
& \begin{array}{cc}
a₁ & b₁ \\
\beta & G
\end{array}
\end{array}
\]

\[
(1)
\]

\[
\begin{array}{cccc}
\begin{array}{cc}
a₀ & b₀ \\
1 & G
\end{array}
& \begin{array}{cc}
a₁ & b₁ \\
G & G
\end{array}
& \begin{array}{cc}
a₀ & b₀ \\
\alpha & G
\end{array}
& \begin{array}{cc}
a₁ & b₁ \\
\beta & G
\end{array}
\end{array}
\Rightarrow
\begin{array}{cccc}
\begin{array}{cc}
a₀ & b₀ \\
1 & G
\end{array}
& \begin{array}{cc}
a₁ & b₁ \\
G & G
\end{array}
& \begin{array}{cc}
a₀ & b₀ \\
\beta & G
\end{array}
& \begin{array}{cc}
a₁ & b₁ \\
\alpha & G
\end{array}
\end{array}
\]

\[
(2)
\]
If $C = |0\rangle$
\[
\text{CNOT} \left\{ y^F \overline{X} = \begin{bmatrix} y_0 & y_1 \end{bmatrix} \begin{bmatrix} |0\rangle & \overline{x_0} \\ |1\rangle & \overline{x_1} \end{bmatrix} \right\} = \begin{bmatrix} y_0 & y_1 \end{bmatrix} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix} = Y^F \overline{X}
\]

If $C = |1\rangle$
\[
\text{CNOT} \left\{ y^F \overline{X} = \begin{bmatrix} y_0 & y_1 \end{bmatrix} \begin{bmatrix} |0\rangle & \overline{x_0} \\ |1\rangle & \overline{x_1} \end{bmatrix} \right\} = \begin{bmatrix} y_1 & y_0 \end{bmatrix} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix} = Y^F \overline{X}
\]

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>00\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>01\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>10\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>11\rangle$</td>
</tr>
</tbody>
</table>

Truth Table of a CNOT gate
Figure 5(b): CNOT Gate Operation for superposition of states

\[ Y_\phi = U_{\phi_Y} Y_i \]

\[
\begin{bmatrix}
Y^0_\phi \\
Y^1_\phi
\end{bmatrix} =
\begin{bmatrix}
I_{4\times4} & 0 & 0 & 0 \\
0 & I_{4\times4} & 0 & 0 \\
0 & 0 & 0 & I_{4\times4} \\
0 & 0 & I_{4\times4} & 0
\end{bmatrix}
\begin{bmatrix}
Y^0_i \\
Y^1_i
\end{bmatrix}
\]

where

\[ Y_i \triangleq \begin{bmatrix}
[1 \ 0 \ 1 \ 0]^T \\
[1 \ 0 \ 0 \ 1]^T \\
[0 \ 1 \ 1 \ 0]^T \\
[0 \ 1 \ 0 \ 1]^T
\end{bmatrix} \begin{bmatrix}
|00\rangle \\
|01\rangle \\
|10\rangle \\
|11\rangle
\end{bmatrix} \]

and

\[ Y_\phi \triangleq \begin{bmatrix}
[1 \ 0 \ 1 \ 0]^T \\
[1 \ 0 \ 0 \ 1]^T \\
[0 \ 1 \ 1 \ 0]^T \\
[0 \ 1 \ 0 \ 1]^T
\end{bmatrix} \begin{bmatrix}
|00\rangle \\
|01\rangle \\
|10\rangle \\
|11\rangle
\end{bmatrix} \]
A swap gate is also a 2-input gate. It takes two qubits as inputs and interchanges them. Thus, it transforms \((\alpha_1 |0\rangle + \beta_1 |1\rangle); (\alpha_2 |0\rangle + \beta_2 |1\rangle) \rightarrow (\alpha_2 |0\rangle + \beta_2 |1\rangle); (\alpha_1 |0\rangle + \beta_1 |1\rangle)\). Graph model for the same is shown in Figure 6.

\[
S\{(\alpha_1 |0\rangle + \beta_1 |1\rangle); (\alpha_2 |0\rangle + \beta_2 |1\rangle)\} \rightarrow \{(\alpha_2 |0\rangle + \beta_2 |1\rangle); (\alpha_1 |0\rangle + \beta_1 |1\rangle)\}
\]
Bell (Entangled States)

Bell states are called as *EPR states* or *EPR pairs* after Einstein, Podolsky and Rosen who first pointed out the strange properties of these states which find use in several quantum applications like quantum teleportation. These states are entangled and have the property that their state $|a\rangle \otimes |b\rangle$ cannot be decomposed as $|ab\rangle$. The four Bell states can also be expressed in graph form as shown in Figure 7. The technique improves upon those presented earlier [1,2] in that no hyper-edges are needed to denote entanglement.

$$|\psi_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\psi_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$
\[ |\beta_0\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \]

\[ |\beta_1\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}} \]

Figure 7
Quantum Teleportation

Quantum teleportation is a technique of moving quantum states around, even in the absence of a quantum communications channel linking the sender of the quantum bit to the recipient. Suppose Alice and Bob are two friends who met a long time ago. Before they separated, they generated an EPR pair and each one of them took one qubit of the EPR pair as they went different ways. After several years, Alice wants to send a qubit $|\psi\rangle$ to Bob. However, she can send only classical information to Bob. The solution lies in quantum teleportation with the quantum circuit employed and the graph models of states of their qubits in the three stages shown in Figure 8 and Figure 9 respectively.
Stages in Quantum Teleportation

Sending End S  Stage 0  Stage 1  Stage 2  Stage 3  Stage 4 Receiving End R
Sending End Stage: Field Terminal Graph

Single Qubit

\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

Energy Excitation at Port 1

Double Qubit

Quantum Entangled Bell State

\[ |\psi_{\text{bo}}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \]

Energy Excitation at Port 2 and Port 3
Boundary Condition: Terminal Representation

\[
Y_{\psi_0'}^{s_1} \triangleq \begin{bmatrix} Y_{\psi_0}^{s_1} \\ Y_{\psi_0}^{s_2} \\ Y_{\psi_0}^{s_3} \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad Y_{\psi_0}^{s_2-s_3} \triangleq Y_{\psi_0}^{s_2} - Y_{\psi_0}^{s_3} \quad \text{QE} \quad Y_{\psi_0}^{s_3} \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} Y_{\psi_0}^{s_3} \\ Y_{\psi_0}^{s_3} \end{bmatrix} \]

Step I

\[
|\psi_0\rangle = |\psi\rangle \beta_{00} \\
= \frac{1}{\sqrt{2}} \left[ \alpha |0\rangle (|00\rangle + |11\rangle) + \beta |1\rangle (|00\rangle + |11\rangle) \right] = \frac{\alpha}{\sqrt{2}} |000\rangle + \frac{\alpha}{\sqrt{2}} |011\rangle + \frac{\beta}{\sqrt{2}} |100\rangle + \frac{\beta}{\sqrt{2}} |111\rangle
\]

Stage 0:

\[
|\psi_0\rangle = |\psi\rangle \beta_{00} = (\alpha |0\rangle + \beta |1\rangle) \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \\
= \frac{\alpha}{\sqrt{2}} |000\rangle + \frac{\alpha}{\sqrt{2}} |011\rangle + \frac{\beta}{\sqrt{2}} |100\rangle + \frac{\beta}{\sqrt{2}} |111\rangle
\]
Field Terminal Graph

$Y_s: a_1^s \rightarrow a_2^s \rightarrow a_3^s \rightarrow G \rightarrow U \rightarrow b_2^s \rightarrow b_3^s \rightarrow G \rightarrow U \rightarrow b_1^s \rightarrow a_2^s \rightarrow a_3^s \rightarrow G \rightarrow U \rightarrow b_1^s \rightarrow b_2^s \rightarrow b_3^s \rightarrow G$
Step II

\[ |\psi_1\rangle = \frac{1}{\sqrt{2}} \left[ \alpha |0\rangle\langle 00| + |1\rangle + \beta |1\rangle\langle 10| + |01\rangle \right] = \frac{\alpha}{\sqrt{2}} |000\rangle + \frac{\alpha}{\sqrt{2}} |011\rangle + \frac{\beta}{\sqrt{2}} |110\rangle + \frac{\beta}{\sqrt{2}} |101\rangle \]

STAGE 1: \[ |\psi_1\rangle \]

Field Terminal Graph

\[ Y_3': \]

\[ G \quad G \quad G \quad G \quad G \quad G \]

\[ U \quad U \quad U \quad U \quad U \quad U \]

Diagram of Field Terminal Graph
Field Terminal Graph

\[ Y_1^{\uparrow} : \frac{\alpha}{\sqrt{2}} \quad \frac{\alpha}{\sqrt{2}} \quad \frac{\alpha}{\sqrt{2}} \quad \frac{\alpha}{\sqrt{2}} \quad \frac{\beta}{\sqrt{2}} \quad \frac{\beta}{\sqrt{2}} \quad \frac{\beta}{\sqrt{2}} \quad \frac{\beta}{\sqrt{2}} \]
Step III

\[ |\psi_2\rangle = \frac{1}{\sqrt{2}} \left[ \alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle) \right] \]  

(depicted in Graph in Figure 9)

\[ = \frac{\alpha}{\sqrt{2}} |000\rangle + \frac{\alpha}{\sqrt{2}} |011\rangle + \frac{\alpha}{\sqrt{2}} |100\rangle + \frac{\alpha}{\sqrt{2}} |111\rangle + \frac{\beta}{\sqrt{2}} |010\rangle + \frac{\beta}{\sqrt{2}} |001\rangle - \frac{\beta}{\sqrt{2}} |110\rangle - \frac{\beta}{\sqrt{2}} |101\rangle \]

Can also be re-arranged as:

\[ |\psi_2\rangle = \frac{1}{\sqrt{2}} \left[ |00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle) \right] = \]
\[ |\psi_1\rangle = \frac{\alpha}{\sqrt{2}} |000\rangle + \frac{\alpha}{\sqrt{2}} |011\rangle + \frac{\alpha}{\sqrt{2}} |100\rangle + \frac{\alpha}{\sqrt{2}} |111\rangle + \frac{\beta}{\sqrt{2}} |010\rangle + \frac{\beta}{\sqrt{2}} |001\rangle - \frac{\beta}{\sqrt{2}} |110\rangle - \frac{\beta}{\sqrt{2}} |101\rangle \]

**STAGE 2 :** \[ |\psi_2\rangle \]

Field Terminal Graph

\[ Y_i^2 : \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ a_1^2 \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ a_2 \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ a_3 \]

\[ b_1^2 \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ b_2 \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ b_3 \]

\[ G \]

\[ U \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ G \]

\[ U \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ G \]

\[ U \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ G \]

\[ U \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ G \]

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\[ \alpha \frac{\sqrt{2}}{} \]

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\[ U \]

\[ \alpha \frac{\sqrt{2}}{} \]

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\[ G \]

\[ U \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ G \]

\[ U \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ G \]

\[ U \]

\[ \alpha \frac{\sqrt{2}}{} \]

\[ G \]
Field Terminal Graph

\[ Y^2_\phi : \]

\[ a_1^2 \quad a_2^2 \quad a_3^2 \quad a_1' \quad b_2^2 \quad b_3^2 \quad b_1^2 \quad a_2^2 \quad a_3^2 \quad b_1^2 \quad b_2^2 \quad b_3^2 \]

\[ G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \]

\[ a_1' \quad b_2' \quad a_3' \quad a_1' \quad a_2' \quad b_3' \quad b_1' \quad b_2' \quad a_3' \quad b_1' \quad a_2' \quad b_3' \]

\[ G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \quad G \]
Stage 3 : $|\psi_3\rangle$

Field Terminal Graph

\[ Y^3_{\phi} \{ \begin{array}{c} a_1^3 \\ a_2^3 \\ a_3^3 \\ b_1^3 \\ a_3^3 \\ b_3^3 \end{array} \} \{ \begin{array}{c} G \\ G \\ U \\ U \\ G \\ G \end{array} \} \{ \begin{array}{c} a_1^3 \\ b_2^3 \\ a_3^3 \\ b_2^3 \\ a_3^3 \\ b_3^3 \end{array} \} \{ \begin{array}{c} G \\ G \\ U \\ U \\ G \\ G \end{array} \} \{ \begin{array}{c} U \\ U \\ U \\ U \end{array} \} \]
Terminal Characteristics
Transfer Function Representation : Measurement Gate $M_1M_2$

\[
Y_s = \begin{bmatrix}
1 \\
1 \\
\alpha \\
\beta \\
1 \\
1 \\
\alpha \\
-\beta \\
1 \\
1 \\
\alpha \\
-\beta \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\sqrt{3}}{\alpha} \\
0 \\
\frac{\sqrt{3}}{\alpha} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\frac{\sqrt{3}}{\alpha} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
Y_{00} \\
Y_{01} \\
Y_{10} \\
Y_{11} \\
Y_{02} \\
Y_{12} \\
Y_{20} \\
Y_{21} \\
Y_{22} \\
Y_{00} \\
Y_{01} \\
Y_{02} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\alpha/\sqrt{2} \\
\end{bmatrix}
\]

TM$_1$M$_2$
Receiving end (Bob): Triple Qubit Input, Single Qubit Output, Pauli Gate

(Convolved Qubit Controlled Pauli (I/X/Y/Z) Gate)

Controlled Pauli I/X/Y/Z Gate

\[ |\psi_3\rangle_I \]

Con Double Qubit CC representing state of the entangled double qubit of sending end (Alice) of EPR pair shared with receiver end (Bob)

\[ |\psi_2\rangle_II \]

(i) : If CC = 00, apply Pauli I gate
(ii) : If CC = 01, apply Pauli X Gate
(iii) : If CC = 10, apply Pauli Z Gate
(iv) : If CC = 11, first apply a Pauli X gate and then a Pauli Z gate

\[ |\psi_4\rangle_III = \alpha|0\rangle + \beta|1\rangle \equiv |\psi\rangle \]

Output Target Qubit of Receiving End (Bob) Teleported from Sending End (Alice)

[ Instructions conveyed from Sending End (Alice) to Receiving End (Bob) over Classical Communication Channel]

Figure 10: Block Diagram for Quantum Teleportation
After Alice conveys her measurement CC to Bob, there are 4 conditions

**Condition (i)**

If CC = $|00\rangle$, apply Pauli I gate

\[ Y_i^3 \]
**Condition (ii)**

If $CC = |01\rangle$, apply Pauli X Gate

\[
|\psi_3\rangle = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}
\]

Pauli X matrix

\[
\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = |\psi_4\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}
\]

C\text{\textsubscript{I}}

\[
|0\rangle \quad |1\rangle
\]

C\text{\textsubscript{II}}

\[
|0\rangle \quad |1\rangle
\]

G

\[
1 \quad 1
\]

\[
\beta \quad \alpha
\]

\[
\Rightarrow \quad \alpha \quad \beta
\]

G

G

G

G

G

G
Condition (iii)
If $CC = |10\rangle$, apply Pauli Z Gate

\[ |\psi_3\rangle = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix} \]

Pauli Z matrix:
\[ \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = |\psi_4\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} |\psi_5\rangle = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix} \]
Condition (iv)

If \( CC = |11\rangle \), first apply Pauli X Gate and then Pauli Z Gate

\[
|\psi_5\rangle = \begin{bmatrix} -\beta \\ \alpha \end{bmatrix}
\]

Pauli Z matrix

\[
\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = |\psi_4\rangle = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \left\{ |\psi_5\rangle = \begin{bmatrix} -\beta \\ \alpha \end{bmatrix} \right\}
\]

Pauli XZ Matrix

\[
\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = |\psi_4\rangle = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \left\{ |\psi_5\rangle = \begin{bmatrix} -\beta \\ \alpha \end{bmatrix} \right\}
\]
Stage 4: $|\psi_4\rangle$

Field Terminal Graph
Terminal Characteristics
Transfer Function Representation: Controlled Pauli (I, X, Z, XZ) Gates

\[
Y^4 = T_{Pauli Gates} Y^3
\]

\[
Y^4_{\phi} = Y^3_{\phi \rightarrow i}
\]

Pauli Gates (I, X, Z, XZ)

\[
Y^4_{\phi} = T_{Pauli Gates} Y^3_i
\]

\[
Y^4_{\phi \rightarrow 3} = \text{Receiving End Teleported Qubit s. t. } \left| \psi_{\phi} \right>_3 = \left| \psi \right>_i
\]
Summary

- The quantum graph theoretic field model is facilitated by considering a qubit and developing a “ket 0” subgraph and a “ket 1” subgraph.

- The system graph is in separate parts corresponding to a qubit \( |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \), consisting of “ket 0” and “ket 1” subsystem graphs.

- Unit “Poynting” vectors \( |0\rangle \) and \( |1\rangle \) behave like quantum across (potential) variables specifying “direction” of energy propagation, as it were, while \( \alpha \) and \( \beta \) behave like quantum through (flow-rate or “force”) variables specifying quantity of energy flow-rate (energy flux or power flow, \( W/m^2 \)).

- For \( n \) independent states, there will be precisely \( n \) basis quantum potential vectors (or unit “Poynting” vectors) to be more precise. The model has been successfully applied for several quantum gates as well as applications such as quantum teleportation.
Theory of ‘Many Things’

• In terms of systems modelling, instead of the system scientists and practitioners getting bogged down with similar dreams of modelling theory of everything, one could perhaps look for ‘modelling theory of many things’.

• There is certainly a possibility of founding such a unified modelling theory of many things, not everything, based on linear graph theory and unified field theory, not just quantum or string theory but perhaps some futuristic M-string theory or whatever that qualifies as the grand unified theory of so-called everything.

• This ‘modelling theory of many things’, will then span perhaps all kinds of systems, including natural systems, designed physical systems, designed abstract systems and human-activity systems [8].
High-End Complexity Systems

- All kinds of systems can give rise to their own peculiar problems, which can be attempted to be resolved to the extent possible.

- Kristy Kitto [1] prepared a **complexity scale** for systems ranging from
  - **Simple** (e.g. projectile motion, billiard balls, thermodynamic equilibrium, microeconomics etc. amenable to Newtonian mechanics, thermodynamics, computational complexity, algorithmic information theory etc.) through
  - **Complicated** (e.g. weather dynamics, food webs etc. amenable to chaos / fractals, statistical mechanics, catastrophe theory, network theory etc.)
  to
  - **Complex** (e.g. bound states, quantum tunneling, electron and photon behaviour, genetic regulatory networks, quark and gluon behaviour, biological development, evolution of mind, language, societies . . . at the high-end amenable to quantum field theory, evolution and natural selection, post modernism etc.)

- It is at this higher end of complexity scale that the unified quantum field graph theory holds considerable promise and potential for modelling systems successfully [8].
Quantum field graph theory approach, which appears only as an alternative in the present work, would emerge as the modelling tool of choice therein.
Conclusions

- A modelling theory founded on linear graph theory and quantum force-fields is indeed capable of modelling a large variety of Quantum Information Processing systems.
References


THANK YOU