Assignment 3: CHM221A
Topic: Intensive properties and equation of states-II

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To students: These assignments are designed to strengthen your understanding in the lessons taught in class as well as develop your problem solving skills. If you copy from other sources without understanding, those purposes will be defeated and you will receive no benefits. **If you cannot solve them, *try* and bring your problems to the instructor. He will definitely help. Also, there may be some typo - discuss with me if you think there are any.**

1. For a solid with N lattice points and l quanta of energy, the entropy is

$$S = k_B \left((l+N)\ln(l+N) - N\ln(N) - l\ln(l)\right)$$

and the total energy is

$$U = \frac{N\epsilon}{2} + l\epsilon$$

where ϵ is a fundamental quantity of energy.

   (a) Find out the ***heat capacity*** of the system given by $C_V = \left(\frac{\partial U}{\partial T}\right)_V$ as a function of temparature T.

   (b) Plot C_V *vs* T.

   (c) Show that

$$\lim_{T\to\infty} C_V = 3Nk_B.$$

2. For a star of volume V and temperature T the energy

$$U = \sigma V T^4$$

and the pressure

$$P = \frac{U}{3V}.$$

σ is the Stefan-Boltzmann constant. Find out the entropy S for this star.

3. For a system with equations of state

$$(P + \frac{a}{v^2})(V - b) = RT$$

and

$$u = cRT - \frac{a}{v}.$$

Here a and b are constants.

(a) Find out the expression of entropy S for this system with respect to a reference state with entropy S_0.

(b) Are these equations of states valid for $T \to 0$?

4. Two containers, kept at constant temperature T, of volume V contain 1 mole of CH_4 and 2 moles of Ne gases, respectively. Suddenly, the wall between them has been made to permeable for Ne only. After a new equilibrium reaches, find out the mole numbers and pressures in each containers.

5. Show that the total pressure P for a mixture of l ideal gases with mole numbers N_i can be written as a sum of ***partial pressure*** $P_i = \frac{N_i RT}{V}$.

6. For a system obeys

$$U = \frac{1}{2}PV$$

and

$$T^2 = \frac{AU^{3/2}}{VN^{1/2}}$$

where A is a positive constant. Find out the fundamental relation.