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## A Receptance Coupling Approach to Optimally Tune and Place Absorbers on Boring Bars for Chatter Suppression

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### Abstract

A receptance coupling based method to optimally tune and place a tuned mass damper (TMD) on slender boring bars is presented. The boring bar and the TMD are treated as separate substructures. Substructural receptances are synthesized while optimizing TMD parameters as well as its location on the bar to target an increase in dynamic stiffness of the boring bar to improve chatter-free cutting conditions. Simulation based results with the proposed approach agree with classical analytical models reported in the literature. Methods presented are generalizable and can be extended to other slender cantilevered tools to increase their resistance against chatter vibrations.

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*Keywords:* Receptance coupling; Boring Bar; Tuned mass damper; Chatter

### 1. Introduction

Deep hole boring necessitates the use of long slender boring bars with length to diameter ratios of six and higher. These boring bars are essentially cantilevered with cutting forces acting on the free end. On account of being long, slender, and cantilevered, and because of their inherently low structural damping, the boring bars tend to vibrate with large amplitudes under the action of cutting forces. Such process induced large amplitude vibrations often result in machining instabilities, i.e. chatter. Chatter deteriorates surface finish and results in high tool wear or breakage; thereby limiting productivity, precision, and material removal rates. To avoid chatter, boring bars must possess improved dynamic stiffness and damping behaviour.

Dynamic stiffness and damping are often characterized using frequency response functions (FRFs), i.e. receptances. Chatter-free and stable turning and boring cutting conditions can be shown to be inversely proportional to the minimum real part of tool point FRF [1]. Hence, the approach adopted in this paper is to attempt reducing the magnitude of the peak in the real part of the tool point FRF by presenting a strategy to optimally tune and place dampers on boring bars.

There exist many methods [2-4] in the literature to

maximize the dynamic stiffness and improve damping, and in turn obtain chatter free boring processes. These methods include solutions using tuned dampers – passive, semi-active, and/or active. Though active systems can sometimes outperform passive ones, in most cases, a well-designed passive damper is preferred due to its simplicity, cheaper costs, and industrial viability. Hence, this paper also focuses on articulating a solution using a passive tuned mass damper (TMD) to increase resistance against chatter.

Design of TMDs include the classical approach of Den Hartog [5], in which analytical closed form solutions were presented to integrate a tuned damper for the case of zero damping in a single degree of freedom (SDOF) vibrating system. Rivin and Kang [6] analyzed the more general case of damped original systems and different forcing functions. Sims [7] presented analytical solutions to tune absorbers to suppress chatter for the case of zero damping in main SDOF systems.

Though effective, these [5-7] tuning methods do not consider the effect of TMD location relative to the free end of tool and model continuous systems as simplified SDOF systems. To address the issue of designing dampers that can be positioned at any arbitrary distance away from the free end

of the tool, we present a novel method to find optimized non-dimensional parameters for different positions of the damper using the receptance coupling approach. Methods presented herein build on our preliminary work [8].

Receptance coupling (RC), a structural modification tool, is used to couple two separate subsystems in the frequency domain [9]. In this study, two subsystems: unmodified boring bar and tuned mass damper, as shown in Fig. 1, are coupled together to obtain the modified FRF of the damped bar.

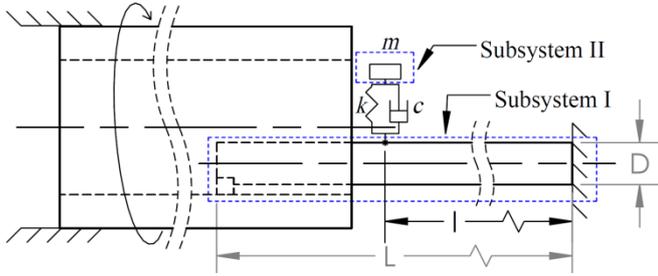


Figure 1: Schematic of long boring bars

The remainder of the paper is organized as follows: the chatter stability model is introduced in Section 2, followed by introducing the receptance coupling model in Section 3. Sensitivity analysis to understand influence of TMD parameters and its location on the dynamic stiffness of the tuned boring bar is carried out in Section 4. Recommended TMD parameters and locations are given in Section 5 which also compares results from the classical analytical models of [5, 7]. The paper is concluded in Section 6.

## 2. Machining Stability Model

Since the purpose of developing damped boring bars is to avoid chatter, the relationship between dynamic stiffness, damping, and the limiting stable, chatter-free cutting parameters (depth of cut) can be characterized as [1]:

$$a_{lim,crit} = -\frac{1}{2K_f \alpha \min(Re(H(\omega_c)))} \quad (1)$$

wherein  $a_{lim,crit}$  is the critical limiting chatter-free depth of cut,  $K_f$  is the cutting force coefficient which depends on the tool geometry and workpiece material combination;  $\alpha$  is the directional factor; and  $Re(H(\omega_c))$  is the real part of the FRF,  $H$  at a chatter frequency,  $\omega_c$ . It is evident that critical depth of cut is inversely proportional to the negative peak in real part of tool point FRF. Thus, a decrease in the magnitude of the negative peak in the real part of tool point FRF is taken as the objective function for tuning of TMD parameters.

## 3. Receptance Coupling

Receptance coupling methods facilitate analysis of complex systems by synthesizing receptances (FRFs) of their individual subsystems. In this study, modified damped boring bar is the final assembled system which has been modelled as an assembly of two sub-systems: the original boring bar as the first subsystem, and the TMD mass as the second subsystem, with a spring and damper coupling the two, see Fig. 1.

The classical approaches to the design of TMDs [5-7] do not account for placement considerations of the TMD along the length of the boring bar. They are limited to only placing the TMD at the free end, which is not feasible in boring, since the free end also corresponds to the cutting end. On the other

hand, the RC method allows designing TMDs to be positioned at different sections along the boring bar.

If the direct receptances of the original boring bar at its free end and the TMD coupling location are known a priori, as are the cross receptances between the free end and the TMD coupling location, the receptance of the TMD can be tuned such that the synthesized receptance,  $H_{11}$ , for the combined model at the free end can be shown to be [8-9]:

$$H_{11} = h_{11} - h_{12a} \left( h_{2a2a} + h_{2b2b} + \frac{1}{k'} \right)^{-1} h_{2a1} \quad (2)$$

$$k' = k + ic\omega$$

Wherein  $h_{11}$  is the receptance at the free end for the original boring bar;  $h_{2a2a}$  is the receptance at the coupling location of the TMD for the original boring bar;  $h_{12a}$  and  $h_{2a1}$  are the cross receptances between the free end and the coupling location of the TMD, again for the original boring bar, and since the structure is symmetric and linear, these cross receptances are the same and equal;  $h_{2b2b}$  is the tunable receptance of the TMD; and  $k'$  is the effective complex stiffness. Though receptances in Eq. (2) are frequency ( $\omega$ ) dependent,  $\omega$  is omitted for brevity. The subsystem level receptances for the original boring bar and the TMD are obtained as discussed below.

### 3.1. Original Boring Bar Model

A long slender boring bar with a length to diameter ratio of 12 is used in this study. Boring bar is modelled with Euler Bernoulli beams using the Finite Element (FE) method. Beam and FE parameters are given in Table 1. The cutting insert and insert cartridge were ignored while modelling the boring bar.

Table 1: Boring Bar FE Model Details

Parameters	Values
Diameter (D)	25mm
Length (L)	300mm
Density ( $\rho$ )	7850 Kg/m <sup>3</sup> (Mild Steel)
Young's Modulus (E)	200 GPa (Mild Steel)
Number of elements	300 (equal length)
Element Type	Linear, 2 nodes per element
Node degree of freedom (DOF)	2 DOF per node ( $u, \frac{\partial u}{\partial x}$ )

An eigenvalue problem form of the equation of motion – obtained using the mass and stiffness matrices from the FE model, was solved for the eigenvalues (natural frequencies) and eigenvectors – which were used as follows to evaluate the beams receptances:

$$h_{pq}(\omega) = \sum_{r=1}^n \frac{\phi_{pr} \phi_{qr}}{-\omega^2 + \omega_r^2 + 2i\xi_r \omega \omega_r} \quad (3)$$

wherein  $h_{pq}$  is the receptance with response at location  $p$  and excitation at  $q$ .  $\phi_{pr}$  and  $\phi_{qr}$  are the mass normalized eigenvectors at  $p, q$  for mode ' $r$ ';  $\omega_r$  is natural frequency for mode ' $r$ '. Damping ratio,  $\xi_r$ , is assumed to be uniform for all modes at 1%.

### 3.2. Tuned Mass Damper (TMD) Model

The TMD mass is modelled as a single degree of freedom (SDOF) system with mass  $m$ . The receptance of this SDOF system can be shown to be:

$$h_{2b2b}(\omega) = -\frac{1}{m\omega^2} \quad (4)$$

**4. Sensitivity Analysis**

To tune the absorber to the optimal parameter set of  $l, m, k, c$  combination, all parameters were converted to a non-dimensional set and varied over a range according to Table 2. Maximum free mass was selected to be ~10% of the beam mass (1.14 kg). Stiffness was selected such that TMD’s natural frequency is close to the first and most flexible mode of the boring bar. The effective modal mass,  $M_{eff}$  of the first mode was evaluated to be 0.287 kg. Length and damping values are selected on the basis of physical feasibility.

Table 2: Absorber parameters used in sensitivity analysis

Parameter	Non-dimensional		Range
Length ( $l$ )	Length ratio ( $l$ )	$l/L$	0.75-0.95
Free Mass ( $m$ )	Mass ratio ( $\mu$ )	$m/M_{eff}$	0.01-0.50
Stiffness ( $k$ )	Frequency ratio ( $f$ )	$\sqrt{k/m}/\omega_{bar}$	0.80-1.20
Damping ( $c$ )	Damping ratio ( $\zeta$ )	$c/2\sqrt{km}$	0.01-0.40

Influence of damper parameters and position of the TMD on the damped response of the boring bar is investigated by sequentially fixing three parameters and varying the other to check the sensitivity of the damped assembled response ( $H_{11}$ ) on the TMD parameters and location.

Fig. 2 shows the influence of mounting the absorber at different locations with different frequency ratios, for a fixed mass and damping ratio. It is evident that increasing the distance between damper and fixed end may not always improve the resistance to chatter (resistance to chatter being inversely proportional to  $\min(Re(H_{11}))$ ) For example, a damper with frequency ratio of 1.05 has better performance when attached at length ratio of 0.75 compared to the length ratio of 0.85 whereas frequency ratio of 0.95 gives better results at length ratio of 0.85.

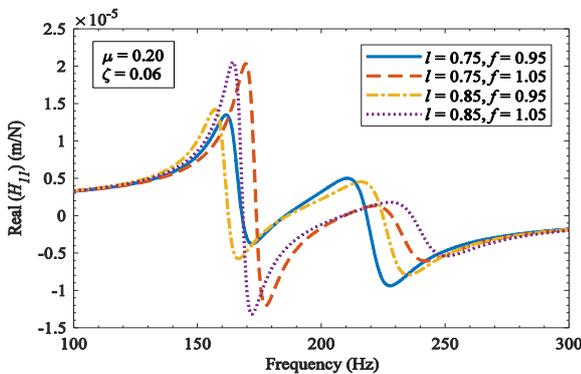


Figure 2: Influence of length and frequency ratio on the real part of the assembled response, for fixed mass and damping ratios

Fig. 3 shows the influence of mass and damping ratio for a fixed length and frequency ratio. It is evident that continuously increasing damping in the absorber may result in inefficiency of the damper. This effect is similar to the fact that energy dissipation for a SDOF increases with the increase in damping, but after a threshold ( $\zeta = 1$ ) this has adverse effects. Same can be inferred from the Fig. 3 wherein response for the  $\mu$  of 0.05 and  $\zeta = 0.05$  outperforms the

response at the  $\zeta = 0.35$ .

Thus it can be observed that there exists an optimized parameter set of damping ratio and frequency ratio for different mass and positions of the absorber.

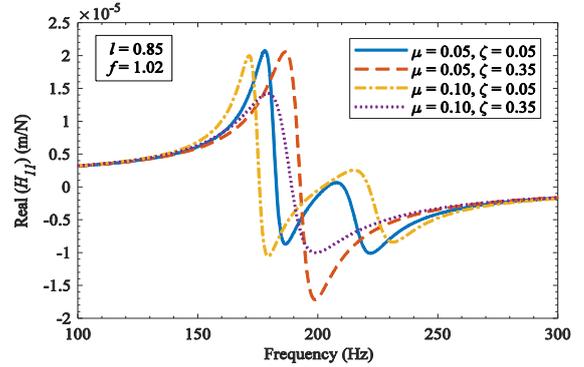


Figure 3: Influence of mass and damping ratio on the real part of the assembled response, for a fixed length and frequency ratio

**5. Recommended TMD Parameters**

Discussions in Section 4 show that each of the four TMD parameters, i.e.  $l, \mu, f$ , and  $\zeta$  influence the damped response of the boring bar differently. Hence, to find the optimum set of parameters, a large 4D-matrix was created such that each of its element stores data points corresponding to the absolute value of the minimum negative peak in the real part of assembled FRF, i.e.  $|\min(Re(H_{11}))|$  corresponding to the different input parameters. This matrix is then scanned to find the optimum set of  $k$ , and  $c$  corresponding to each value of mass ( $m$ ) and length ( $l$ ).

The procedure to scan the 4D-matrix to find the optimum data set can be understood from Table 3, which reports  $|\min(Re(H_{11}))|$  for a selective subset of absorber parameters given in Table 2. For example, if we consider the Bold Italicized data point ( $7.666 \times 10^{-6}$  m/N), this data point represents  $|\min(Re(H_{11}))|$  of the assembly response for a mass ratio of 0.15, frequency ratio of 1.06, damping ratio of 0.1 and assumed placement at 0.75 times of the total length.

Optimal stiffness and damping were found by varying  $f$  and  $\zeta$  in the specified range for a fixed value of free mass and joining length, and the real part of assembly response was analysed to obtain  $|\min(Re(H_{11}))|$ . This procedure can be again explained using Table 3. For example, if we were to find the optimum parameter set for a damper with mass ratio of 0.15 and placed at  $l = 0.85$ , data points corresponding to the fourth row need be scanned. On scanning this row,  $|\min(Re(H_{11}))|$  is observed to be  $4.242 \times 10^{-6}$  m/N, which corresponds to a frequency ratio of 1.04 and a damping ratio of 0.20. Similarly other absolute minimum values of the negative part of the real part ( $|\min(Re(H_{11}))|$ ) corresponding to each value of the free mass for different locations are obtained, and these data points have been underlined in Table 3. The above procedure is followed for all parameter levels of Table 2 with fine variation in parameters so as to get the actual trend. These results are summarized in Fig. 4.

Table 3: Absolute value of negative peak in real part (m/N X  $10^{-5}$ ) of modified receptance for different TMD parameters

S. No.	Length Ratio ( $l$ )	Mass Ratio ( $\mu$ )	$\zeta = 0.10$			$\zeta = 0.15$			$\zeta = 0.20$		
			Frequency Ratio( $f$ )								
			1.02	1.04	1.06	1.02	1.04	1.06	1.02	1.04	1.06
1	0.75	0.050	0.9399	<u>0.8441</u>	1.1076	1.1481	1.1894	1.3042	1.4635	1.5096	1.5876
2		0.150	0.5818	0.5939	<b>0.7666</b>	0.5151	<u>0.4767</u>	0.5904	0.5442	0.5662	0.6436
3	0.85	0.050	0.7872	<u>0.7095</u>	0.9094	0.8366	0.8490	0.9662	1.0770	1.1163	1.1885
4		0.150	0.5128	0.6508	0.7948	0.4203	0.4338	0.5490	0.4050	<u>0.4242</u>	0.5104

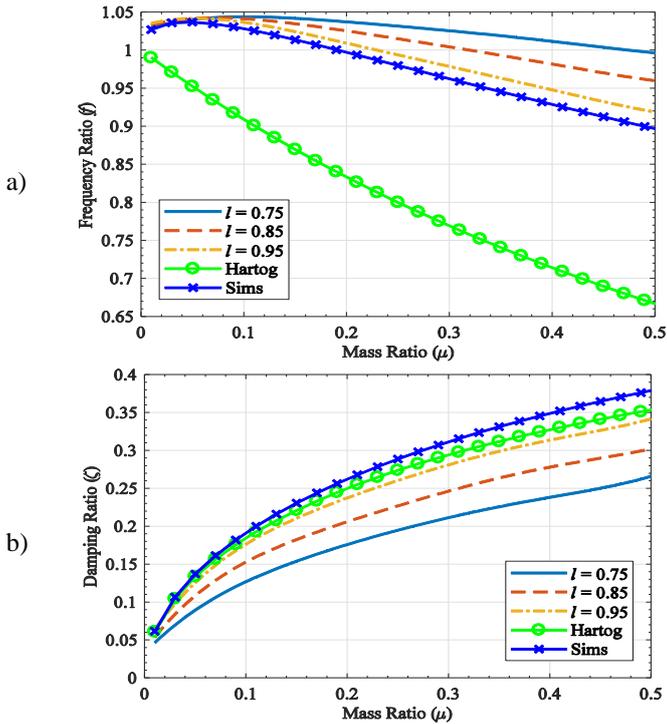


Figure 4: (a) Recommended values of Frequency ratio ( $f$ ), (b) Recommended values of Damping ratio ( $\zeta$ ) for a given Mass

Fig. 4 compares results obtained using the proposed approach with those obtained using the classical analytical models of Hartog [5] and Sims [7]. It is evident from Fig. 4 that as the length ratio is increased, the optimized damping and stiffness ratios approach optimized parameters as given by Sims [7], which is expected, since the approaches in [5, 7] assumes placement of the TMD at the free end. The constant offset in the frequency ratio, evident in Fig. 4(a) using the proposed approach can be explained by the fact that [7] assumes the main system as undamped whereas in this study, a damping ratio of 1% is considered in the original boring bar. Deviations in the frequency ratio using the proposed model from the model in [5] are consistent with those reported in [7].

Similarly, as evident from Fig. 4(b), deviations observed at higher mass ratios in optimal damping ratios estimated using the proposed approach, when compared with the analytical models [5, 7] is thought to be because of the assumed simplifications in those models [5, 7].

The recommended TMD parameters using the proposed approach are used to obtain  $|\min(\text{Re}(H_{11}))|$  for the assembled system for different length ratios as shown in Fig. 5.

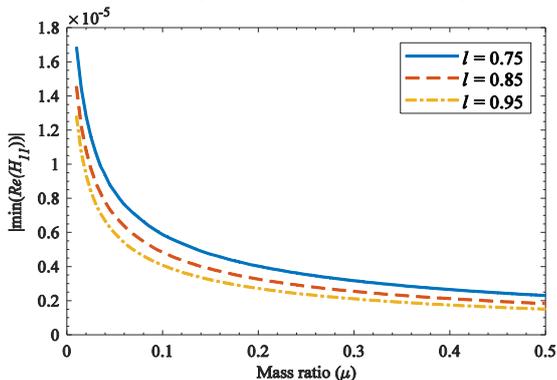


Figure 5: Absolute value of negative peak in real part of tool point FRF with recommended  $f$  and  $\zeta$  parameters

It is evident from Fig. 5 that response of the boring bar improves as we move the damper close to the free end and increase the mass of the damper. However it should also be noted that beyond a threshold value of mass, increased damping does not necessarily translate to reduction in amplitude of the absolute minimum of the negative part of the real part of the FRF.

Finally, the influence of the optimized TMD on the critical chatter-free stable depth of cut is evaluated for the case of cutting soft Aluminium with an effective cutting force coefficient ( $K_f$ ) of  $600e6 \text{ N/m}^2$  and with the directional factor ( $\alpha$ ) of 0.5. For the original (unmodified) boring bar, the critical chatter free depth is evaluated to be 0.03 mm (with the negative peak in real part of receptance being  $5.623e-05 \text{ m/N}$ ); whereas for the boring bar with a TMD at  $l = 0.75$  that has a mass of 0.029kg ( $\mu = 0.10$ ), frequency ratio of 1.044 and damping ratio of 0.124, the chatter-free critical depth of cut is evaluated to increase to 0.28 mm (with receptance peak magnitude of  $5.84e-06 \text{ m/N}$ ), this is a significant improvement.

## 6. Conclusions

This paper presents a new receptance coupling based approach to optimally tune and place absorbers on boring bars for chatter suppression. Boring bar was modelled as an Euler Bernoulli beam, and the absorber was modelled as a damped single degree of freedom spring-mass system. The receptance coupling model allows integration of the absorber at arbitrary locations along the length of the boring bar, something not possible with the earlier reported classical methods. Receptances of the separate substructures were synthesized to obtain the damped response of the boring bar at its free (cutting) end. Optimal absorber stiffness and damping parameters were obtained as a function of its free mass and joining position. Absorber effectiveness was observed to saturate beyond certain levels of damping values i.e. size of damper. Chatter-free depth of cut for boring was found to increase to 0.28 mm with the tuned bar as compared to being 0.03 mm for the original bar – a significant improvement.

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