

# Learning machining stability using a bayesian model

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## ABSTRACT

### KEYWORDS

Machining Stability,  
Bayesian Learning,  
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*Instabilities in machining can be detrimental. Usually, analytical model-predicted stability charts guide selection of cutting parameters to ensure stable processes. However, since inputs to the model seldom account for the speed-dependent behaviour of the cutting process or the dynamics, models often fail to guide stable cutting parameter selection in real industrial settings. To address this issue, this paper discusses how real experimentally classified stable and unstable cutting data with all its vagaries and uncertainties can instead be used to learn the stability behaviour using a supervised Bayes' learning approach. We expand previously published work to systematically characterize how probability distributions, training data size, and thresholding influence the learning capacity of the Bayesian approach. Prediction accuracies of up to 95% are shown to be possible. We also show how the approach nicely extends itself to a continuous learning process. Results can hence inform further development towards self-optimizing and autonomous machining systems.*

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## 1. Introduction

Selection of stable cutting parameters for high performance machining is often guided by knowledge of stability diagrams. Analytical models usually predict these diagrams. Inputs to the model like cutting force coefficients and measured dynamics influence the quality of prediction. However, since inputs seldom account for the speed-dependent behaviour of the cutting process or the dynamics, and since models make several linearizing assumptions, models, though useful, often fail to guide stable cutting parameter selection in real industrial settings. Since instabilities are detrimental to the part, the tool, and to elements of the machine, there is need for better quality predictions. To address this need, this paper discusses the use of a supervised Bayesian machine learning (ML) technique that can 'learn' the stability diagram from real experimentally classified stable and unstable data points without relying on an analytical model. Since real data captures all experimental vagaries and uncertainties in the cutting processes and/or in the dynamics, the learnt stability diagram is expected to be more accurate and useful in guiding the selection of stable cutting parameters.

In the context of machine tool systems, use of ML models has been shown to be useful for tool wear analysis, thermal error compensation, monitoring and classifying states of machining as being stable or unstable, identifying dynamics, and to learn the stability diagram. Succinct summaries are available in this review paper (Aggogeri et al., 2021).

Prior research on learning stability using ML models has discussed the use of artificial neural networks (ANN) (Friedrich et al., 2018), support vector machines (SVM) (Denkana et al., 2020; Friedrich et al., 2018), the k-nearest neighbourhood (kNN) method (Friedrich et al., 2017), and Bayesian methods (Chen et al., 2021; Karandikar et al., 2020; Schmitz et al., 2022). Given that training models requires that experiments be done to obtain unstable data points, and since those experiments can be destructive due to the nature of instabilities, an accurate model that can be trained with less data, and one that extends itself to a continual learning scheme should be preferred. The Bayes' method fits these criteria. It is hence our preferred method for implementation.

The Bayes' method to learn stability was in its original form intended to be agnostic to the process physics (Karandikar et al., 2020). However, in other related work, physics-informed modifications have been reported to work well

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(Chen et al., 2021; Schmitz et al., 2022). Since it is desirable to develop a generalized ML model that is agnostic to the process physics and that can work with many different data sets, this study follows the approach reported in (Karandikar et al., 2020) that was blind to the physics of the process.

For given data that is pre-classified as stable/unstable, the goal with the Bayesian approach is to calculate the posterior probability of stability at each grid point on the stability map. Though this was done in prior work (Chen et al., 2021; Karandikar et al., 2020; Schmitz et al., 2022), that work did not systematically characterize the influence of changing Gaussian likelihoods, or the influence of changing threshold of the stability contour on the learning accuracy. Moreover, there was no discussion on how to characterize and quantify the learning accuracy of the prediction. Nor was the continual learning aspect of the method discussed. The aim of this paper is to address these unreported issues, and in doing so, we further show the strength of the Bayes' method for learning machining stability.

The remainder of the paper is organized as follows. At first, in Section 2, we discuss how we gather data that we feed to our learning model. We then briefly overview the Bayes' method in Section 3. In Section 4, we discuss results to show how changing likelihoods and threshold influence learning as well as discuss the continuous learning capacity of the method. Main conclusions follow.

## 2. Gathering Data for the ML Model

Since the experimental pathway to gather data that is needed to train a Bayesian model is costly due to the destructive nature of unstable experiments, this paper trains and tests the ML model using data obtained from emulations on an in-house developed hardware-in-the-loop (HiL) simulator that was built to study machining instabilities (Sahu & Law, 2022; Sahu et al., 2020). Experiments on the HiL simulator are used to classify combinations of depths of cuts and spindle speeds that result in unstable conditions. The process is akin to procedures in real cutting experiments. Emulations were carried out with assumed linear cutting force characteristics. 30 experimental data points were recorded and plotted to obtain stability boundaries shown in Fig. 1. Though the Bayesian model can be adequately trained with this data, testing the model for its learning capacity needs more data than we have obtained. As such, we synthesize

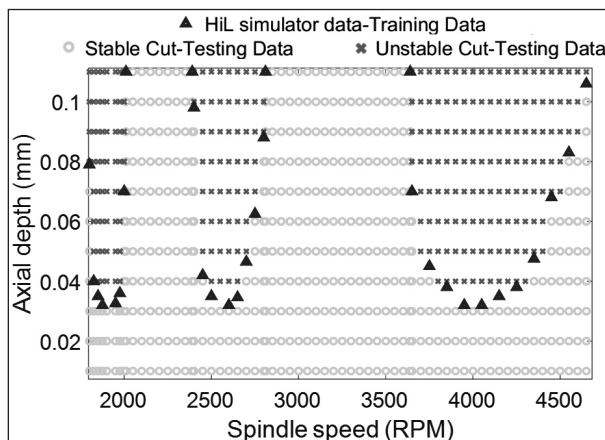


Fig. 1. Data obtained from emulations on a HiL simulator (▲), along with synthesized data for stable (o) and unstable conditions (\*).

the emulated data with more data. Since the region below the boundary is stable and that above unstable, we add data points at depth of cut intervals of choice to pad the emulated data. In this manner, we generate additional 561 points. These data are also shown in Fig. 1.

## 3. Bayesian Learning for Machining Stability

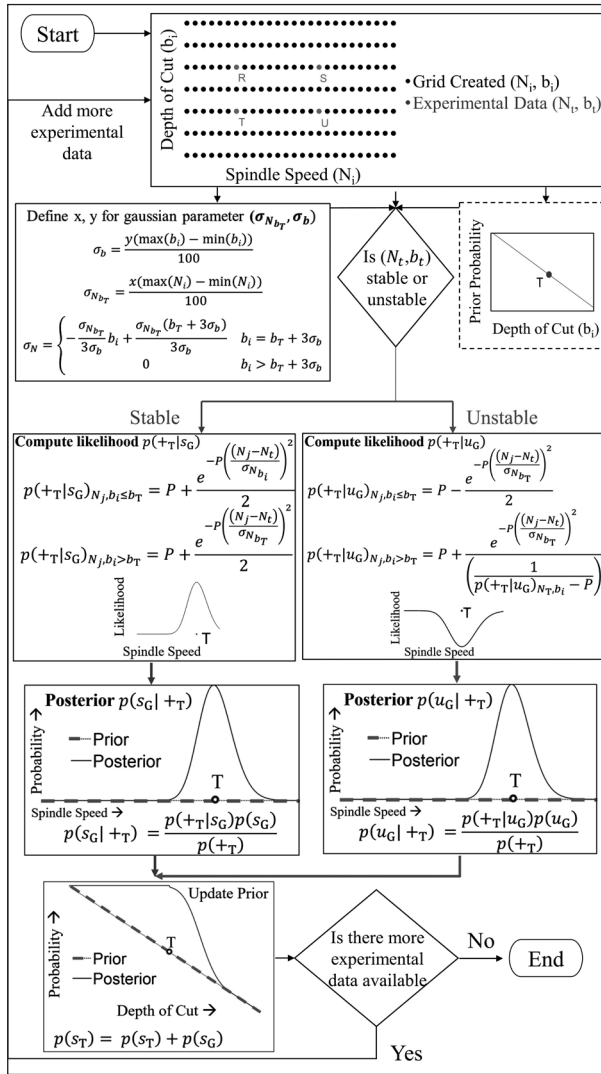
This section outlines the Bayes' procedure to learn machining stability diagrams. We only provide an overview and direct the reader to the original source (Karandikar et al., 2020) for details. Bayes' rule updates probabilities when new information is made available. Mathematically, it can be stated as:

$$p(A | B) = \frac{p(B | A)p(A)}{p(B)} \dots\dots\dots(1)$$

wherein A and B are separate events. p(A | B) is the probability of event A occurring given that B is true. This is also known as the posterior probability of A given B. p(B | A) is the probability of event B occurring given that A is true. This is also known as the likelihood of A given a fixed B. p(A) and p(B) are probabilities of A and B occurring and are known as the prior probabilities. In the context of machining stability, the Bayes' rule becomes:

$$p(\text{stability} | \text{experimental data}) = \frac{p(\text{experimental data} | \text{stability})p(\text{stability})}{p(\text{experimental data})} \dots\dots\dots(2)$$

wherein p(stability) is an assumed prior probability of stability, p(experimental data) is the known probability of a data point being stable or not, p(experimental data | stability) is the likelihood probability of a stable result at the



**Fig. 2.** An overview of our implementation of the bayes’ rule to learn stability diagrams.

given experimental data point, and  $p(\text{stability}|\text{experimental data})$  is the evaluated posterior probability of a stable data point for the given experimental condition. Likewise, it is also possible to evaluate the posterior probability of an unstable data point for the given experimental condition and given an assumed prior probability of instability,  $p(\text{instability})$ .

For the given data points on the stability diagram (in Fig. 1) the goal with the Bayesian approach is to calculate the posterior probability of stability at each grid point on the axial depth of cut – spindle speed map. The procedure to do so is outlined in a flowchart in Fig. 2. The nomenclature is described in Table 1.

For every data point, we first evaluate the prior probability. This is done by assuming that as the depths of cut increase at any spindle speed, the

**Table 1**

Nomenclature for bayesian learning approach, as also used in (Karandikar et al., 2020).

A	uncertain event
B	experimental result
N	spindle speed
b	axial depth of cut
i	axial depth of cut grid point index
j	spindle speed grid point index
p	probability
s	stable
u	unstable
G	arbitrary grid point in the domain
T	test grid point
$U_g$	total grid uncertainty
+	stable result
-	unstable result
$\sigma_N$	standard deviation in spindle speed
$\sigma_{NbT}$	standard deviation in spindle speed at test axial depth
$\sigma_b$	standard deviation in axial depth of cut

likelihood of encountering instabilities increases. A linear distribution for the prior probability is assumed as shown in an inset in the flowchart in Figure 2. This prior probability remains the same for all spindle speeds.

The next step is to evaluate the likelihood probabilities. For this, it is assumed that the influence of a test result will reduce as the distance in terms of depths of cut or speeds from the test point increases. And, since the nature of cutting is such that data points below a point that is stable will remain stable, and those above a point that is unstable will remain unstable, the influence of a test point is not symmetric along the axial depths of cut and speeds. Moreover, since the boundary between the stable and unstable limits is not known a priori, the influence of test result being stable at a particular depth of cut will also reduce as the depth of cut increases. Furthermore, since knowledge that the width of the stability region is usually greater at smaller depths of cut than at larger depths of cut, that behaviour too must inform the distribution of the likelihood probability. Characterizing such behaviour is best done using Gaussian distributions, which was the genius of insight proffered in (Karandikar et al., 2020), that makes

the Bayesian approach suitable for predicting machining stability.

The influence of a test result along spindle speeds at the depth of cut of interest,  $b_T$  is defined as  $\sigma_{N_{b_T}}$  with the mean being the test spindle speed of interest, and the influence of the test result being restricted to  $3\sigma_{N_{b_T}}$ . The subscript T refers to the test result under consideration. The likelihood probability of a stable result at T ( $p(+_T)_{N_T, b_T}$  given G, another test result on the depth of cut – spindle map that is stable, is one, i.e.,  $p(+_T | s_G)_{N_T, b_T} = 1$ . For the same depth of cut,  $b_T$ , the likelihood probability of a stable result at T given G is stable reduces for other spindle speeds  $N_j$  following a Gaussian distribution, as shown in the schematic and in the equation in the insets in the flowchart in Fig. 2.

For a changing depth of cut at the same spindle speed,  $N_T$ , for example, when  $b_i < b_T$ , and the data point  $b_T$ ,  $N_T$  is stable, the likelihood probability will remain one since for the case of the binary type of classification, every point below  $b_T$  will remain stable. However, the probability of a data point remaining stable reduces as  $b_i > b_T$ , and is assumed to have no influence beyond  $b_T + 3\sigma_b$ , wherein  $\sigma_b$  is the standard deviation along the axial depth of cut, and  $b_T + 3\sigma_b$  is the mean for a non-normalized Gaussian probability density. Since the influence of stable results will be higher at lower axial depths of cut,  $\sigma_{N_{b_T}}$  is different for the cases of  $b_i < b_T$  and for  $b_i > b_T$ . This is shown in the equations within the flowchart in Fig. 2. Procedures to obtain the likelihood probability of instabilities follows the same logic as for the case of obtaining the likelihood probability of a stable result. The Gaussian distribution however is inverted for the case of the unstable result.

Using procedures outlined above, a posterior probability is computed using the Bayes' conditional probability theorem using the equations shown in the insets in Fig. 2. To do so, the likelihood and prior probabilities are both used. For a representative test result T, the posterior probability is overlaid on the prior assumption within the inset in Fig. 2, and it clearly shows that probabilities change using the Bayes' rule. This new probability becomes the prior probability for the subsequent experimental data point, and the process is repeated until all grid points on the spindle speed – axial depth of cut map are evaluated.

After probabilities are updated using all test results, a stability lobe prediction from the

Bayes' approach can be made at axial depths of cut when the probability of stability is equal to a user-defined threshold. In prior work (Karandikar et al., 2020), that threshold was fixed at 0.5. However, since the threshold could influence the accuracy of the decision boundary, its influence the decision boundary is investigated in this work.

Learning capacity of the Bayesian model is governed not only by the assumed prior distribution of probabilities, and the assumed Gaussian distributions for the likelihood probabilities, but also the choice of  $\sigma_N$  and  $\sigma_b$  within those distributions.  $\sigma_N$  and  $\sigma_b$  will both determine the influence of a test point and the posterior probabilities, which in turn, influences the width and amplitude of the stability boundaries after each update. In prior work on the use of the Bayes' approach (Karandikar et al., 2020),  $\sigma_N$  and  $\sigma_b$  and were selected as 3% of the spindle speed range and as 10% of the axial depth of cut range, respectively. Though these values resulted in good prediction accuracies, systematic investigations about the influence of different values of  $\sigma_N$  and  $\sigma_b$  remains unexplored and will be addressed in this work. To quantify the goodness of predictions, we use a confusion matrix and evaluate accuracies and F1 scores.

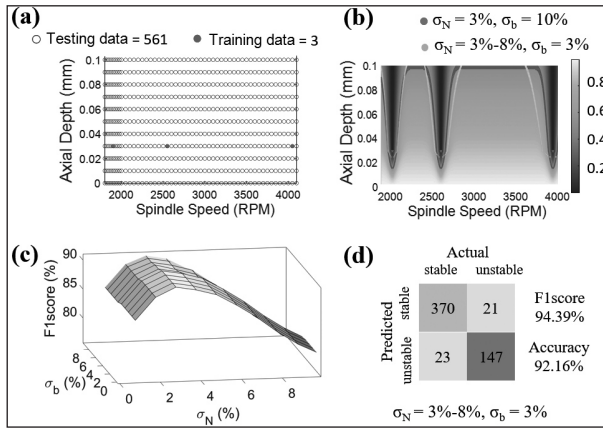
#### 4. Parameters Influencing Learning Capacity

This section characterizes the role of changing Gaussian distributions and thresholds and discusses continual learning using Bayes' processes.

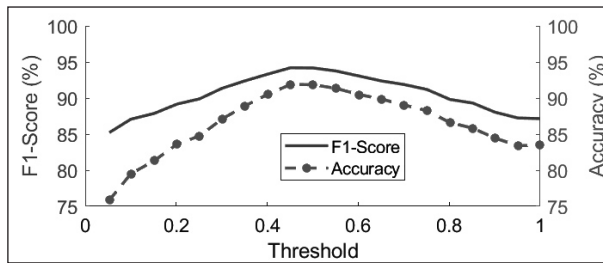
##### 4.1. Influence of changing gaussian distributions

Three data points were used to train the Bayesian model. These are marked in Fig. 3(a). The threshold for evaluating the stability boundary was taken to be 0.5, and  $\sigma_N$  and  $\sigma_b$  were selected as 3% of the spindle speed range and as 10% of the axial depth of cut range, respectively, i.e., as per what was reported in prior work (Karandikar et al., 2020). As is evident from Fig. 3(b), with these parameters, the stability boundary is well-predicted. The color map in Fig. 3(b) depicts the probability of stability, with blue representing a very low probability of stability and yellow indicating a high probability of stability. Although the stability boundary is determined by the threshold value, the color map gives an overview of the distribution of the evaluated posterior probabilities.

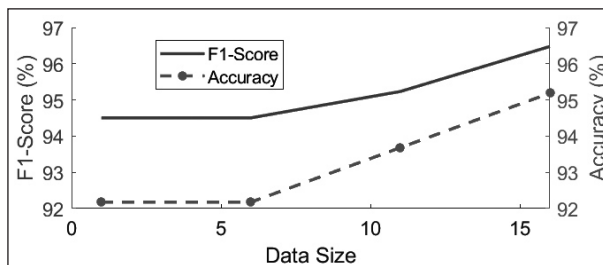




**Fig. 3.** (a) Data for training and testing  
 (b) Two stability boundaries with and without tuning the gaussian distribution,  
 (c) The influence of  $\sigma_N$  and  $\sigma_b$  on F1 score,  
 (d) Confusion matrix for tuned gaussian distribution.



**Fig. 4.** The influence of threshold on F1 score and accuracy.



**Fig. 5.** The influence of training data size on F1 score and accuracy.

We tested the predicted boundary against 393 synthesized stable data points and another 168 unstable data points. The accuracy was found to be 86.63%, and the F1 score was found to be 90.91%. Though this predicted accuracy is in line with observations in (Karandikar et al., 2020), there is still a 13.37% chance that an incorrect selection of cutting parameters might be made. To check if adjustments to the model that can further improve the quality of prediction, we evaluated how predictions change with  $\sigma_N$  and  $\sigma_b$ . These results are summarized in Fig. 3(c). Interestingly, the learning capacity characterized by the F1 score suggests that the model is more sensitive to a change in  $\sigma_N$  than in  $\sigma_b$ .

Instead of tuning these parameters as constants, if knowledge of the stability behavior is used to inform their selections, then, since the width of the stability boundary is usually greater at higher spindle speeds than at lower spindle speeds, the Gaussian distribution for the spindle speed can characterize this behavior by potentially linearly increasing  $\sigma_N$  from a low value at lower spindle speeds to a higher value at high spindle speeds. This was tried and it was found that if  $\sigma_N$  is linearly increased from 3% to 8% of spindle speed range within the speed range of interest, the accuracy and the F1 score further improve to 92.16% and to 94.39%, respectively. These results are summarized as a confusion matrix in Fig. 3(d). The stability boundary also changes shape slightly with for this linearly varying  $\sigma_N$  with a fixed  $\sigma_b$  – as is evident from Fig. 3(b). This analysis suggests that the size of the Gaussian distribution must be tuned as per the data set of interest, and that it is not necessary that a fixed  $\sigma_N$  and  $\sigma_b$  are better than a linearly varying  $\sigma_N$ .

#### 4.2. Influence of changing the threshold

Analysis herein was informed by the analysis in Section 4.1, i.e., the same number of data points were used to train-test, and  $\sigma_N$  was linearly increased from 3% to 8% of spindle speed range, and  $\sigma_b$  was taken to be fixed at 3% of the axial depth of cut range. Analysis for varying thresholds is shown in Fig. 4, and it is evident that the highest F1 score is not obtained for thresholds being fixed at 0.5, as they were in prior reports in (Karandikar et al., 2020), but for a threshold of 0.45. This suggests that this parameter must also be tuned as necessary.

#### 4.3. Continuous learning using a bayesian model

Since the very nature of the Bayes' rule is that the posterior probability updates with every new data point that is provided to the model, the model extends itself naturally to a continuous learning process wherein new experimental data when available can be fed to the model to update the posterior and its predictive capability. To test this capacity of the Bayesian model, we train the model with four different data set sizes. Learnings from Sections 4.1 and 4.2 inform analysis in this section. Results are summarized in Fig. 5, and as is evident, in general, learning capacity improves with larger data sets, even if marginally.

## 5. Conclusion

This paper demonstrated successfully that the stability diagram can be learnt from experimental data using a supervised Bayesian learning model. We observed that the Gaussian distributions that make up the likelihood probabilities for spindle speeds affect learning capacity more than the assumed distribution for depths of cuts. These must be tuned for the data set of interest, and not kept fixed as was done in prior work. We also observe that the thresholds used to evaluate stability contours must also be tuned for the data set of interest, and not kept fixed as was done in prior work. Accuracy was observed to improve with increasing size of training data, even if marginally. And, since wrong predictions may result in incorrect selection of cutting parameters that may result in instabilities that may result in damage, small changes are meaningful since mistakes are costly. As such, since learning stability behaviour from data includes in it the vagaries of experiments, methods discussed herein can overcome shortcomings of analytical model-predictions. Moreover, since the Bayes' method extends itself easily to a continual learning processes, it can find use in self-optimizing machining systems in which cutting parameter selection can be adapted autonomously and in real-time based on predictions.

## References

- Aggogeri, F., Pellegrini, N., & Tagliani F. L. (2021). Recent advances on machine learning applications in machining processes. *Applied sciences*, 11(18), 8764. <https://doi.org/10.3390/app11188764>
- Chen, G., Li, Y., Liu X., & Yang, B. (2021). Physics-informed bayesian inference for milling stability analysis. *International Journal of Machine Tools and Manufacture*, 167. <https://doi.org/10.1016/j.ijmachtools.2021.103767>
- Denkana, B., Bergmann, B., & Reimer, S. (2020). Analysis of different machine learning algorithms to learn stability lobe diagram. *Procedia CIRP*, 88. <https://doi.org/10.1016/j.procir.2020.05.049>
- Friedrich, J., Hinze C., Renner A., Verl A., & Lechler A. (2017). Estimation of stability lobe diagrams in milling with continuous learning algorithms. *Robotics and Computer-Integrated Manufacturing*, 43, 124-134. <https://doi.org/10.1016/j.rcim.2015.10.003>
- Friedrich, J., Torzewski, J., & Verl, A. (2018). Online learning of stability lobe diagrams in milling. *Procedia CIRP*. <https://doi.org/10.1016/j.procir.2017.12.213>
- Karandikar, J., Honeycutt, A., Schmitz, T., & Smith, S. (2020). Stability boundary and optimal operating parameter identification in milling using Bayesian learning. *Journal of Manufacturing Processes* 56. <https://doi.org/10.1016/j.jmapro.2020.04.019>
- Sahu, G. N., & Law, M., (2022). Hardware-in-the-loop simulator for emulation and active control of chatter. *HardwareX*. <https://doi.org/10.1016/j.ohx.2022.e00273>
- Sahu, G. N., Vashisht, S., Wahi, P., & Law, M. (2020). Validation of a hardware-in-the-loop simulator for investigating and actively damping regenerative chatter in orthogonal cutting. *CIRP Journal of Manufacturing Science and Technology*, 29, 115-129. <https://doi.org/10.1016/j.cirpj.2020.03.002>
- Schmitz T., Cornelius A., Karandikar J., Tyler C. & Smith S. (2022). Receptance coupling substructure analysis and chatter frequency-informed machine learning for milling stability. *CIRP Annals*, 71(1), 321-324. <https://doi.org/10.1016/j.cirp.2022.03.020>



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