DNS of the Turbulence Decay in Liquid Metal Flow Under an Increasing Magnetic Field

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Department of Mechanical and Manufacturing Engineering
University of Cyprus

International conference on MHD turbulence
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- The blanket concept is based on **liquid-metal flow**.
- The plasma is confined using a **helical magnetic field**.
- The section of the pipes is expected to be **circular**.
Circular Pipe Turbulent Flow

Turbulence
- Generally three-dimensional
Circular Pipe Turbulent Flow

Turbulence

- Generally three-dimensional
- Generated by wall instabilities
Circular Pipe Turbulent Flow

Turbulence
- Generally three-dimensional
- Generated by wall instabilities
- Mixing of heat, momentum and mass is enhanced.
Circular Pipe Magnetohydrodynamic flow

Effect of the magnetic field on electrically conducting fluid

- Induced currents
  \[ \mathbf{j} \rightarrow -\nabla \phi + \mathbf{u} \times \mathbf{B} \]
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- The current loops define the local effect of the Lorentz force

- Turbulence is systematically suppressed by magnetic fields \((Ha/Re > 0.025)\)
Objective: Magnetohydrodynamics & Turbulence

The action of strong magnetic fields suppresses turbulence
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Fringing magnetic fields are related to three-dimensional effects
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- Three-dimensional effects increase the complexity and instabilities of the flow
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- The action of **strong** magnetic fields suppresses turbulence
- Fringing magnetic fields are related to three-dimensional effects
- Three-dimensional effects increase the complexity and instabilities of the flow
- Coexistence of the two phenomena is here investigated
Experimental Configuration. 
ALEX facility, 
Argonne National Lab., Illinois, USA.
Case

Case addressed by current work. Analyzed for the very first time. Role of inertia disqualifies the use of asymptotic methods. There are also important experimental accuracy problems. Is DNS an option?
Background (i)

Experimental work, decreasing magnetic field. Upstream conditions $Ha = 6640, N = 10700$ (ALEX Results: Picologlou et al.. 1986, Reed et al.. 1987 )
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- Attention to the accuracy in Experiments!!

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- The ERROR signal output of the local magnetic field becomes very important (negative value!)... The last ACCEPTABLE value is at $Ha = 60!$ ($17R$ downstream of the high mag. field area)
- For a Increasing Magnetic field: Experimental analysis present enormous difficulties to provide accurate information about the transition from (turbulent) HD flow to strong MHD flow ($Ha/Re > 0.025$, $Ha \approx 100$ turbulence decayment)
Background (ii). Size of the problem?

How to characterize this area? What is the most reliable description of the evolution of the magnetic field from HD conditions to $Ha \approx 50$ and beyond?
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  - Computational domain of at least $50R$ for conducting cases, even more for insulating cases! (development length!, three-dimensional effects of $R \times O(Ha^{1/2})$) R. Holroyd and J. S. Walker, Journal of Fluid Mechanics 84, 79 (1978)
Governing Equations

Conservation of mass, momentum and charge:

\[ \nabla \cdot \mathbf{u} = 0 \]  
\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + N(\mathbf{j} \times \mathbf{B}) \]  
\[ \nabla \cdot \mathbf{j} = 0 \]

Ohm’s Law:

\[ \mathbf{j} = \sigma ( -\nabla \phi + \mathbf{u} \times \mathbf{B}) \]  
\[ \nabla^2 \phi = \nabla \cdot (\mathbf{u} \times \mathbf{B}) \]

\[ \nu \text{ viscosity} \]
\[ \sigma \text{ conductivity} \]
\[ \rho \text{ density} \]
\[ p \text{ pressure} \]
\[ t \text{ time} \]
\[ \phi \text{ electric potential} \]
\[ \mathbf{B} \text{ magnetic field} \]

\[ Re = \frac{U_b R}{\nu} \]
\[ N = \frac{R \sigma B^2}{\rho U_b} \]
\[ Ha = RB \sqrt{\frac{\sigma}{\rho \nu}} \]
Boundary Conditions

- **Wall:**
  - \( \vec{u}_{wall} = 0 \), (No slip)
  - \( \frac{\partial p}{\partial n} |_{wall} = 0 \),
  - \( \frac{\partial \phi}{\partial n} |_{wall} = -[\nabla \tau \cdot (c \nabla \tau \phi_w)] \), J. S. Walker, Journal de Mécanique 20, 79 (1981).

- **Inlet:**
  - \( \vec{u}_{inlet} = \text{defined} \)
  - \( p_{inlet} \rightarrow \text{inlet penalty b.c.} \),
  - \( \frac{\partial \phi}{\partial n} |_{inlet} = 0 \)

- **Outlet:**
  - \( \frac{\partial \vec{u}_{outlet}}{\partial t} + U_{conv} \frac{\partial \vec{u}_{outlet}}{\partial n} = 0 \), (Convective)
  - \( p_{outlet} \rightarrow \text{outlet penalty b.c.} \),
  - \( \frac{\partial \phi}{\partial n} |_{outlet} = 0 \)
Numerical Method, Grid Generation

- Unstructured Nodal-based Finite Volume Code
- Collocated scheme
- Fractional-step method (Chorin, 1968)
- Time discretization: Crank-Nicholson / Semi-implicit / Explicit integration of the Lorentz Force
- Conservative computation of the Lorentz force (Ni, 2007)
- Direct Numerical Simulation (no model used)

- Hybrid quads/triangles b.layer/core
- Very strong near-wall stretching to resolve *Ha layers*
- Core designed to resolve turbulence scales
Numerical Method, Grid Generation

- Very strong near-wall stretching to resolve $Ha$ layers (up to $Ha \approx 10000$)
  \[ \Delta r_{min}^+ = 0.013 \]
  \[ \Delta r_{min}/R = 0.00005 \]
- Core $\rightarrow$ turbulence scales at $Re_\tau \approx 520$ \[ \Delta x^+ = 10.4, \] \[ \Delta \theta_{max}^+ = 7.2, \Delta r_{max}^+ \sim 7 \]
Numerical Method, Grid Generation

SUMMARY

- Explicit Lorentz Force...

  TIME-STEPING REQUIREMENTS!
  \[ \Delta t \leq \frac{2}{\sigma B^2} \rightarrow Ha = 7000 \rightarrow 3 \cdot 10^5 \text{ time-steps are needed to compute one turn-over time} \]

- (development length = 50R)

  ESTIMATED MESH REQUIREMENTS! \[\rightarrow 17 M^6\text{ elements} \]

Something needs to be done... impossible to address...
Numerical Strategy

**Case & Background**
- $Ha=500$, $N=62.7$
- $Ha=50$, $N=0.62$

**Introduction**

**Formulation & Methodology**

**Results**

**Concluding remarks**
Numerical Strategy

Magnets’ magnetic field generates $Ha = 50$ conditions

$$B_y = 50 \frac{1 + \tanh[0.45(x/R)]}{2}$$

Magnets’ magnetic field generates $Ha = 500$ conditions

$$B_y = 500 \frac{1 + \tanh[0.45(x/R)]}{2}$$

it is possible to find a shift to match both values without assuming too much... why?
Numerical Strategy

\[
\frac{\partial (\tanh(x))}{\partial x} = 1 - \tanh^2(x) \quad \text{so...}
\]

\[
\frac{\partial (\tanh(x))}{\partial x} \rightarrow 0 \quad \text{faster than} \quad \tanh(x) \quad \text{for} \quad x \rightarrow -\infty
\]

we choose appropriate \( x/R \) which, in combination with the shift (while matching both magnetic field values), produces a difference in the slope that is small ...

This feature allows to reuse computational information and extend further downstream our domain, very useful to assess inertia effects!!!
Numerical Strategy

Flow

\[ \begin{align*}
\text{AB} & \quad \text{Hydrodynamic flow} \quad \text{Re}_b \approx 4000 \\
\text{BC} & \quad \text{Rapid transition to MHD flow, decayment of turbulence} \\
\text{CD} & \quad \text{Constant magnetic field, fully-developed MHD flow?}
\end{align*} \]
Numerical Strategy

- Flow
- Periodic Boundary-conditions
- Hydrodynamic flow $Re_b \approx 4000$
- Time-history...t, u,v,w
Numerical Strategy

Flow: inlet defined in time and position

$D \quad 20R \quad R \quad B$
Flow: inlet defined in time and position

Rapid transition to MHD flow (I) (low N, low Ha)

Rapid transition to MHD flow (II)

Constant magnetic field, fully-developed MHD flow?
Numerical Strategy

Flow: inlet defined \((u,v,w)\) in time and position

\[ C'C'' \] Rapid transition to MHD flow (II)

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Results
Verification. Connecting magnetic fields

- The information is accurately transferred to the new domain, identical magnetic fields

- Evaluation of the injection into higher magnetic field under process...
Validation & Verification. Hydrodynamic flow.

- Accurate resolution of the Fully-developed turbulent flow at $Re_{\tau} \approx 520 \rightarrow$
- Resolution of the turbulence scales, near-wall viscous sublayer

The injection profile is also assessed...while evaluating inlet/outlet effects...perfect agreement regarding momentum!!
**Validation. MagnetoHydrodynamic flow.**

- Accurate resolution of the fully-developed MHD flow up to $Ha = 7000$

Resolution of the $Ha$ layers and side layers

$Ha = 10, Ha = 50, Ha = 2000, Ha = 7000$

![Graphs showing validation of magneto-hydrodynamic flow](image-url)
Transition to low MHD conditions ($Ha = 50$) (1/3).
Transition to low MHD conditions \((Ha = 50)\) (2/3).

- The mean velocity evolves to a "flatter" profile, \(Ha\) and \(Roberts\) layers effects.
- The MHD pressure drop is higher than the turbulent one. Conductivity effects increase the pressured-drop requirements due to the net braking Lorentz force.
- The turbulent fluctuating levels decrease non-homogeneously! Overall agreement with experimental observation for the turbulence decay \((Ha/Re > 0.025)\), as the turbulence indicators are not completely dampened out.
The reduction of the fluctuating statistics is observed for $Ha > 25$, $N > 0.3$...rapid slope of the magnetic field?

The reduction of the fluctuating statistics takes places in a in-homogenous way, being the conducting $Ha$ layer more effective to organize the flow and reduce the turbulence levels.

The inertial range of the power spectra is reduced by the magnetic field ($-5/3 \rightarrow -3$) (agreement with spectral analysis...)

Power Spectrum (4 turn-over time) $z/R=0.95$
Transition to moderate MHD conditions (1/4)

Wall-conductivity effects \((c = 0.027, c = 0)\)
Transition to moderate MHD conditions (1/4)
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\(c = 0.0\)

\(c = 0.027\)
Transition to moderate MHD conditions (2/4)
Wall-conductivity effects \((c = 0.027, c = 0)\)
Transition to moderate MHD conditions (3/4)

\[(Ha = 500, c = 0.0)\]

- Overspeed-areas are generated due to the braking Lorentz force. A fully-developed profile is not completely achieved (inertial terms...)

- Important three-dimensional effects take place. The MHD pressure drop is one order of magnitude larger than the HD one.

- The (turbulent) fluctuations are completely dampened out for values \( Ha >\approx 300 \)
  \( N >\approx 25 \)...However, important fluctuating levels take place... LAMINAR magnetic flow instabilities!!!
A rapid transition to the fully-developed flat profile is achieved by the moderate magnetic field. Important jets take place, but they are smaller respect the insulating ones (wall-conductivity effect)!

The HD pressure drop is negligible respect to the MHD one (2 orders!×HD) due to the strong braking Lorentz force in the downstream area. Inertial effects are masked.

The (turbulent) fluctuations are completely dampened out for values $Ha >\approx 300$, $N >\approx 25$, the fluctuating effect of insulating configurations does not take place... (wall-conductivity effect)
Concluding Remarks

- Present results agree with experiments \((Ha/Re > 0.025)\) when predicting the decayment of the turbulence produced by the magnetic field.
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- Important over-speed areas are generated by the **moderate and strong** increasing magnetic fields. Conductivity reduces the intensity of the jets and mask inertial terms role. Three-dimensional effects are quickly dissipated
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- Pulsating phenomena are generated close to the edge-pole face of the magnet for the insulating case. Turbulence is completely suppressed for \(N \approx 25, Ha \approx 300\). Additional moderate mag. fields should be analyzed...