



# Identifying Joint Dynamics in Bolted Cantilevered Systems Under Varying Tightening Torques and Torsional Excitations

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## Abstract

**Objectives** This paper characterizes how joint stiffness and damping parameters for a bolted cantilevered beam change with different number of bolts, different tightening torques on each, and with different levels of torsional excitations. Such characterization is unaddressed in the literature and is our modest contribution to the state-of-the-art. Another modest contribution is to propose and demonstrate the use of an alternative method of identifying joint characteristics using the harmonic balance method.

**Methods** A cantilevered beam was torsionally excited at its free end using a modal shaker and the relative displacements across the bolted connection were measured. Experiments were conducted in a temperature-controlled environment. Experiments for all configurations were repeated in a randomized order.

**Results** System dynamics were observed to be strongly dependent on the level of excitation, tightening torques, and number of bolts at the interface. Parameters identified using the harmonic balance method were found to agree with those identified using the standard and established method of using hysteresis loops. Identified parameters were used to predict the dynamic behavior of another assembled system which had a similar bolted connection, thus validating the identification.

**Conclusions** Experimental setup and procedures described are simple and the methods of identification robust. Our results suggest that the harmonic balance method is a viable alternate method for joint parameter identification that is not very sensitive to signal processing and conditioning. Our methods and findings can guide other researchers and practitioners interested in the dynamics of assemblies with bolted joints.

**Keywords** Bolted joint · Damping · Harmonic balance · Hysteresis · Receptance coupling · Stiffness

## Introduction

Slender bolted cantilevered systems vibrate. Their vibrations are influenced by the characteristics of the bolted joint. For instance, the dynamic response of slender cantilevered boring bars at the free end is influenced by the number of bolts, the tightening torque on each, the order of tightening the bolts, and the levels of excitation [1]. Since slender cantilevered boring bars are very flexible, they tend to vibrate uncontrollably under cutting process-based excitations [2].

Since such large amplitude vibrations have no redeeming qualities, the boring bar is usually damped by integrating a tuned mass damper within it. The damper is tuned to vibrate at the dominant mode of the cantilevered boring bar. However, these tuned dampers often underperform in practice, as the boring bar's dynamics change with changing boundary conditions and excitations. To help instruct the design of better tuned dampers for cantilevered tooling systems, it is important to characterize the influence of bolted joints on their response. This is indeed the main aim of this paper.

Bolted joints are ubiquitous, and since up to 90% of the total damping in assemblies may be provided by joints [3], dynamics of assemblies with bolted joints have long interested researchers and practitioners. Detailed summaries are available in these review reports [4–6]. Though bolted joints are meant to rigidly connect two subsystems, there is always some relative motion at the contacting surfaces which helps dissipate energy. There is friction due to shearing and

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torsional forces, which in turn is governed by loads, materials in contact, and their surface roughness characteristics that make the contact statistical in nature. There are also regimes of macro- and/or microslip and these change with contact pressure, which in turn decreases away from the bolt's central axis.

Since friction and slip are nonlinear and nonuniform across the contacting interfaces, modeling these effects is non-trivial. Despite some exemplary modeling efforts [6–9], since it is difficult for models to account for bolts that exhibit parametric uncertainty [5], and/or bolts exhibiting geometrical and material nonlinearities, the experimental route of identifying joint characteristics is instead preferred by many.

Experimental methods fall under two broad categories. One extracts joint stiffness and damping from measured dynamics characterized by modal parameters [10] and/or by frequency response functions (FRFs) used in clever implementations of frequency-based substructuring methods [11–13]. These methods extract joint parameters using FRFs of the substructures in their free–free, and assembled configurations. Though effective, these methods are prone to experimental errors due to noise and to issues of controllability and observability.

The other category extracts joint parameters from measured hysteresis loops [14–19], in which the relative response across a joint is measured under harmonic loading at the natural frequency of the system. The resulting force–displacement curves result in loops, the area under which equals the energy dissipated, i.e., the damping. The stiffness is also easily estimated. The identification of joint parameters using this approach is, hence, more straightforward and is also one of the methods preferred in this paper. These experimental methods seldom make any assumptions about the dynamical model describing the underlying physical phenomena at the bolted interface and are concerned more with simply extracting joint stiffness and damping under different conditions.

Notwithstanding the usefulness of earlier research using hysteresis methods [14–17], those results are not directly applicable to cantilevered tooling systems of interest in this paper. The stability of bolted cantilevered boring bars of interest in this paper is primarily governed by the dynamics in the radial direction [20], i.e., for the case of bolts being loaded in torsion, and for the cantilevered system being excited at its free end. Hence, though reports in [14, 15] are seminal, since those results were limited to tangential and/or normal excitations only, those findings are not directly relevant to this research. Similarly, though the findings reported in [16] are interesting, those were for the specific case of two beams assembled via a bolted joint, with one of them being treated as fixed, and not for the case of a simple bolted cantilevered system as is of interest herein.

The case of identification of joint stiffness and damping for a simple bolted cantilever system under torsional excitation,

for different number of bolts and different levels of tightening torques used to fasten the joint, remains unaddressed. Such a characterization is the main objective of this research.

The identification of bolted joint characteristics using an experimental setup that approximates the use case of a slender bolted cantilevered boring bar is discussed in Section “[Experimentation](#)” of the paper. Since bolted joints can be designed to produce adequate damping and rigidity, the behavior of the joint under different tightening torques is discussed. As the same cantilevered system can be excited differently such as in the real use case of the cutting forces at the free end being different, the behavior of the joint under different excitations is also discussed. Since the cantilevered system is stiff about its axis, and since damping under normal loadings is usually negligible [6], and since cantilevered tooling systems are usually loaded in torsion only, characterization of the behavior of the joint under normal and/or tangential excitations is not performed, and investigations are limited to torsional excitations. The response across the joint is characterized by sinusoidal excitations at the fundamental natural frequency of the system. Furthermore, since the cantilevered system can be fixed with one or more bolts, the response of the joint with different number of bolts is also discussed. Section “[Experimentation](#)” details the experimental procedure, and Section “[Experimental Observations](#)” discusses the preliminary observations from those experiments.

Methods to identify joint characteristics are detailed in Section “[Identifying joint Stiffness and damping](#)”. In addition to describing the hysteresis approach therein, a novel implementation of the harmonic balance method (HBM) to identify joint dynamics is also discussed. Since the excitation and response are harmonic, the joint forces may also be assumed to be harmonic. These forces can, hence, be replaced by amplitude-dependent equivalent stiffness and damping terms, whose coefficients depend on the amplitude of the relative displacement across the joint [6]. The HBM is often used in models that linearize the nonlinear behavior in joints [21–23]. Its use is extended to identify joint characteristics, and in doing so, an alternate method is suggested.

Section “[Discussion and Analysis of Identified Joint Characteristics](#)” of the paper discusses the identified characteristics. To check if the identified joint characteristics are correct, identified parameters are used to predict the dynamic behavior of a different assembled system—as discussed in Section “[Validation](#)” of the paper. The paper is finally concluded in Section “[Conclusion](#)”.

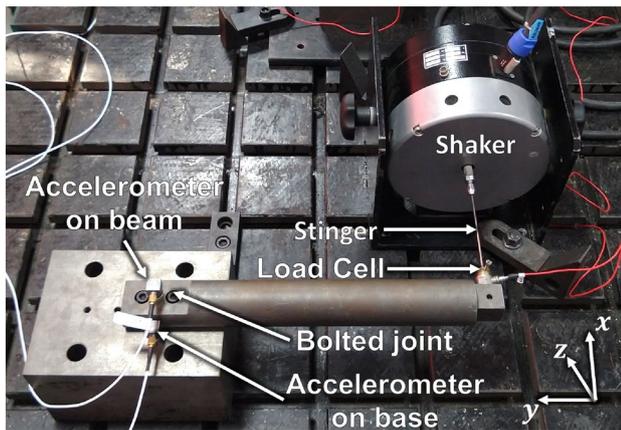


Fig. 1 Test setup used to identify bolted lap joint characteristics

## Experimentation

### Experimental Setup

To characterize the influence of bolted joints on the dynamics of cantilevered systems, experiments were carried out on the setup shown in Fig. 1. A cantilever beam of 50 mm diameter, and an overhang of 360 mm were mounted on a relatively rigid block using M8 bolt(s). The lap joint had a nominal contact area of  $\sim 40 \text{ cm}^2$ . Both surfaces of the joint interface were milled. The beam and the block were both made of steel. The block was mounted on a rigid cast iron table using four M12 bolts. The beam was excited at its free end using a modal shaker (make: B&K, type: 4824). The shaker was connected to the beam using a stinger of length 100 mm. The shaker was driven by its amplifier (make: B&K, type: 2732). The amplifier was provided a signal generated in LabVIEW through a digital to analog converter (make: NI, type: 9263). A load cell (make: Dytran, type: 1051V4) was attached at the end of the stinger to measure the force applied at the free end of the beam. This excitation will produce a torque at its bolted connection in the same plane, i.e., there will be torsional excitation.

The response of the beam to this torsional excitation was measured at its fixed end and at the block near the bolted connection—as shown in Fig. 1. This was done to obtain a sense of relative motion across the joint. Response at both locations were measured in the direction of excitation using magnetically mounted accelerometers (make: Meggitt, type: 4A16-1032). Accelerations and force were acquired using an analog to digital converter (make: NI, type: 9234), and were processed in LabVIEW on a computer.

## Experimental Procedure

Since identification of joint characteristics based on the hysteresis loop approach and on the use of the harmonic balance method is preferred, and since both methods require the system to be subjected to harmonic excitation at the natural frequency, first a modal hammer (make: Dytran, type: 5800B4) test was conducted to identify the main natural frequencies of the system. The modal tests were conducted using CutPRO's MALTF module, and modal parameters corresponding to the main mode of the system were extracted using its Modal module [24]. The main natural frequency of the system, which is also the most flexible [20], corresponds to the first bending mode of the beam—which were confirmed using roving hammer tests. After identifying the main natural frequency, the system was harmonically excited using the shaker with a linear slow sine-sweep around the main mode of interest. Each frequency was excited for 5 s, and the input force and the output response were logged. A representative example of such input–output measured data is shown in Fig. 2. All data were sampled at the rate of 51.2 kHz using a 24-bit analog to digital converter.

To minimize the influence of noise and other sources of measurement error corrupting the analysis, measured time series data was band-pass filtered with cut-offs of 160 Hz and 240 Hz, respectively. Displacements were obtained by integrating the measured accelerations in the frequency domain. These displacements were back-transformed to the time domain for further analysis. Use of displacements obtained from measured accelerations for joint parameter characterization is routine [15, 16].

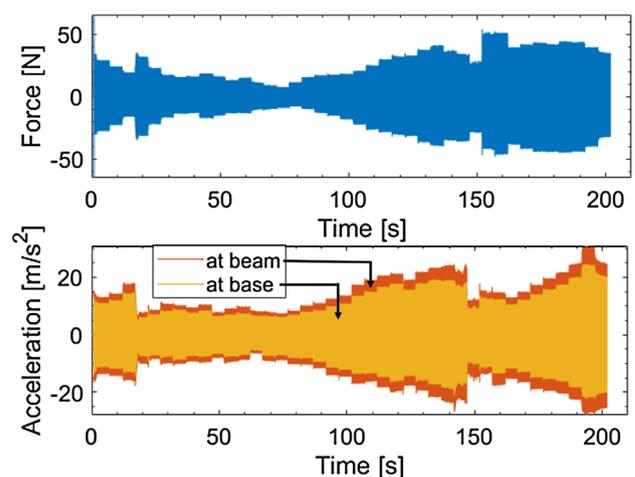


Fig. 2 Representative case of measured excitation and response signals for the configuration with two bolts tightened to a torque of 40 Nm each under a high level of excitation. Responses shown were measured at the beam and base

To evaluate the system's response under different loading scenarios, voltage levels supplied to the shaker were adjusted through its amplifier. Two levels of voltages were chosen and classified as low (1.5 V) and high (3 V). Different boundary conditions were emulated by (i) having one and two bolts respectively at the joint, (ii) varying the tightening torques of the M8 bolts to being low (30 Nm), and high (40 Nm). These levels were chosen as per recommendations for the grade of the bolts. Hence, for the case of the beam connected to the block with 1 and 2 bolts, respectively, four separate experiments were performed with two levels of tightening torques and two levels of torsional excitations—making for a total of eight different experiments. Though experiments with additional levels of torques and/or excitations may have helped to uncover the general trends in the joint characteristics, since tightening torques higher than recommended for the grade of the bolt in use may damage the threads and torque lower than 30 Nm resulted in a loose joint, only two levels of torque were considered in the study.

The above-described procedure to harmonically excite and measure response was repeated for all eight different configurations. Each experiment was also repeated in a randomized order. These experiments were conducted over several weeks. Each experiment lasted for approximately ~ 200 s. All the experiments were conducted in a temperature controlled laboratory.

## Experimental Observations

For each experimental configuration, the filtered input–output time series data were decomposed to the frequency domain and modal parameters were extracted using the 'peak-picking' method [25]. These parameters—stiffness ( $k$ ), damping coefficient ( $\zeta$ ) and natural frequency ( $f_n$ )—are listed in Table 1. The receptances constructed from these modal parameters, shown in Fig. 3, are for the cross response between the excitation at the beam's free end and the response measured at the beam's fixed end.

As is evident from Table 1 and from Fig. 3, the dynamics change with changing loading scenarios and with changing

boundary conditions. For the 1-bolt configuration in which the excitation level is low, an increase in the torque increases the stiffness, and hence there is an associated increase in the natural frequency. For all other experimental configurations, the natural frequencies were observed to be similar under changing excitations and tightening torques. However, the peak magnitude in the receptances for the 'low' level of torque were observed to be strongly dependent on the excitation level. The system appears to dynamically stiffen with increasing levels of excitation, and this could potentially be because of larger excitations resulting in larger dissipation at the joint. Even though receptances for the 'low' level of torque change with excitation levels, for the higher level of tightening torque, the response does not appear to change with excitations. This is potentially due to the higher torque arresting any potential micro slip at the joint interface. This may also explain the dynamic stiffening at resonance.

For the case of the 2-bolt configuration, there was a change in the dominant mode's frequency for all cases—which is different than the 1-bolt case. In general, for the 2-bolt case, the natural frequency reduces slightly with an increase in the excitation voltage and increases slightly with an increase in the tightening torque.

Comparison of the 1- and 2-bolt configurations suggests that the system stiffens with additional number of bolts at the interface. This stiffening is evident from an increase in the natural frequencies in the 2-bolt configuration from the 1-bolt configuration. From the peak magnitudes of the receptances being higher in the case of the 2-bolt configuration, it is evident that the damping has reduced. The dynamics of the bolted cantilevered beam being influenced by the loading and boundary conditions may all be attributable to its joint characteristics. Identifying these from the measured dynamics is discussed next.

## Identifying Joint Stiffness and Damping

Stiffness and damping are evaluated using the hysteresis area approach—discussed in Section “[Hysteresis Area Approach](#)”, and the harmonic balance method—discussed in Section “[Harmonic Balance Method](#)”. Identification presumes that the joint is massless, that the beam is linear, that there is no macroslip at the joint, and that for every experimental configuration, the joint can be characterized by linear, spring-viscous damper system.

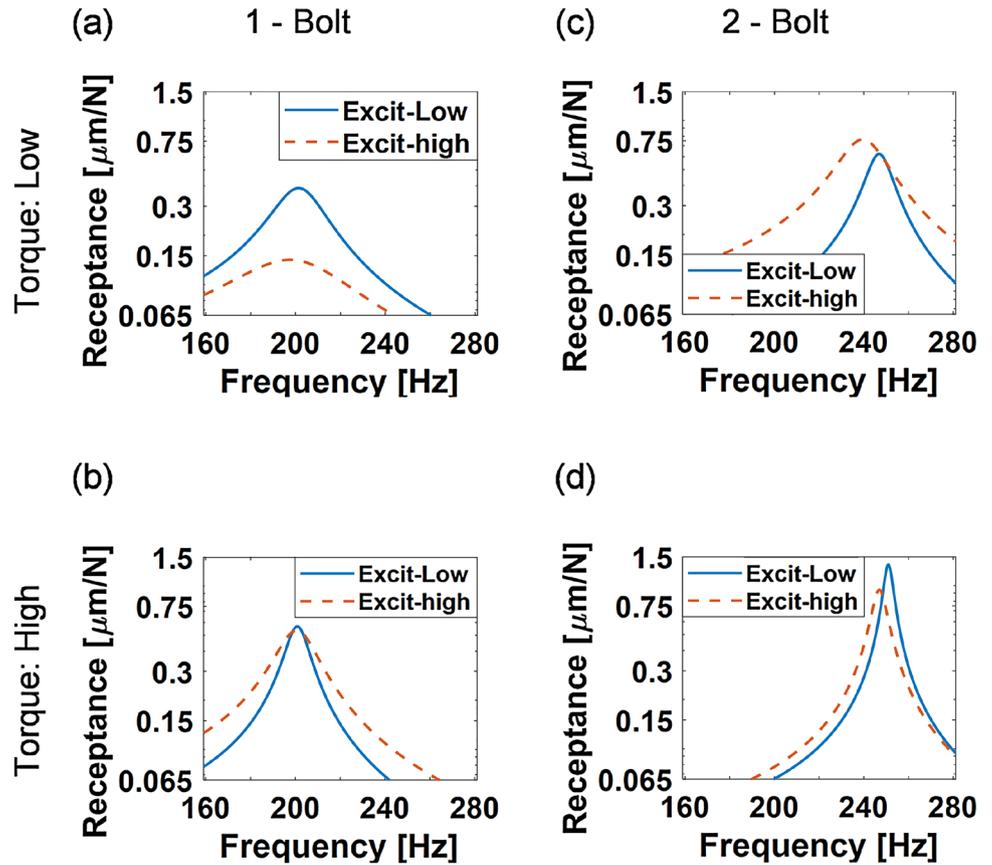
## Hysteresis Area Approach

Since bolted joints display a hysteresis loop for the force occurring in the joint versus the relative local displacement at the joint when subjected to torsional loading [16], the joint's characteristics were evaluated using the hysteresis

**Table 1** Estimate of modal parameters for the different configurations

Bolt	Torque Excitation level	Low		High	
		Low	High	Low	High
1	$f_n$ (Hz)	202	201	207	201
1	$k$ ( $10^7$ N/m)	2.28	2.71	3.39	2.09
1	$\zeta$	0.057	0.131	0.026	0.044
2	$f_n$ (Hz)	247	240	251	247
2	$k$ ( $10^7$ N/m)	3.31	1.42	4.18	3.74
2	$\zeta$	0.024	0.046	0.009	0.014

**Fig. 3** Receptances with different loading and boundary conditions: **a, b** configurations with 1 bolt with different torques and excitations, **c, d** configurations with 2 bolts with different torques and excitations



area approach. The area enclosed by a hysteresis curve,  $E$  is considered equivalent to the energy dissipated by the system and is indicative of the damping present in the system. Assuming the damping force to be viscous in nature, i.e.,  $F = C_{\text{hyst}}\dot{x}$ , the energy dissipated is:

$$E = \oint Fdx = \oint C_{\text{hyst}}\dot{x}dx = \oint C_{\text{hyst}}\dot{x}^2 dt, \tag{1}$$

wherein  $C_{\text{hyst}}$  is the damping coefficient and  $\dot{x}$  is the velocity. Since the response is harmonic, the displacement is assumed to be of the form:  $x = A_x \sin(\omega t - \phi)$  when substituted in Eq. (1) results in:

$$E = \int_0^{2\pi/\omega} C_{\text{hyst}}\omega^2 A_x^2 \cos^2(\omega t - \phi) dt = \pi\omega A_x^2 C_{\text{hyst}}. \tag{2}$$

Hence, the damping in the system becomes:

$$C_{\text{hyst}} = E/\pi\omega A_x^2. \tag{3}$$

Stiffness can be obtained directly as the ratio of the value of force  $f_s$  at which the displacement is highest:

$$K_{\text{hyst}} = f_s/A_x. \tag{4}$$

In the above analysis, displacements are obtained across the joint interface by integrating the accelerations measured at two locations near the fixed end of the beam in Fig. 1.  $x_1$  is obtained at the beam, and  $x_2$  at the block. The difference between  $x_1$  and  $x_2$  describes the relative motion at the interface, which is defined as  $A_x = |x_1 - x_2|$ . Above procedures were used to identify the joint characteristics for all eight experimental configurations.

### Harmonic Balance Method

For a dynamical system being excited harmonically, if the response is also harmonic, the opposing force from all structural parts is also harmonic—suggesting that the forces at the joint must also be harmonic. Since the beam is excited harmonically, the opposing force,  $F$  at the joint can be written as:

$$F(t) \approx K_{\text{hbm}}x + C_{\text{hbm}}\dot{x}, \tag{5}$$

wherein  $K_{\text{hbm}}$  and  $C_{\text{hbm}}$  are the equivalent joint stiffness and damping, and  $x$  and  $\dot{x}$  are the relative displacements and velocities across the joint. Since the force and response are both harmonic,  $F(t)$  can be written as a Fourier series:

$$F(t) = a_0 + \sum_{j=1}^{\infty} a_j \cos(j\omega t) + b_j \sin(j\omega t), \tag{6}$$

wherein  $a_i$  and  $b_i$  are Fourier coefficients, and  $\omega$  is the natural frequency at which the system is being excited.

Due to the assumption that the joint can be described as a linear, spring-viscous damper for every experimental configuration of interest, the resulting motion across the joint to steady-state harmonic excitation is harmonic, and as such higher harmonic terms of the response are expected to have smaller amplitudes relative to the fundamental harmonic [24]. Hence, if the static term  $a_0$  is neglected, as are the higher harmonics, the equivalent stiffness and damping from Eqs. (5) and (6) become:

$$K_{\text{hbm}} = \omega / \pi A_x \int_0^{2\pi/\omega} F(t) \cos(\omega t) dt, \tag{7}$$

$$C_{\text{hbm}} = -1 / \pi A_x \int_0^{2\pi/\omega} F(t) \sin(\omega t) dt, \tag{8}$$

wherein  $A_x = |x_1 - x_2|$ , and  $F(t)$  is the instantaneous harmonic excitation—both of which are measured. Since  $F(t)$

and  $A_x$  are both known, the joint stiffness ( $K_{\text{hbm}}$ ) and the joint damping ( $C_{\text{hbm}}$ ) can be identified for all eight different experimental configurations.

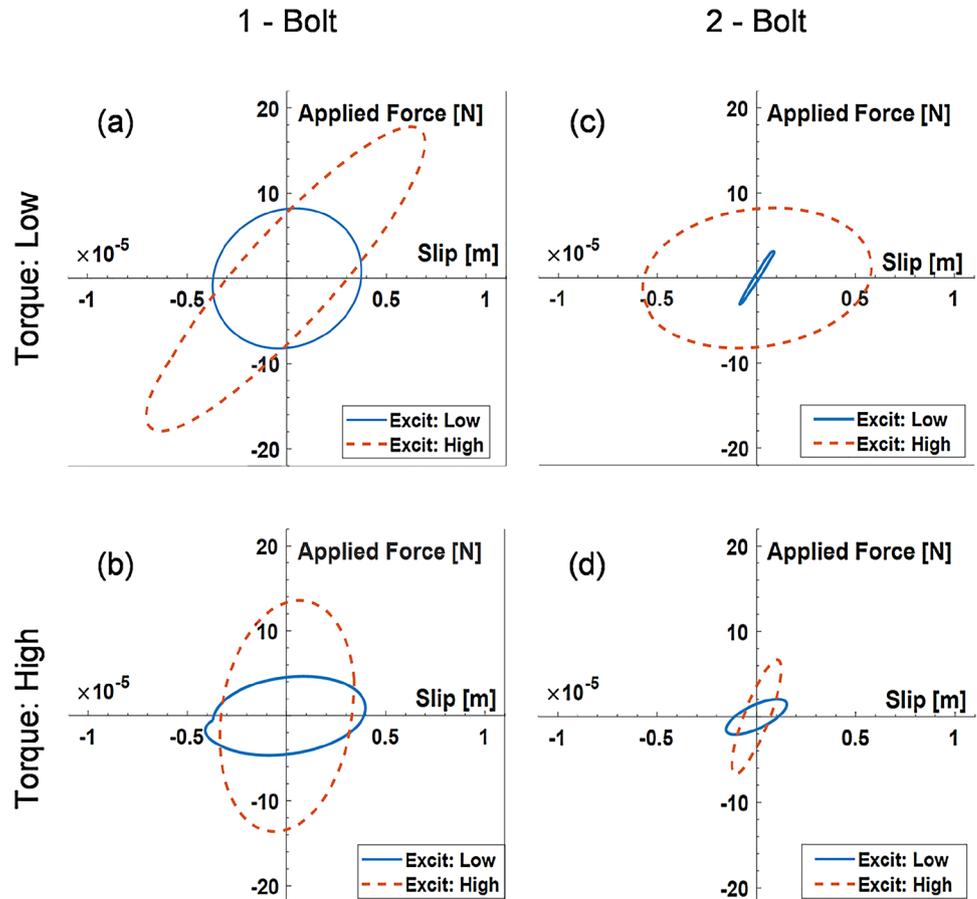
### Discussion and Analysis of Identified Joint Characteristics

This section first discusses the hysteretic behavior in Section “Hysteresis Loops”, followed by the force and response characteristics used in the harmonic balance method in Section “Harmonic Force and Response”. Identified joint characteristics are compared in Section “Identified stiffness and Damping”—which also discusses issues of repeatability and addresses how our assumptions condition our results.

#### Hysteresis Loops

The measured time series data of the force and response for excitation at the natural frequency of the system were used to obtain the hysteresis loops for all eight experimental configurations. Since the hysteresis loop is obtained for every cycle of loading, they are averaged over a period of 1 second. The resulting loops are shown in Fig. 4. The joint damping

**Fig. 4** Hysteresis loops for **a, b** configurations with 1 bolt with different torques and excitations, **c, d** configurations with 2 bolts with different torques and excitations



and stiffness estimated from these loops using Eqs. (3) and (4) are listed in Table 2. As is evident for most cases, the hysteresis loop tends towards an inclined ellipse—confirming that our assumption of the joint being characterized as a linear, spring-viscous damper system is valid.

## Harmonic Force and Response

The measured time series data of the force and response that were used to obtain the hysteresis loops, which in turn were used to evaluate joint characteristics, are used directly in the harmonic balance method to estimate joint characteristics. The time series data, spanning 0.1 s, for all eight experimental configurations are shown in Fig. 5. Since only a single cycle's data are necessary to evaluate the joint damping and stiffness, data are averaged appropriately. The joint stiffness and damping, thus, evaluated are listed in Table 2.

## Identified Stiffness and Damping

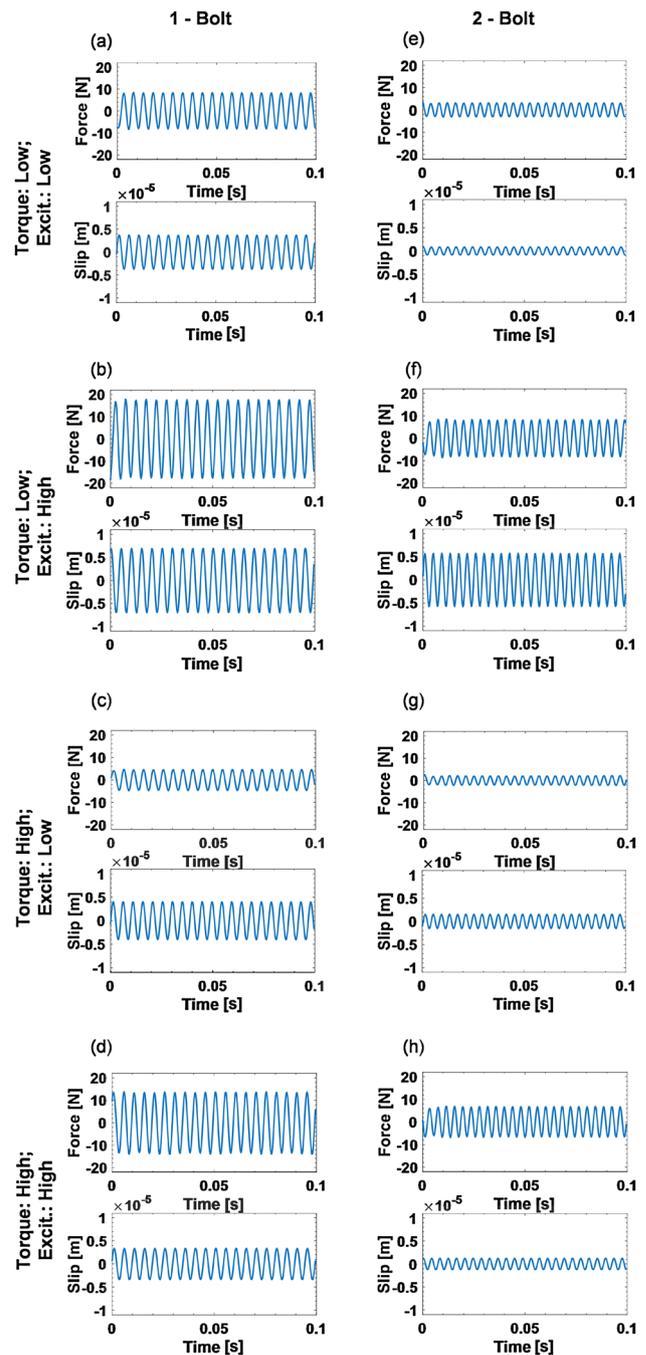
Results in Table 2 suggest that changes in the excitation and boundary conditions cause changes in the stiffness and damping at the joint. Results in Table 2 also suggest that the parameters estimated with the proposed harmonic balance method for joint characterization agree with the traditional approach of using hysteresis loops.

From Table 2, it is also clear that in the case of the 1-bolt condition, when the excitation level is increased from low to high, the damping appears to decrease for a low joint preload, and damping increases for a high level of preload. However, for the 2-bolt case, damping appears to increase for both low and high levels of preload.

When the joint preload is high and when the excitation level is increased, a higher force is exerted at the joint. If this higher force causes more slip, then the damping may increase as seen from the results. Also, when the bolt preload is low enough, smaller excitations may cause slip which increases the damping. However, when the preload value is high, low levels of excitation may not be enough to cause enough slip, hence potentially reducing damping.

In the case of the 1-bolt configuration, the stiffness appears to increase with increase in excitation level for both low and high levels of preload. Whereas, in a 2-bolt configuration, when the excitation level is increased from low to high, the stiffness appears to decrease for a low joint preload, and stiffness increases for a high level of preload. Although stiffness was observed to change with boundary conditions, the variation in stiffness was not as considerable as the variation in damping. It also appears that the damping is higher in the 1-bolt configuration, whereas stiffness is higher in the 2-bolt configuration.

Results presented in Table 2 are from a single data set for each of the eight different experimental configurations.



**Fig. 5** Time series data used in the harmonic balance method for **a–d** 1-bolt configuration with different loading and boundary conditions; **e–h** 2-bolt configuration with different loading and boundary conditions

Though experiments were repeated, and the order of experimentation was randomized, due to the statistical nature of contact at the interface, joint parameters evaluated were not repeatable across repetitions. Since the joint is potentially nonlinear, results not being repeatable is unsurprising. Even though results were strictly not repeatable, they were observed

**Table 2** Identified joint stiffness and damping

Bolt	Torque	Exc.	Hysteresis		Harmonic	
			Area Approach		Balance Method	
			$k \times 10^5$	c	$k \times 10^5$	c
			(N/m)	(Ns/m)	(N/m)	(Ns/m)
1	Low	Low	2.21	1739.2	2.18	1647.1
1	Low	High	23.7	928.8	26.2	952.9
1	High	Low	2.22	821.5	1.89	845.3
1	High	High	2.82	3246.4	2.82	3314.6
2	Low	Low	34.9	288.3	34.6	380.0
2	Low	High	3.46	902.7	2.42	904.6
2	High	Low	10.9	583.7	10.8	607.8
2	High	High	45.1	1481.3	45.7	1722.9

to follow similar trends as evaluated and listed in Table 2. If, however, experiments with the same configuration were repeated without changing the configuration, results were found to be repeatable. Results were also found to be repeatable for different data sets within the same time series data. Results were also found to be repeatable with changing periods of averaging data to obtain the hysteresis loops and/or balance the harmonics.

Though the joint parameters evaluated using the hysteresis area approach and harmonic balance method agree with each other, results are conditioned by our assumptions. In the hysteresis area approach, the energy dissipated by the system was taken as the area under the hysteresis curve, which was assumed to be solely due to damping. However, the dissipation could also be caused friction that is not modeled, and/or other unmodeled phenomena. In the harmonic balance method, the higher harmonics were ignored, thus neglecting any role that frictional macroslip may play at the interface.

Furthermore, since the energy dissipated is a function of damping and the amplitude of displacement (see Eq. 2), an increase in the area may not necessarily result in an increase in the damping value if the amplitude of displacement also increases. This might explain why for cases with bigger hysteresis loops (see Fig. 4), the damping is evaluated to be low (see Table 2).

Whether the methods of identifying joint parameters and/or the identified parameters are useful depends on how well these parameters can predict the dynamic behavior of another assembled system which has a similar bolted connection. This validation is discussed in the next section.

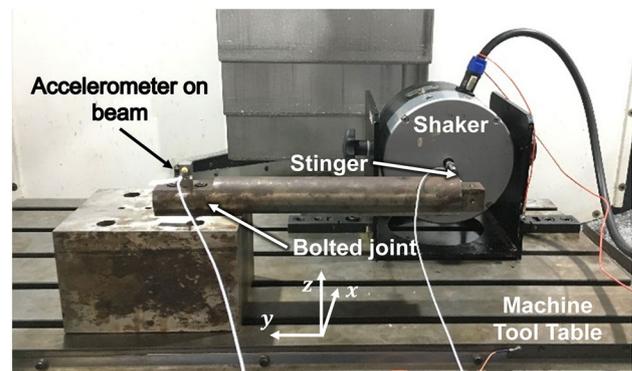
## Validation

To verify and validate if the identified joint characteristics are correct and useful, the identified joint parameters were used in a model of a cantilevered beam. The agreement

between the predicted response and experiments on a beam bolted at one end on to a block which is mounted on a different base—in this case, a machine tool table is observed. The experimental setup for this is shown in Fig. 6, and the schematic for the model of the beam is shown in Fig. 7. The setup is like the setup shown in Fig. 1, except for the block being mounted on a machine tool table.

The beam was bolted on to the block with two bolts, both tightened with a 'high' level of tightening torque. The level of excitation was kept 'low'. Experiments on this setup followed the same procedure as described in Section "Experimentation". The dynamic response for this system is also characterized by a single dominant bending mode in the direction of excitation—as shown in Fig. 8.

Modeled response was predicted using a frequency-based substructuring (FBS) approach. The FBS approach affords modularity and, hence, was preferred. The beam modeled as a Euler–Bernoulli beam was treated as one substructure. Its receptance was obtained as described in [26]. The geometry of the beam is the same as described in the experimental setup in Section "Experimental setup", i.e., the beam has



**Fig. 6** Experimental setup of a beam bolted on to a block mounted on machine tool table

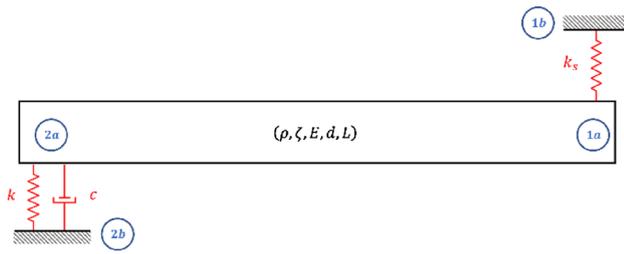


Fig. 7 Schematic for predicting response of a bolted beam—for validation purposes

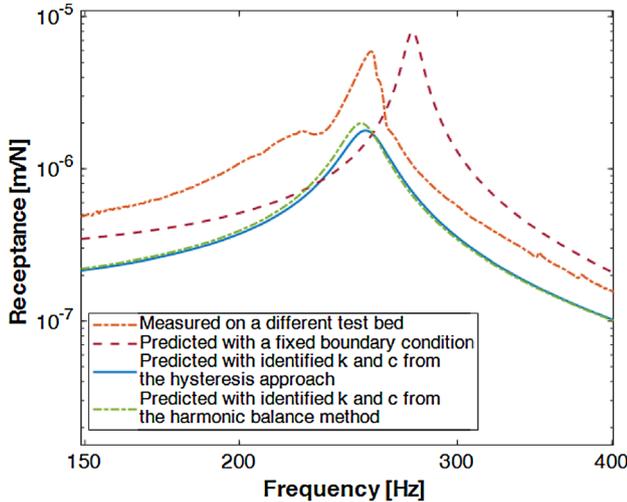


Fig. 8 Schematic for predicting response of a bolted beam—for validation purposes

a 50 mm diameter ( $d$ ), and an overhang ( $L$ ) of 360 mm. It is made of steel, with a modulus ( $E$ ) of 205 GPa, and a density ( $\rho$ ) of 7840 kg/m<sup>3</sup> and with a damping constant ( $\zeta$ ) of 0.03. The substructural receptance of the beam, i.e., its FRF ( $h_{2a2a}$ ) was coupled at the fixed end with a spring ( $k$ ) and damper ( $c$ )—parameters of which were identified as discussed in Section “Identified Stiffness and damping”. To faithfully approximate the experimental setup in which the beam is excited with a shaker coupled to it with a stinger, the model also treats the stinger as a separate substructure modeled as a spring of stiffness  $k_s = 3.22 \times 10^6$  N/m. The beam’s receptance at its free end ( $h_{1a1a}$ ) is coupled to this stinger in the frequency domain. The model schematic is shown in Fig. 7.

Since the measured receptance is a cross FRF between the excitation location and the fixing location, the model predicted cross receptance too can be shown to be [26]:

$$H_{21} = H_{2a1} - H_{2a1}(H_{2a2a} + h_{2b2b} + 1/k')^{-1}H_{2a2a}, \quad (9)$$

wherein

$$k' = k + i\omega c, \quad (10)$$

$$H_{2a1} = h_{2a1a} - h_{2a1a}(H_{1a1a} + h_{1b1b} + 1/k_s)^{-1}h_{1a1a}, \quad (11)$$

and

$$H_{2a2a} = h_{2a2a} - h_{2a1a}(H_{1a1a} + h_{1b1b} + 1/k_s)^{-1}h_{1a2a}, \quad (12)$$

respectively.  $h_{2a1a}$  in the above equations is the cross receptance between the free and fixed ends of the beam.

Joint stiffness ( $k$ ) and damping ( $c$ ) parameters in Eq. (9) are taken from Table 2 for the 2-bolt configuration with the tightening torque being ‘high’ and the excitation being ‘low’. Since these parameters evaluated from the hysteresis area approach and the harmonic balance method were slightly different, predictions were made using parameters evaluated from both methods. In addition to predicting the response of the beam using the identified joint parameters, response was also predicted with the case of the beam with a fixed (rigid) boundary condition at the bolted end and with a stinger attached to it at the free end. These modeled receptances are compared with the measured receptance in Fig. 8.

It is evident from Fig. 8 that the receptance predicted with a fixed boundary condition does not agree with the measured behavior. Since the rigid boundary condition cannot account for damping at the joint, response predicted with it is naturally more flexible. And since the joint is rigid, the stiffness too is higher than in practice, and this results in a higher natural frequency. On the other hand, response predicted with identified joint characteristics using both the hysteresis approach and the harmonic balance method compares better with the measured response—at least in getting the natural frequency correct. Since damping at the interface is a complex phenomenon, and since the damping is idealized to be of the viscous nature while ignoring any potential macroslip, both methods underestimate the damping. Even though damping is overestimated, since the natural frequency is correctly estimated, the identified characteristics are deemed correct, and the model valid.

Moreover, since one of the aims of this paper was to characterize the influence of joint dynamics on the response of slender cantilevered tooling systems to help instruct the tuning of dynamic vibration absorbers, and since absorber tuning depends on the natural frequency of the primary cantilevered system, and since that behavior was captured correctly, the approach presented herein will indeed be useful.

## Conclusion

This paper was concerned with experimentally characterizing the dynamics of bolted cantilevered systems under changing tightening torques, torsional excitations, and

with different number of bolts at the fixing interface. A simple test setup was conceived to harmonically excite a slender cantilevered beam in torsion and to measure relative displacements across the interface. Since excitation and response were harmonic, the use of the harmonic balance method was proposed to identify joint characteristics. Identified characteristics were benchmarked with the established hysteretic approach of joint parameter identification. Following observations were made:

- (i) System dynamics were observed to be strongly dependent on the level of excitation, tightening torques, and number of bolts at the interface.
- (ii) In general, since the hysteresis loops tended towards an inclined ellipse, our assumption of the joint being characterized as a linear, spring-viscous damper system was deemed valid
- (iii) Signal processing and conditioning were not observed to influence identification.
- (iv) The 1-bolt configuration exhibited higher dissipative behavior in contrast to the 2-bolt configuration. The stiffness however was higher for the two 2-bolt configurations. These observations suggest that the bolted connection can be designed to provide adequate damping as without compromising on rigidity.
- (v) In general, since the harmonic balance method directly uses measured time series data to estimate joint characteristics and does not use an indirect method such as the area contained in a hysteresis loop, the harmonic balance method may be considered as a viable alternative for joint parameter estimation.

Though identified parameters were validated using them in a model of a beam and by comparing predicted response with measured in a different configuration, and even though our results are conditioned by our assumptions during identification, the setup and procedure described are simple, and the methods robust, and as such can serve as a guide for other researchers and practitioners interested in the dynamics of assemblies with bolted joints.

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## Declarations

**Conflict of interest** The authors declare that they have no conflict of interest.

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