Viscosity in Accretion Discs

Owen Matthews
Kanpur - 22nd December 2009
Why study accretion discs?

• Accretion discs are ubiquitous
  – AGN
  – Interacting binaries such as dwarf novae
  – Around young stars
Why study accretion discs?

- Accretion discs are ubiquitous
  - AGN
  - Interacting binaries such as dwarf novae
  - Around young stars
Why study accretion discs?

- Accretion discs are ubiquitous
  - AGN
  - Interacting binaries such as dwarf novae
  - Around young stars
Why study viscosity in accretion discs?

- Viscosity is an ill-understood but important process which regulates:
  - Accretion rate
  - Disc emission and observational signature
  - And therefore controls disc structure and so:
    - Planet formation
    - Planet migration…
  - And the disc time evolution and so:
    - Outburst behaviour
    - Stellar spin…

“Viscosity”: Fighting angular momentum

• Keplerian discs live forever unless some mechanism redistributes or removes angular momentum

• For now, we will use the term viscosity for such a mechanism
  • Need not be molecular viscosity
  • Need not be a purely local effect
  • Need not even be a kinematic viscosity

• What it does, without asking how:
  • Kinetic energy is radiated from the disc
  • The net result is a flux of mass inwards
  • Accretion material loses angular momentum
  • To conserve global angular momentum there must be a flux of angular momentum outwards
"Viscosity": Fighting angular momentum

- Keplerian discs live forever unless some mechanism redistributes or removes angular momentum
- For now, we will use the term viscosity for such a mechanism
  - Need not be molecular viscosity
  - Need not be a purely local effect
  - Need not even be a kinematic viscosity
- What it does, without asking how:
  - Kinetic energy is radiated from the disc
  - The net result is a flux of mass inwards
  - Accreting material loses angular momentum
  - To conserve global angular momentum there must be a flux of angular momentum outwards
Illustration: a spreading ring
Illustration: a spreading ring
Illustration: a disc in equilibrium
Illustration: a disc in equilibrium
Shakura-Sunyaev Viscosity

  - Viscosity is given by:

- All our ignorance is now collected in the a parameter
  - This is a very powerful assumption
  - Simulations using a viscosity have been extremely successful in reproducing outburst behaviour
  - Even complex phenomena such as superhumps
  - Gives a full disc model including temperature, scale height...
  - Makes no assumptions about the nature of the viscosity

- Standard T Tauri accretion rates of $\sim 10^{-7} \text{M}_{\text{sun}}/\text{yr}$ require a
Shakura-Sunyaev Viscosity

  - Viscosity is given by:
    \[ n = \text{mean speed} \times \text{mean free path} \]

- All our ignorance is now collected in the \( a \) parameter
  - This is a very powerful assumption
  - Simulations using a viscosity have been extremely successful in reproducing outburst behaviour
  - Even complex phenomena such as superhumps
  - Gives a full disc model including temperature, scale height…
  - Makes no assumptions about the nature of the viscosity

- Standard T Tauri accretion rates of \( \sim 10^{-7} \, M_{\text{sun}}/\text{yr} \) require a
Shakura-Sunyaev Viscosity

  - Viscosity is given by:
    \[ n = \text{mean speed} \times \text{mean free path} \]
    \[ n < c_s \times \text{mean free path} \]

- All our ignorance is now collected in the \( a \) parameter
  - This is a very powerful assumption
  - Simulations using a viscosity have been extremely successful in reproducing outburst behaviour
  - Even complex phenomena such as superhumps
  - Gives a full disc model including temperature, scale height…
  - Makes no assumptions about the nature of the viscosity

- Standard T Tauri accretion rates of \( \sim 10^{-7} \text{M}_{\odot}/\text{yr} \) require a
Shakura-Sunyaev Viscosity

  - Viscosity is given by:
    \[ n = \text{mean speed} \times \text{mean free path} \]
    \[ n < c_s \times \text{mean free path} \]
    \[ n < c_s H \]

- All our ignorance is now collected in the a parameter
  - This is a very powerful assumption
  - Simulations using a viscosity have been extremely successful in reproducing outburst behaviour
  - Even complex phenomena such as superhumps
  - Gives a full disc model including temperature, scale height…
  - Makes no assumptions about the nature of the viscosity

- Standard T Tauri accretion rates of \( \sim 10^{-7} \, M_{\odot}/\text{yr} \) require a
Shakura-Sunyaev Viscosity

  - Viscosity is given by:
    
    $$n = \text{mean speed} \times \text{mean free path}$$
    $$n < c_s \times \text{mean free path}$$
    $$n < c_s H$$

    $$\nu = \alpha c_s H$$

- All our ignorance is now collected in the $\alpha$ parameter
  - This is a very powerful assumption
  - Simulations using a viscosity have been extremely successful in reproducing outburst behaviour
  - Even complex phenomena such as superhumps
  - Gives a full disc model including temperature, scale height…
  - Makes no assumptions about the nature of the viscosity

- Standard T Tauri accretion rates of $\sim 10^{-7} M_{\text{sun}}/\text{yr}$ require a
Shakura-Sunyaev Viscosity

  - Viscosity is given by:
    \[ n = \text{mean speed} \times \text{mean free path} \]
    \[ n < c_s \times \text{mean free path} \]
    \[ n < c_s H \]
    \[ \nu = \alpha c_s H \]

- All our ignorance is now collected in the a parameter
  - This is a very powerful assumption
  - Simulations using a viscosity have been extremely successful in reproducing outburst behavior
  - Even complex phenomena such as superhumps
  - Gives a full disc model including temperature, scale height…
  - Makes no assumptions about the nature of the viscosity

- Standard T Tauri accretion rates of \(~10^{-7} M_{\text{sun}}/\text{yr}\) require a
Shakura-Sunyaev Viscosity

  - Viscosity is given by:
    \[
    n = \text{mean speed} \times \text{mean free path} \\
    n < c_s \times \text{mean free path} \\
    n < c_s H
    \]
    \[
    \nu = \alpha c_s H
    \]

- All our ignorance is now collected in the a parameter
  - This is a very powerful assumption
  - Simulations using a viscosity have been extremely successful in reproducing outburst behaviour
  - Even complex phenomena such as superhumps
  - Gives a full disc model including temperature, scale height…
  - Makes no assumptions about the nature of the viscosity

- Standard T Tauri accretion rates of \(~10^{-7} M_{\odot}/yr\) require a
Molecular viscosity

Classical (kinematic) viscosity:

$$\nu = \frac{(\text{mean free path})^2}{\text{collision time}} = \text{mean free path} \times \text{mean velocity} = \frac{2 \sqrt{kT}}{3\pi r_0^2 m^{1/2} n}$$
Molecular viscosity

Classical (kinematic) viscosity:

\[ \nu = \frac{(\text{mean free path})^2}{\text{collision time}} = \text{mean free path} \times \text{mean velocity} = \frac{2 \sqrt{kT}}{3\pi r_0^2 m^{1/2} n} \]

Observed parameters (inner disk):

T \sim 600 \text{ K}; \ n \sim 3 \times 10^{-9} \text{ g/cm}^3
Molecular viscosity

Classical (kinematic) viscosity:

\[ \nu = \frac{(\text{mean free path})^2}{\text{collision time}} = \text{mean free path} \times \text{mean velocity} = \frac{2 \sqrt{kT}}{3\pi r_0^2 m^{1/2} n} \]

Observed parameters (inner disk):

T \sim 600 \text{ K}; \ n \sim 3 \times 10^{-9} \text{ g/cm}^3

This yields \( \nu \sim 3 \times 10^5 \text{ cm}^2/\text{s} \) – far too small to explain the observed accretion rates!
Molecular viscosity

Classical (kinematic) viscosity:

\[ \nu = \frac{(\text{mean free path})^2}{\text{collision time}} = \text{mean free path} \times \text{mean velocity} = \frac{2 \sqrt{kT}}{\pi r_0^2 m^{1/2} n} \]

Observed parameters (inner disk):

\[ T \sim 600 \text{ K}; \quad n \sim 3 \times 10^{-9} \text{ g/cm}^3 \]

This yields \( \nu \sim 3 \times 10^5 \text{ cm}^2/\text{s} \) – far too small to explain the observed accretion rates!

Save the concept (Newtonian fluids):

- Increase \( n \) (molecular \( \rightarrow \) eddy, driven by gravitational instability for example)
Classical (kinematic) viscosity:

\[ \nu = \frac{(\text{mean free path})^2}{\text{collision time}} = \text{mean free path} \times \text{mean velocity} = \frac{2\sqrt{kT}}{3\pi r_0 m^{1/2} n} \]

Observed parameters (inner disk):
- T \sim 600 \text{ K}; n \sim 3 \times 10^{-9} \text{ g/cm}^3

This yields \( \nu \sim 3 \times 10^5 \text{ cm}^2/\text{s} \) – far too small to explain the observed accretion rates!

Save the concept (Newtonian fluids):
- Increase n (molecular \( \rightarrow \) eddy, driven by gravitational instability for example)
- Introduce resistivity h, replace HD by MHD (driven by magneto-rotational instability for example...
Magneto-rotational instability

- The MRI is an instability which occurs in magnetised, differentially rotating discs
- It is probably widespread in circumstellar discs since it requires
  - Only weak initial field required (but very strong fields may kill it)
  - Ideal instability, but robust against resistivity
- It is driven by angular momentum transfer but
  - It soon saturates
  - Drives MHD turbulence and hence dissipation through viscosity and resistivity
Magneto-rotational instability
Magneto-rotational instability
Magneto-rotational instability
Magneto-rotational instability
Magneto-rotational instability
Magneto-rotational instability
Magneto-rotational instability


- Accretion disc turbulence, similar to that expected from MRI
- The MRI can now be reproduced in laboratory experiments (eg. Goodman & Kageyama, 2001)
- But simulations are restricted to shearing boxes and/or very short integration times! Not suitable for disc evolution
Magneto-rotational instability

- Accretion disc turbulence, similar to that expected from MRI
- The MRI can now be reproduced in laboratory experiments (eg. Goodman & Kageyama, 2001)
- But simulations are restricted to shearing boxes and/or very short integration times! Not suitable for disc evolution
A possible solution to resolution issues?

- The simulation of an accretion disc requires an enormous dynamic range.
- LES models small scale dissipation by renormalising viscous and resistive terms.
- Simulations in k space produce values for viscosity and resistivity as a function of resolution, magnetic Prandtl number and the dissipation rate.
- Dissipation rates are well known from observations.
Code implementation

Smoothed Particle Hydrodynamics (A Lagrangian particle method)
• Conserves angular momentum so that MHD viscous and resistive effects are easily identified
  
  BUT

• Full MHD is still experimental so MHD turbulence problematic
• Kelvin-Helmholtz type instabilities can be difficult so MRI potentially tricky
• Renormalisation of viscosity has been attempted but is not a well established technique

Eulerian Grid Based Codes (Eg, Flash* – University of Chicago)
• MHD very well developed in these codes; turbulence traditionally modelled in k space
• Kelvin-Helmholtz type instabilities well treated with AMR so MRI feasible
• Renormalisation of viscosity and resistivity well understood and implemented (mainly in k space codes)
  
  BUT

• Does not conserve angular momentum so well, so that numerical viscosity can be confused with ‘real’ viscous effects
Smoothed Particle Hydrodynamics (A Lagrangian particle method)

• Conserves angular momentum so that MHD viscous and resistive effects are easily identified

BUT

• Full MHD is still experimental so MHD turbulence problematic
• Kelvin-Helmholtz type instabilities can be difficult so MRI potentially tricky
• Renormalisation of viscosity has been attempted but is not a well established technique
Eulerian Grid Based Codes
(Eg, Flash* – University of Chicago)

• MHD very well developed in these codes; turbulence traditionally modelled in k space

• Kelvin-Helmholtz type instabilities well treated with AMR so MRI feasible

• Renormalisation of viscosity and resistivity well understood and implemented (mainly in k space codes)

BUT

• Does not conserve angular momentum so well, so that numerical viscosity can be confused with ‘real’ viscous effects
Smoothed Particle Hydrodynamics (A Lagrangian particle method)

- Conserves angular momentum so that MHD viscous and resistive effects are easily identified

  BUT

- Full MHD is still experimental so MHD turbulence problematic
- Kelvin-Helmholtz type instabilities can be difficult so MRI potentially tricky
- Renormalisation of viscosity has been attempted but is not a well established technique

Eulerian Grid Based Codes (Eg, Flash* – University of Chicago)

- MHD very well developed in these codes; turbulence traditionally modelled in k space
- Kelvin-Helmholtz type instabilities well treated with AMR so MRI feasible
- Renormalisation of viscosity and resistivity well understood and implemented (mainly in k space codes)

  BUT

- Does not conserve angular momentum so well, so that numerical viscosity can be confused with ‘real’ viscous effects
Outlook

- We have performed the first simulations of the MRI in SPH.
- We are adding renormalized viscosity and resistivity to the FLASH code.
- The strength of the dissipative terms can be estimated from observations.
- We plan to:
  - Perform self consistent long timescale simulations using realistic dissipative terms
  - Reproduce the complete disc instability outburst cycle by adding ionization dependent cooling functions
  - Add dust distribution and cooling
Turbulent (eddy) viscosity

The MHD equations

\[(\partial_t + u \nabla)u = \frac{\mu_0}{\rho} \mathbf{j} \times \mathbf{b} + \nu \Delta \mathbf{u}\]

\[\partial_t \mathbf{b} = \nabla \times \mathbf{u} \times \mathbf{b} + \eta \Delta \mathbf{b}\]

can in Fourier space be written as

\[(-i \omega + D_0) f_i(k) = T_{ijm}(k) \int d\mathbf{k}' f_j(k) f_m(k - k')\]

where \(\mathbf{f} = (\mathbf{u}, \mathbf{b})\) and \(D_0 = k^2 \begin{pmatrix} \nu & 0 \\ 0 & \eta \end{pmatrix}\).

Goal: replace the non-linear interaction of small scales by an ‘equivalent’
Subgrid modeling: renormalization of $\nu$ and $\eta$

(Wilson, 1975, Advances in Mathematics, 16, 170; Zhou, Vahala & Hossain, 1988, Physical Review)

- Symbolically, $(\partial_t + D_0) \ f = T \ f \ f \ \ldots \ldots \ (1)$
Subgrid modeling: renormalization of \( \nu \) and \( \eta \)

(Wilson, 1975, Advances in Mathematics, 16, 170; Zhou, Vahala & Hossain, 1988, Physical Review)

- Symbolically, \((\partial_t + D_0) f = T f f \) ........ .... (1)
- Split \( f = f^\uparrow + f^\downarrow \) for \(|k| \not< k_1\):
Subgrid modeling: renormalization of $\nu$ and $\eta$
(Wilson, 1975, Advances in Mathematics, 16, 170; Zhou, Vahala & Hossain, 1988, Physical Review

- Symbolically, $(\partial_t + D_0) f = T f f$ ............ (1)

- Split $f = \langle f \rangle + f^< \text{ for } |k| \leftrightarrow k_1$:

  $(\partial_t + D_0) f^< = T (f^< + \langle f \rangle) (f^< + \langle f \rangle) ............ (2)$

  $(\partial_t + D_0) \langle f \rangle = T (f^< + \langle f \rangle) (f^< + \langle f \rangle)$
Subgrid modeling: renormalization of $\nu$ and $\eta$
(Wilson, 1975, Advances in Mathematics, 16, 170; Zhou, Vahala & Hossain, 1988, Physical Review

- Symbolically, $(\partial_t + D_0) f = T f f$ ............ (1)
- Split $f = \hat{f} + f^<$ for $|k| \gg k_1$:
  
  $(\partial_t + D_0) f^< = T (f^< + \hat{f}) (f^< + \hat{f})$ ............ (2)

  $\hat{f} = (\partial_t + D_0)^{-1} T (f^< + \hat{f}) (f^< + \hat{f})$
Subgrid modeling: renormalization of $\nu$ and $\eta$

(Wilson, 1975, Advances in Mathematics, 16, 170; Zhou, Vahala & Hossain, 1988, Physical Review

- Symbolically, $(\partial_t + D_0) f = T f f$ .......... (1)

- Split $f = f^> + f^<$ for $|k| <> k_1$:

  $(\partial_t + D_0) f^< = T (f^< + f^>) (f^< + f^>)$ .......... (2)

  $f^> = (\partial_t + D_0)^{-1} T (f^< + f^>) (f^< + f^>)$
Subgrid modeling: renormalization of \( \nu \) and \( \eta \)

(Wilson, 1975, Advances in Mathematics, 16, 170; Zhou, Vahala & Hossain, 1988, Physical Review)

- Symbolically, \((\partial_t + D_0) \mathbf{f} = \mathbf{T} \mathbf{f} \mathbf{f} \) .......... (1)

- Split \( \mathbf{f} = \mathbf{f}^\uparrow + \mathbf{f}^\downarrow \) for \( |\mathbf{k}| \leftrightarrow k_1 \):

  \[(\partial_t + D_0) \mathbf{f}^\downarrow = \mathbf{T} (\mathbf{f}^\downarrow + \mathbf{f}^\uparrow) (\mathbf{f}^\downarrow + \mathbf{f}^\uparrow) \) .......... (2)

  \[\mathbf{f}^\uparrow = (\partial_t + D_0)^{-1} \mathbf{T} (\mathbf{f}^\downarrow + \mathbf{f}^\uparrow) (\mathbf{f}^\downarrow + \mathbf{f}^\uparrow)\]

- Average Eq. (2) over the 1\(^{st}\) shell \( k_1 <|\mathbf{k}|<k_0 \) (no effect on \( \mathbf{f}^\downarrow \)). Assume Gaussian \( \mathbf{f}^\uparrow \) with \( \langle \mathbf{f}^\uparrow \rangle = 0 \) and isotropic Kolmogorov spectrum \( \langle \mathbf{f}^\uparrow \mathbf{f}^\downarrow \rangle \). Neglect \( \langle \mathbf{f}^\downarrow \mathbf{f}^\downarrow \mathbf{f}^\downarrow \rangle \).
Subgrid modeling: renormalization of $\nu$ and $\eta$

(Wilson, 1975, Advances in Mathematics, 16, 170; Zhou, Vahala & Hossain, 1988, Physical Review)

- Symbolically, $(\partial_t + D_0) f = T f f$ ............ (1)

- Split $f = \mathcal{P} + f^< \text{ for } |k| \leftrightarrow k_1$:

$$
(\partial_t + D_0) f^< = T (f^< + \mathcal{P}) (f^< + \mathcal{P}) ............ (2)
$$

$$
\mathcal{P} = (\partial_t + D_0)^{-1} T (f^< + \mathcal{P}) (f^< + \mathcal{P})
$$

- Average Eq. (2) over the 1st shell $k_1 < |k| < k_0$ (no effect on $f^<$). Assume Gaussian $\mathcal{P}$ with $\langle \mathcal{P} \rangle = 0$ and isotropic Kolmogorov spectrum $\langle \mathcal{P} \mathcal{P} \rangle$. Neglect $\langle f^< f^< f^< \rangle$.

- Result: $(\partial_t + D_1) f^< = T f^< f^< \text{ with } D_1 = D_0 + \text{fct}(\langle \mathcal{P} \mathcal{P} \rangle)$. Looks like the original Equation (1), but with $D_0$ replaced by $D_1$!
Subgrid modeling: renormalization of $\nu$ and $\eta$

(Wilson, 1975, Advances in Mathematics, 16, 170; Zhou, Vahala & Hossain, 1988, Physical Review)

- Symbolically, $(\partial_t + D_0) f = T f f$ ............. (1)

- Split $f = f^> + f^<$ for $|k| \leftrightarrow k_1$:

  $$(\partial_t + D_0) f^< = T (f^< + f^>) (f^< + f^>)$$ ............. (2)

  $$f^> = (\partial_t + D_0)^{-1} T (f^< + f^>) (f^< + f^>)$$

- Average Eq. (2) over the 1st shell $k_1 <|k|<k_0$ (no effect on $f^<$). Assume Gaussian $f^>$ with $\langle f^> \rangle = 0$ and isotropic Kolmogorov spectrum $\langle f^> f^> \rangle$. Neglect $\langle f^< f^< f^< \rangle$.

- Result: $(\partial_t + D_1) f^< = T f^< f^<$ with $D_1 = D_0 + fct(\langle f^> f^> \rangle)$. Looks like the original Equation (1), but with $D_0$ replaced by $D_1$!

- **Iterate** the above elimination procedure while re-scaling $k$ and $\nu$ until $D_n(k) \rightarrow D^*(k)$ (RG fixpoint): eddy viscosity/resistivity.