

# Vibration suppression of a slender boring bar by an impact damper

Arjun Patel, Mohit Law\*, Pankaj Wahi

Indian Institute of Technology Kanpur, Kanpur, India

Presented in International Conference on Precision, Micro, Meso and Nano Engineering (COPEN - 12: 2022)

December 8 - 10, 2022 IIT Kanpur, India

---

## ABSTRACT

### KEYWORDS

Impact Damper,  
Boring Bar,  
Vibration Suppression,  
Chatter.

*This paper presents an analytical model to characterize the role of an impact damper in suppressing vibrations of slender boring bars that are used in deep hole boring processes. The boring bar is modelled as a Euler-Bernoulli beam that interacts with a ring impact damper through a spring and damper combine. Parametric analysis suggests that smaller gaps in between the bar and the damper result in higher reduction in the vibration response. Analysis also suggests that for maximum vibration suppressions the damper's stiffness should be less than that of the bar, and its damping should be greater. Though the vibration suppression capacity of this impact damped boring bar is found slightly wanting when compared to a boring bar damped with an optimally tuned mass damper, model-based analysis as is presented herein is new and generalized and can hence guide further design and development of optimal impact damped boring bars.*

---

## 1. Introduction

The boring of deep holes requires the use of slender boring bars. Slenderness reduces the bar's bending stiffness and makes it vulnerable to vibrations during cutting. These vibrations may grow and result in chatter. These process-induced large amplitude chatter vibrations can damage tools and parts of the machine and can destroy part surface quality.

Vibrations of boring bars are usually suppressed by integrating tuned mass dampers within them (Munoa et al., 2016; Yadav et al., 2020). However, if/when the dynamics of the boring bar deviate with speed (Patel et al., 2022) and/or boundary conditions (Thekkepat et al., 2021), tuning becomes non-trivial. In such cases, other hybrid solutions using eddy current dampers in conjunction with a detuned absorber are preferred (Patel, Yadav et al., 2022). Since optimal tuning is difficult, as is making hybrid dampers, this paper instead explores the vibration suppression potential of boring bars using an impact damper.

Impact dampers function by dissipating energy due to collisions and an exchange of momentum between the object to be damped and the mass

damper. Collisions happen when the object or the mass move more than the gap between them. Before collision, the object, and the mass move in opposite directions. And, after collision, the mass reverses direction and the velocity of the main system is attenuated due to its relatively larger inertia (Ibrahim, 2009).

Despite their simple construction and working principle, the use of impact dampers in vibration suppression of cutting tools is limited. The notable exceptions are the works (Thomas et al., 1973) and (Ema & Marui, 2000). Though seminal, these reports being experimental could not completely characterize the role of the gap between the impacting mass and the main system, or the role of the coefficient of restitution, or the role of impacting interface characteristics, or the role of the mass-ratio, stiffness, and damping of the two systems. Such analysis is only possible using analytical and/or numerical models. Presenting such an analytical model that allows for systematic parametric analysis to characterize which parameters govern vibration suppression of the boring bar is the main objective and new technical contribution of this paper.

Though there exist classical analytical models that help understand the behaviour of an impact damper (Masri, 1970), those modelled the impact damper and the primary system as lumped masses. Since the boring bar is akin to a slender

---

\*Corresponding author E-mail: mlaw@iitk.ac.in

cantilevered beam, a more appropriate method to model it would be to approximate it as a Euler-Bernoulli beam. Doing so would overcome the limitation of assuming the bar as a lumped mass that would presume placement of the impact damper at the free end which is also the cutting end.

Models for impact damping for cantilevered beams do exist. In (Cheng & Wang, 2003), free vibrations of a cantilevered beam with an impact damper attached outside of it were investigated. Impacting interfaces were modelled as a spring and a viscous damper system. In other related work reported in (Geng et al., 2020), impacts between a cantilevered beam and the impacting mass were modelled using Hertzian contact mechanics.

This paper will build on these prior models (Cheng & Wang, 2003; Geng et al., 2020) to develop one for a boring bar with an impact damper. However, since our application of interest is different than earlier reports, suitable modifications are made to the model as discussed in Section 2 of this paper. Section 3 presents parametric analysis to discuss the role of the gap between the boring bar and the impact damper and to understand the role of the mass, stiffness, and damping ratios between the two systems. We benchmarked our results with a boring bar integrated with a tuned absorber. Main conclusions follow.

## 2. Model of a Boring Bar with an Impact Damper

This section first discusses a possible design of a boring bar with an impact damper. We follow this by presenting the governing equations of motion for the system.

### 2.1. Possible design of a boring bar with an impact damper

A boring bar with diameter  $D$  and length  $L$  is assumed to be impacting a ring impact damper with mass  $m$  positioned at a distance of  $z_d$  from the fixed end as is shown in Fig. 1(a). The ring is in turn assumed to be placed in an outer housing that is fixed. The support stiffness of the ring in its housing is presently ignored. Though the impact damper should be ideally housed within the bar such as to not reduce the bar's effective overhang, for the preliminary investigations herein, the damper is assumed to be outside the bar. The gap between the bar and the ring is  $d$  – as shown in Fig. 1(b). The interface between the ring

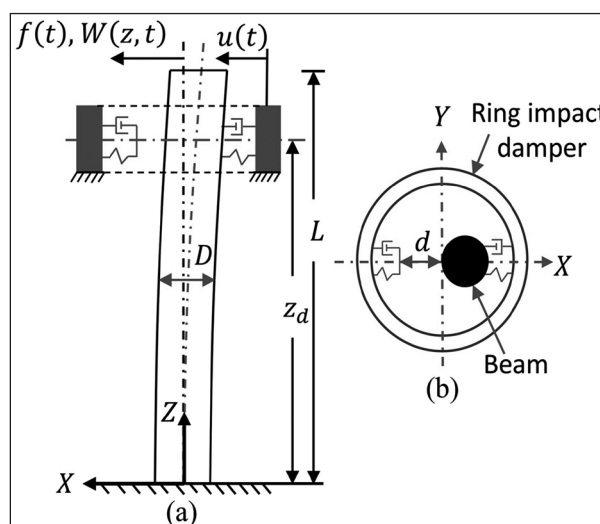


Fig. 1. (a) Possible design of an impact damped boring bar, (b) Plan showing the interface between the two systems.

and the bar is assumed to behave like a spring ( $k_d$ ) and damper ( $c_d$ ) combine. This design is inspired by the design concept in (Cheng & Wang, 2003). Since the bar is more flexible in its radial direction than along its axis (Yadav et al., 2020), impacts are assumed to take place only in the  $X$  direction, and as such the stiffness and damping at the interface are also assumed only active in the  $X$  direction. The bar is assumed to be linear, symmetric, and isotropic. The boring bar vibrates with amplitude  $W$  when excited by the cutting process, and when its motion exceeds the gap, i.e., when  $d \leq W(z_d, t) - u(t) \leq 0$ , wherein  $u$  is the motion of the mass  $m$ , it will impact the ring's housing. This collision and exchange of momentum through the interface will result in dissipation of energy of the boring bar which in turn will reduce its vibration. The equations governing this system are described next.

### 2.2. Governing equations of motion

The boring bar is modelled as a cantilevered Euler-Bernoulli beam, and the impact damper as a lumped mass. To obtain the governing equations of motion, we use notions of kinetic and potential energies of the system along with work done by the external and the damping force on the system. We then apply the extended Hamilton's principle (Hagedorn & Dasgupta, 2007). Though the mechanics of impact are complex and are governed by nonlinear equations, since we model energy dissipation from impacting interfaces between the boring bar and the ring impact damper as a spring-damper system, we obtain piecewise linear governing equations as discussed below.

There are two situations; the first corresponds to no impact, and the second corresponds to when impact takes place. There are two sets of equations for the two cases.

For the first situation, i.e., when there is no impact, i.e., when  $0 < W(z_d, t) - u(t) < d$ , the beam and the ring damper move freely, and the governing equations become:

$$\rho A \left( \frac{\partial^2 W(z, t)}{\partial t^2} \right) + \frac{C}{L} \left( \frac{\partial W(z, t)}{\partial t} \right) + EI \left( \frac{\partial^4 W(z, t)}{\partial z^4} \right) = f(t) \delta(z - L), \quad \dots(1)$$

$$m \left( \frac{d^2 u(t)}{dt^2} \right) = 0, \quad \dots\dots\dots(2)$$

wherein  $\rho, A, L, C, E$  and  $I$  are mass density, cross-sectional area, length, damping coefficient, elastic modulus, and area moment of inertia of beam, respectively.  $f(t)$  is external force on beam acting at the free end, i.e., the cutting end.  $\delta$  is a Dirac delta function. The first, second and third terms on the left side of the Eq. (1) denote the kinetic, damping, and potential energies of the beam, respectively. Term on the left side in Eq. (2) represents the kinetic energy of ring damper.

For the second case, i.e., when  $d \leq W(z_d, t) - u(t) \leq 0$ , the beam impacts the damper, and the governing equations become:

$$\begin{aligned} & \left( \frac{\partial^2 W(z, t)}{\partial t^2} \right) + \frac{C}{L} \left( \frac{\partial W(z, t)}{\partial t} \right) + EI \left( \frac{\partial^4 W(z, t)}{\partial z^4} \right) + \\ & \left[ k_d (W(z, t) - u(t)) + c_d \left( \frac{\partial W(z, t)}{\partial t} - \frac{du(t)}{dt} \right) \right] \end{aligned} \quad \dots\dots(3)$$

$$\delta(z - z_d) = f(t) \delta(z - L),$$

$$m \left( \frac{d^2 u(t)}{dt^2} \right) - [k_d (W(z, t) - u(t))] \delta(z - z_d)$$

$$- \left[ c_d \left( \frac{\partial W(z, t)}{\partial t} - \frac{du(t)}{dt} \right) \right] \delta(z - z_d) = 0, \quad \dots\dots(4)$$

wherein the first three terms in Eq. (3) have the same meaning as they did in Eq. (1), and the third and fourth terms in Eq. (3) denotes the potential and damping energy offered by the spring and damper system that characterize the interface between the beam and the ring. Since the spring and damper will apply equal and opposite reactions on the ring, the damper will have potential and damping energies. These are described in the second and third terms in Eq. (4)

The first term in Eq. (4) remains that of the kinetic energy of the damper.

Displacement of the beam is obtained by superposing all modes of the beam as (Hagedorn & Dasgupta, 2007):

$$W(z, t) = \sum_i q_i(t) \psi_i(z), \quad \dots\dots\dots(5)$$

wherein  $i$  represents the number of modes,  $q_i(t)$  are the time-dependent displacements, and  $\psi_i(z)$  are mode shapes of the cantilever beam, which are given as:

$$\begin{aligned} & \psi_i(z) = (\cos \eta_i z - \cosh \eta_i z) \\ & - \frac{\cos \eta_i L + \cosh \eta_i L}{\sin \eta_i L + \sinh \eta_i L} (\sin \eta_i z - \sinh \eta_i z). \end{aligned} \quad \dots\dots(6)$$

wherein  $\eta_i$  is  $1.8751/L$  for the first bending mode of vibration. Since the first bending mode of the boring bar is usually the most flexible (Yadav et al., 2020), we reduce Eqs. (5-6) by considering  $i=1$ .  $W(z, t)$  from Eq. (5) is then substituted into Eqs. (1-4) and the new equations are multiplied by  $\psi_j(z)$  on both sides and integrated over the length. Using orthogonal properties (Hagedorn & Dasgupta, 2007) and rearranging, we can write modified Eqs. (1-2) and Eqs. (3-4) in matrix form as follows:

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{u}(t) \end{Bmatrix} + \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{q}_1(t) \\ \dot{u}(t) \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} q(t) \\ u(t) \end{Bmatrix} = \begin{Bmatrix} f(t) \psi_1(z_d) \\ 0 \end{Bmatrix}, \quad \dots\dots\dots(7)$$

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{u}(t) \end{Bmatrix} + \begin{bmatrix} C + c_d \psi_1^2(z_d) & -c_d \psi_1(z_d) \\ -c_d \psi_1(z_d) & c_d \end{bmatrix} \begin{Bmatrix} q(t) \\ u(t) \end{Bmatrix} = \begin{Bmatrix} f(t) \psi_1(z_d) \\ 0 \end{Bmatrix}, \quad \dots\dots\dots(8)$$

wherein  $M = \int_0^L \rho A \psi_1(z) \psi_1(z) dz$ ,  $C = \int_0^L L \psi_1(z) \psi_1(z) dz$ , and  $K = \int_0^L EI \beta^4 \psi_1(z) \psi_1(z) dz$ .

Having discussed the procedures to obtain the governing equations of motions, we use these to systematically characterize the role of the gap, and the mass-ratio, stiffness, and damping of the two systems.

### 3. Parametric Analysis with Free and Forced Response

This section discusses the free and forced vibration response of the beam for different gaps and for different ratios of mass, stiffness, and

**Table 1**

Parameters of the boring bar.

Boring bar with a L/D ratio of 8. M, C, and K are for the first mode	
Length, $L = 0.2$ m	Damping ratio, $\zeta = 0.03$
Diameter, $D = 0.025$ m	Mass, $M = 0.765$ kg
Mass density, $\rho = 7800$ kg/m <sup>3</sup>	Stiffness, $K = 5.93 \times 10^6$ N/m
Young's modulus, $E = 200$ MPa	Damping, $C = 127.8$ N-s/m

damping between the two systems. For analysis herein, the impact damper is assumed attached at the middle ( $z_d=L/2$ ) of the boring bar. Placing it any nearer the cutting edge will reduce the working length. Displacements are obtained as discussed above, and to facilitate meaningful comparisons, the root mean square amplitude of the beam at its free end is defined as:

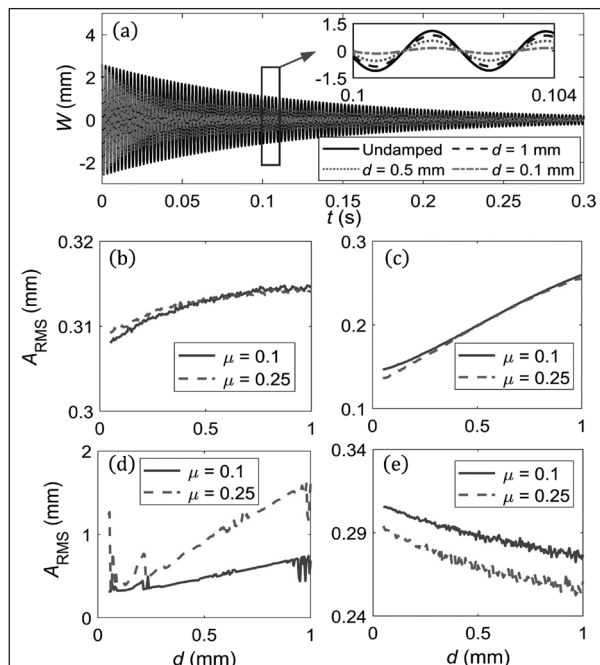
$$A_{RMS} = \left( \frac{1}{T} \int_0^T (q(t)\psi_1(L))^2 dt \right)^{\frac{1}{2}}, \dots\dots\dots(9)$$

wherein T is the observation period.

**3.1. Free vibration analysis**

The free vibration response of the boring bar is shown in Fig. 2(a) for the representative case of the mass-ratio  $\mu$ , which is defined as the ratio of the impacting mass to the modal mass of the beam, being 0.1. These results are obtained with the parameters of boring bar as listed in Table 1. Results shown in Fig. 2(a) are for the case of the stiffness ( $k_d$ ) of the interface being less than that of the boring bar, and the damping ( $c_d$ ) greater, i.e.,  $k_d < K$ ;  $c_d > C$ . Results for different gaps are shown and contrasted with the boring bar without a damper. An initial displacement of 0.5 mm is given to the middle of the boring bar at the location of action of the impact damper, and the free vibration response is tracked. And as is evident, a maximum vibration suppression of ~76% is achieved for when the gap is the small.

Since Fig. 2(a) provides only a local picture, we present summary results characterized by the  $A_{RMS}$  for four cases in Fig. 2(b-e). In the first case shown in Fig. 2(b), the stiffness ( $k_d$ ) and damping ( $c_d$ ) of the interface are less than that of the boring bar, i.e.,  $k_d < K$ ;  $c_d < C$ . In the second case shown in Fig. 2(c),  $k_d < K$ ;  $c_d > C$ . In the third case shown



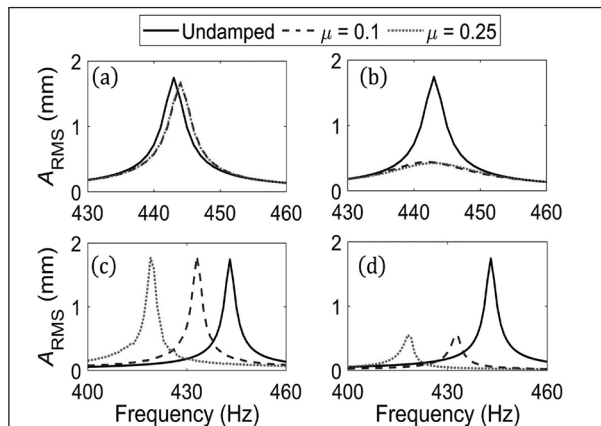
**Fig. 2.** (a) Displacement of the beam for  $k_d=0.01K, c_d=2C, \mu=0.1$ . (b-e) Free response characterized by  $A_{RMS}$  for different mass ratios ( $\mu$ ) and gap values (d): (b)  $k_d=0.01K, c_d=0.5C$ , (c)  $k_d=0.01K, c_d=2C$ , (d)  $k_d=100K, c_d=0.5C$ , (e)  $k_d=100K, c_d=2C$ .

in Fig. 2(d),  $k_d > K$ ;  $c_d < C$ . In the final case shown in Fig. 2(e),  $k_d > K$ ;  $c_d > C$ . For each of these cases, analysis is conducted with two different  $\mu$  and for gaps of up to 1 mm.

As is evident from Fig. 2(b-e) response is consistently low for small gaps, except for the case shown in Fig. 2(e), in which response decreases with increasing gaps. Interestingly, for the cases with  $k_d < K$ , i.e., cases shown in Fig. 2(b-c), the damping ratio and/or the mass ratio appear to play no role. However, when  $k_d > K$ , the damping and mass ratios both appear to play a role – as is evident from Fig. 2(d-e). Of the cases investigated, though the  $A_{RMS}$  is least for the case with  $k_d < K$ ;  $c_d > C$  for when the gap is low, there could be other scenarios in which further suppression might be possible. The model proposed facilitates such analysis and suggests that the gap and the mass-ratio, stiffness, and damping of the two systems do indeed play a significant role in the vibration suppression potential of the impact damped boring bar.

**3.2. Forced vibration analysis**

Since the free vibration analysis, in general, suggested that vibration suppression with smaller gaps is better, for the forced vibration analysis herein, only a representative case for the gap



**Fig. 3.** Forced response characterized by  $A_{RMS}$  for different mass ratios ( $\mu$ ) and  $d=0.5$  mm:  
 (a)  $k_d < K, c_d < C$ , (b)  $k_d < K, c_d > C$ ,  
 (c)  $k_d > K, c_d < C$  (d)  $k_d > K, c_d > C$ .

being 0.5 mm is discussed. And like in Section 3.1, herein too four cases are discussed: in the first case, response for which is shown in Fig. 3(a),  $k_d < K$ ;  $c_d < C$ . In the second case shown in Fig. 3(b),  $k_d < K$ ;  $c_d > C$ . In the third case shown in Fig. 3(c),  $k_d > K$ ;  $c_d < C$ . In the final case shown in Fig. 3(d),  $k_d > K$ ;  $c_d > C$ . For each of these cases, analysis is conducted with two levels of  $\mu$  and with the forcing frequency changing in the range around the natural frequency of the solid boring bar without the impact damper. For analysis herein too, the parameters of the boring bar are taken as those listed in Table 1. To characterize response, the forcing function is assumed to be of the form of  $f(t)=100 \sin \omega t$ , wherein  $\omega$  is excitation frequency. The gain of 100 is arbitrary to the analysis.

As is evident from Fig. 3, the largest reduction in the forced vibration response of  $\sim 76\%$  over the undamped case is achieved for the case when  $k_d < K$ ;  $c_d > C$ , i.e., the case shown in Fig. 3(b). Interestingly for this case, the mass ratio appears to play no role. And, since the only difference between this case and the case shown in Fig. 3(a) is that  $c_d < C$  in Fig. 3(a), it further suggests that the stiffness of the interface being less than the that of the boring bar also plays no role, and just the damping at the interface influences response. This is furthermore clear from Fig. 3(c) in which  $c_d < C$ . The shift in the natural frequency in this case is only due to the change in stiffness and mass ratios. The same reason can explain the shifts in in Fig. 3(d). However, in this case too, since  $c_d > C$ , there is an improvement in the vibration suppression. Forced response behavior is like that observed for free response. And, since the system is assumed to be linear, this is not surprising. Analysis for different gaps was also conducted, and

a change in the gap was also found to significantly influence response. Those results however are not shown for brevity.

To contextualize our results, we evaluated the vibration suppression potential of an optimally tuned mass damper integrated with a boring bar with parameters as listed in Table. 1. We followed the tuning approach in (Yadav et al., 2020), and found that for a  $\mu$  of 0.1, the maximum suppression possible was  $\sim 88\%$ . Though this is slightly greater than what our proposed impact damper can achieve, since our model is generalized, it can guide further design, development, and improvement.

#### 4. Conclusion

This paper presented an analytical model for an impact damped boring bar. The model facilitated parametric analysis to characterize the role of the gap between the impact damper and the boring bar, and to understand the influence of the ratios of the mass, stiffness, and damping between the two systems. Our analysis suggested that smaller gaps result in better vibration attenuation. Our analysis also suggested that when the stiffness of the interface between the boring bar and the damper was less than that of the boring bar's, it plays no significant role, and that damping of the interface when greater than that of the boring bar, plays a significant role in vibration suppression. Since the impact damper was placed outside the boring bar in this preliminary model and since that limits the boring bar's effective operating length, future work could focus on integrating it within.

#### References

Cheng, C. C., & Wang, J. Y. (2003). Free vibration analysis of a resilient impact damper. *International Journal of Mechanical Sciences*, 45(4), 589-604.

Ema, S., & Marui, E. (2000). Suppression of chatter vibration of boring tools using impact dampers. *International Journal of Machine Tools and Manufacture*, 40(8):1141-1156

Geng, X., Ding, H., Wei, K., & Chen, L. (2020). Suppression of multiple modal resonances of a cantilever beam by an impact damper. *Applied Mathematics and Mechanics*, 41(3), 383-400.

Hagedorn, P., & DasGupta, A. (2007). *Vibrations and waves in continuous mechanical systems*. England: John Wiley & Sons, Ltd.

Ibrahim, R. A. (2009). Vibro-Impact Dynamics. *LNACM - Lecture Notes in Applied and Computational Mechanics*, 43.

Masri, S. F. (1970). General motion of impact dampers. *The Journal of the Acoustical Society of America*, 47(1B), 229-37.

Munoa, J., Beudaert, X., Dombovari, Z., Altintas, Y., Budak, E., & Brecher, C. (2016). Chatter suppression techniques in metal cutting. *CIRP Annals*, 65(2), 785-808.

Patel, A., Talaviya, D., Law, M., & Wah, P. (2022). Optimally tuning an absorber for a chatter-resistant rotating slender milling tool holder. *Journal of Sound and Vibration*, 520, 116594. 10.1016/j.jsv.2021.116594.

Patel, A., Yadav, A., Law, M., Bhattacharya, B., & Wah, P. (2022). Damped chatter resistant boring bar integrated with an absorber working in conjunction with an eddy current damper. *Journal of Vibration Engineering & Technologies*.

Thekkepat, A. A., Devadula, S., & Law, M. (2021). Identifying joint dynamics in bolted cantilevered Systems Under Varying Tightening Torques and Torsional Excitations. *Journal of Vibration Engineering & Technologies*.

Thomas, M.D., Knight, W.A., & Sadek, M.M. (1973). The impact damper boring bar and its performance when cutting. *Proc Thirteen Int Mach Tool Des Res Conf*. 47-51.

Yadav, A., Talaviya, D., Bansal, A., & Law, M. (2020). Design of chatter-resistant damped boring bars using a receptance coupling approach. *Journal of Manufacturing Material Processing*, 4(2), 53.



**Arjun Patel** is a Ph.D student at the Department of Mechanical Engineering at the Indian Institute of Technology Kanpur, India. His research interests centre on cutting tools dynamics, vibrations and controls of cutting tools such as boring bars, milling tool holders. (E-mail: [aspatel@iitk.ac.in](mailto:aspatel@iitk.ac.in))



**Mohit Law** is an Associate Professor at the Department of Mechanical Engineering at the Indian Institute of Technology Kanpur, India. He received his Ph.D from the University of British Columbia, Canada. His research interests centre on understanding how and why machine tools vibrate, measuring those vibrations, and on how best to mitigate them.



**Pankaj Wah** is a Professor at the Department of Mechanical Engineering at the Indian Institute of Technology Kanpur, India. He received his Ph.D. from the Indian Institute of Science, India. His research interests centre on nonlinear dynamics, vibrations and controls. (E-mail: [wahi@iitk.ac.in](mailto:wahi@iitk.ac.in))