# Bayesian Learning Model for Predicting Stability of System with Nonlinear Characteristics

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**Abstract.** Instabilities in machining are detrimental. Usually analytical model-predicted stability charts guide selection of cutting parameters to ensure stable processes. However, since models often fail to account for how inputs to them such as the cutting force coefficients and dynamics change with speed and/or time, and because models make several linearizing assumptions, charts often fail to guide stable cutting in industrial praxis. As an alternate way to guide stable cutting, this paper demonstrates how stability charts can be learnt from experimental data using a supervised Bayes' learning approach. We build on prior work related to learning stability for processes with linear characteristics and demonstrate herein that the model can be trained and tested on datasets for processes exhibiting nonlinear characteristics, thus showing how the prediction model is agnostic to the process or to any potential nonlinearities in it. Factors affecting the training capacity of Bayesian model like the likelihood probability distributions and the thresholds of probability necessary to decide on a stability contour, are tuned to give maximum accuracy possible. Predictions to learn the stability were accurate up to 96.5%. Since data that is used to train the model includes in it all the vagaries and uncertainties associated with the cutting process, results herein can inform further development towards self-optimizing and autonomous machining systems.

**Keywords:** Machining stability · Bayesian learning · Machine learning

## 1 Introduction

Selection of stable cutting parameters for high performance machining is often guided by knowledge of stability diagrams. These diagrams chart the boundaries between cutting parameters that might result in stable and unstable cutting and further guide selection that will likely result in higher material removal rates. Analytical models usually predict these diagrams. Quality of prediction is governed by inputs to the models. These include measured cutting force coefficients and measured dynamics. However, since inputs seldom account for the speed-dependent behaviour of the cutting process or the dynamics,


N. Khare et al. (Eds.): MIND 2022, CCIS 1762, pp. 190–200, 2022.
https://doi.org/10.1007/978-3-031-24352-3_16

and since models make several linearizing assumptions, models, though useful, often fail to guide stable cutting parameter selection in real industrial settings. Since instabilities are detrimental to the part, the tool, and to elements of the machine, there is need for better quality predictions. To address this need, this paper discusses the use of a supervised machine learning (ML) technique that can 'learn' the stability diagram from real experimentally classified stable and unstable data points without relying on an analytical model. Since the real data is expected to better capture the vagaries and uncertainties in the cutting processes and/or in the dynamics, the 'learnt' stability is expected to be accurate.

Use of ML models in the domain of machine tools and machining process related research has gained traction over the recent years, as is nicely and succinctly summarized in the review paper [1]. Since learning from real and/or simulated data has value, ML has been shown to be useful for tool wear analysis, thermal error compensation, monitoring and classifying states of machining as being stable and/or unstable, identifying dynamics, and to learn the stability diagram – which is of direct relevance to this research.

Prior research on learning stability using ML models has discussed the use of artificial neural networks (ANN) [2], support vector machines (SVM) [2, 3], the k-nearest neighbourhood (kNN) method [4], and Bayesian methods [5–8]. Given that training models requires that experiments be done to obtain unstable data points, and since those experiments can be destructive due to the nature of instabilities, an accurate model that can be trained with less data, and one that extends itself to a continual learning scheme should be preferred. The Bayes' method fits these criteria. It is hence our preferred method for implementation.

The Bayes' method to learn stability was in its original form intended to be agnostic to the process physics [5]. However, in other related work, physics-informed modifications have been reported to work well [6, 7]. Since it is desirable to develop a generalized ML model that is agnostic to the process physics and that can work with many different data sets, this study follows the approach reported in [5, 8] that was blind to the physics of the process.

For given data that is pre-classified as stable/unstable, the goal with the Bayesian approach is to calculate the posterior probability of stability at each grid point on the stability map. Though this was done in prior work [5–7], that work did not systematically characterize the influence of changing Gaussian likelihoods, or the influence of changing threshold of the stability contour on the learning accuracy. Moreover, there was no discussion on how to characterize and quantify the learning accuracy of the prediction. In our concurrent research [8], we report on the influence of changing Gaussian likelihoods, on the influence of changing threshold of the stability contour, and on the influence of data size in a continuous learning scenario.

Although, in [5–8], the Bayes' method is clearly agnostic to the physics of the system, that research was limited to learning stability for processes with linear characteristics. The use of the Bayes' method for learning stability with nonlinear characteristics remains previously unexplored and forms the focus of this paper. We train and test the Bayes' method for a process exhibiting non-linear force characteristics resulting in bistabilities and for a process exhibiting the interesting process damping phenomena. In doing so, we further show the strength of the Bayes' method for learning the machining stability behavior directly from the data, even when the data embeds nonlinearities within it. This is the main new technical contribution of this paper to the state-of-the-art in the use of ML methods for learning stability.

The remainder of the paper is organized as follows. At first, in Sect. 2, we discuss how we gather data that we feed to our learning model. We then briefly overview the Bayes' method in Sect. 3. In Sect. 4, we demonstrate the results obtained from the model which learn stability for processes with nonlinear characteristics and for those with process damping. This is followed by the main conclusions.

## 2 Gathering Data for the ML Model

Since the experimental pathway to gather data that is needed to train a Bayesian model is costly due to the destructive nature of unstable experiments, this paper trains and tests the ML model using data obtained from emulations on an in-house developed hardware-in-the-loop (HiL) simulator that was built to study machining instabilities [9–12]. Experiments on the HiL simulator are used to classify combinations of depths of cuts and spindle speeds that result in unstable conditions. The process is akin to procedures in real cutting experiments. When the process has nonlinear force characteristics, there exist regions of conditional stabilities that are characterized by the process being stable for small perturbations and unstable for larger ones for cutting at parameters within the conditionally stable regions. The procedure to find the global unstable limits in this case is the same as for the case of cutting with linear force characteristics. And, to find the find the global stable limits, i.e., to find the lower limits of the bistable regions, for every speed of interest, the depths of cuts were decreased in the same step size as they were increased. And the last but one depth of cut at which the finite amplitude instabilities disappear, was recorded as the lower limit of the bistable region. Experimental data characterizing the stability boundaries for cutting with a process having non-linear force characteristics is shown in Fig. 1(a) and is obtained as detailed in [11].

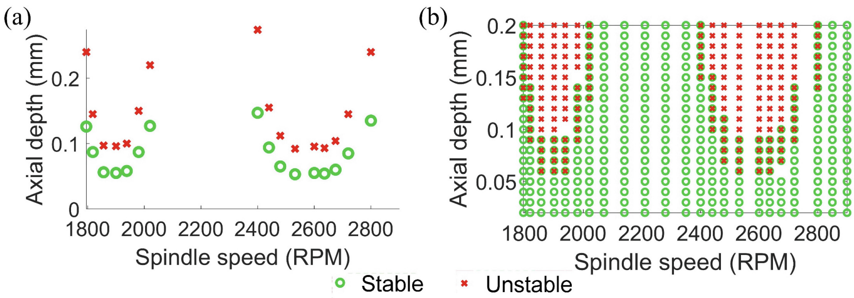



**Fig. 1.** (a) Experimental data corresponding to unstable cutting conditions obtained from emulations on a HiL simulator for the case of a process with nonlinear characteristics i.e., bistability, (b) synthesized data used for testing the model.

When the cutting process exhibits process damping due to interference of a worn tool's flank face with a previously cut surface, the critical chatter-free depths of cuts at low-speeds are observed to be higher than those at high-speeds. Experimental data characterizing the stability boundaries for cutting with a process having linear force characteristics and exhibiting process damping is shown in Fig. 2(a). Data shown in Fig. 2(a) is obtained as detailed in [12].

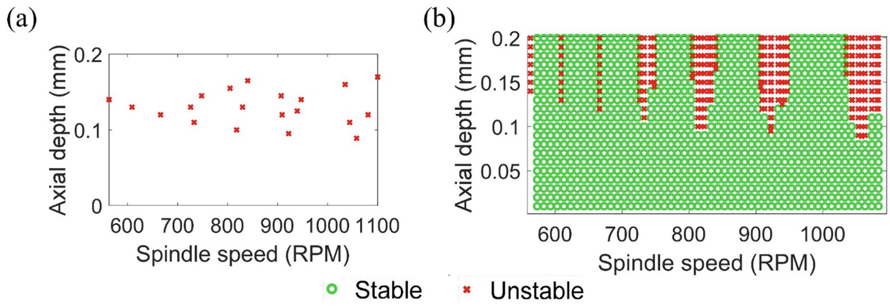



**Fig. 2.** (a) Experimental data corresponding to unstable cutting conditions obtained from emulations on a HiL simulator for the case of a process exhibiting process damping, (b) synthesized data used for testing the model.

Since the main idea of fitting a model to data is for the model to be agnostic to the data type, a comparison between the above two cases is not intended and/or recommended. What is however clear is that for the case with the nonlinear force characteristics there exists a clear bistable region with there being a globally stable boundary as well as a globally unstable boundary. This case will evidently need to be addressed differently due to the ternary types of classification of the data with some data being stable, some being conditionally stable, and some being unstable. Previous studies [5–8] using the Bayesian approach were limited to cases with linear force characteristics in which the classification was of the binary type with some data being stable and some being unstable.

The case of the nonlinear force characteristics has a total of 32 (16 each for global stable and global unstable case) data points (Fig. 1(a)). And the case of the process exhibiting process damping has a total of 20 data points (Fig. 2(a)). Though the Bayesian model can be adequately trained with this data, testing the model for its learning capacity needs more data than we have obtained. As such, we synthesize the emulated data with more data. Since the region below the boundary is stable and that above unstable, we add data points at depth of cut intervals of choice to pad the emulated data. In this manner, we generate additional 1227 and 434 data for the case of a process with nonlinear characteristics (Fig. 1(b)) and a process with linear characteristics exhibiting process damping (Fig. 2(b)), respectively. Synthesized data allows us to systematically quantify the learning capacity of the model, something that was missing from previous investigations [5–7]. More on how we quantify the learning accuracy is discussed in the next section.

## 3 Bayesian Learning for Machining Stability

This section outlines the Bayes' procedure to learn machining stability diagrams. We only provide an overview and direct the reader to the original source [5] for details. Bayes' rule updates probabilities when new information is made available. Mathematically, it can be stated as:

$$p(\mathrm{A}|\mathrm{B}) = \frac{p(\mathrm{B}|\mathrm{A}).p(\mathrm{A})}{p(\mathrm{B})} \tag{1}$$

wherein $A$ and $B$ are separate events. $p(A|B)$ is the probability of event $A$ occurring given that $B$ is true. This is also known as the posterior probability of $A$ given $B$. $p(B|A)$ is the probability of event $B$ occurring given that $A$ is true. This is also known as the likelihood of $A$ given a fixed $B$. $p(A)$ and $p(B)$ are probabilities of $A$ and $B$ occurring and are known as the prior probabilities. In the context of machining stability, the Bayes' rule becomes:

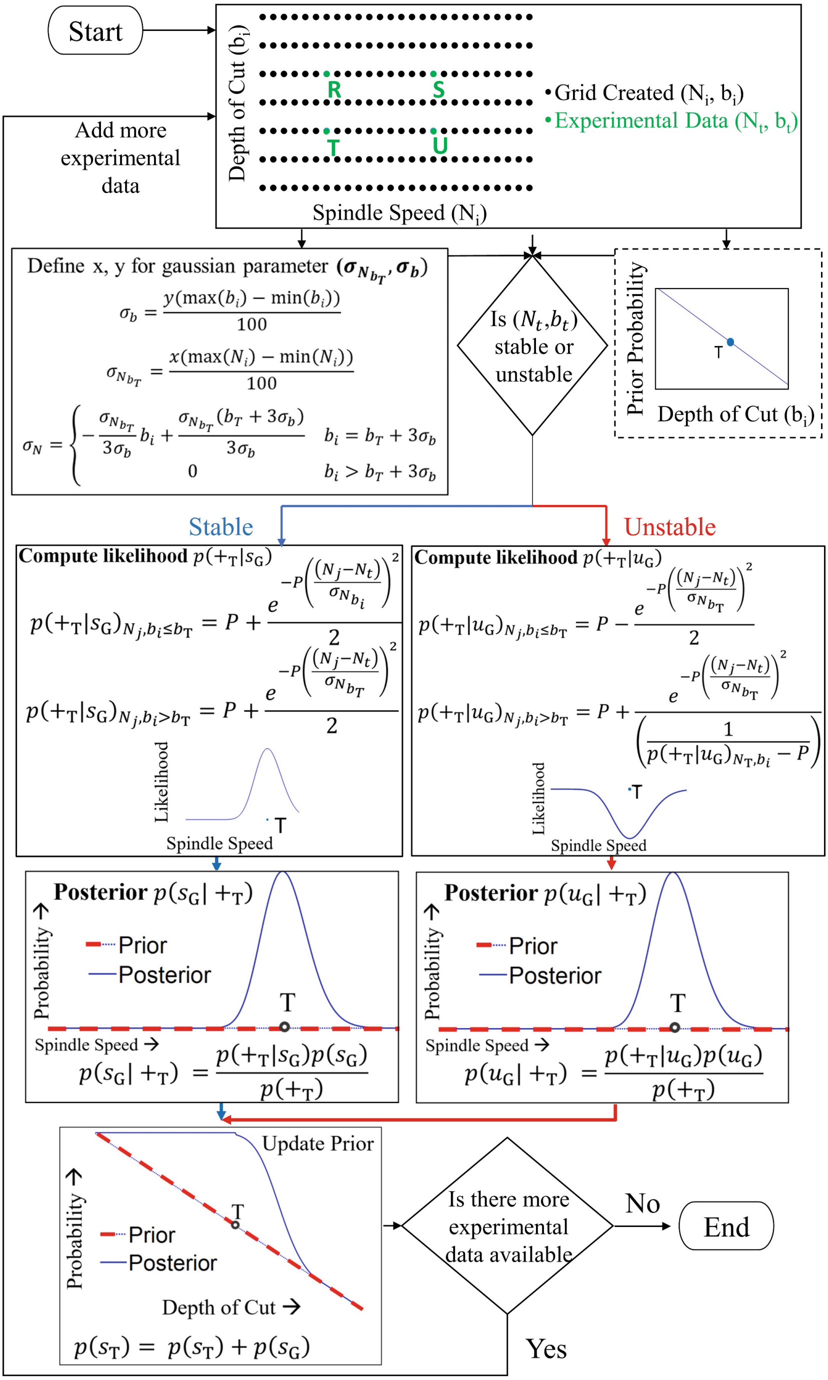
$$p(\text{stability} \mid \text{experimental data}) = \frac{p(\text{experimental data} \mid \text{stability}).p(\text{stability})}{p(\text{experimental data})} \tag{2}$$

wherein $p$(stability) is an assumed prior probability of stability, $p$(experimentaldata) is the known probability of a data point being stable or not, $p$(experimentaldata|stability) is the likelihood probability of a stable result at the given experimental data point, and $p$(stability|experimentaldata) is the evaluated posterior probability of a stable data point for the given experimental condition. Likewise, it is also possible to evaluate the posterior probability of an unstable data point for the given experimental condition and given an assumed prior probability of instability, $p$(instability).

For the given data points on the stability diagram (in Figs. 1 and 2) the goal with the Bayesian approach is to calculate the posterior probability of stability at each grid point on the axial depth of cut – spindle speed map. The procedure to do so is outlined in a flowchart in Fig. 3.

For every data point, we first evaluate the prior probability. This is done by assuming that as the depths of cut increase at any spindle speed, the likelihood of encountering instabilities increases. A linear distribution for the prior probability is assumed as shown in an inset in the flowchart in Fig. 3. This prior probability remains the same for all spindle speeds.

The influence of a test result along spindle speeds at the depth of cut of interest, $b_{\mathrm{T}}$ is defined as $\sigma_{N_{b_T}}$ with the mean being the test spindle speed of interest, and the influence of the test result being restricted to 3 $\sigma_{N_{b_T}}$. The subscript T refers to the test result under consideration. The likelihood probability of a stable result at T $(p(+_{\mathrm{T}})_{N_{\mathrm{T}},b_{\mathrm{T}}})$ given G, another test result on the depth of cut – spindle map that is stable, is one, i.e., $p(+_{\mathrm{T}}|s_{\mathrm{G}})_{N_{\mathrm{T}},b_{\mathrm{T}}} = 1$. For the same depth of cut, $b_{\mathrm{T}}$, the likelihood probability of a stable result at T given G is stable reduces for other spindle speeds $N_j$ following a Gaussian distribution, as shown in the schematic and in the equation in the insets in the flowchart in Fig. 3.




$$\sigma_b = \frac{y(\max(b_i) - \min(b_i))}{100}$$

$$\sigma_{N_{b_T}} = \frac{x(\max(N_i) - \min(N_i))}{100}$$

$$\sigma_N = \begin{cases} -\frac{\sigma_{N_{b_T}}}{3\sigma_b} b_i + \frac{\sigma_{N_{b_T}}(b_T + 3\sigma_b)}{3\sigma_b} & b_i = b_T + 3\sigma_b \\ 0 & b_i > b_T + 3\sigma_b \end{cases}$$

$$p(+_T|s_G)_{N_j, b_i \le b_T} = P + \frac{e^{-P\left(\frac{(N_j - N_t)}{\sigma_{N_{b_i}}}\right)^2}}{2}$$

$$p(+_T|s_G)_{N_j, b_i > b_T} = P + \frac{e^{-P\left(\frac{(N_j - N_t)}{\sigma_{N_{b_T}}}\right)^2}}{2}$$

$$p(+_T|u_G)_{N_j, b_i \le b_T} = P - \frac{e^{-P\left(\frac{(N_j - N_t)}{\sigma_{N_{b_T}}}\right)^2}}{2}$$

$$p(+_T|u_G)_{N_j, b_i > b_T} = P + \frac{e^{-P\left(\frac{(N_j - N_t)}{\sigma_{N_{b_T}}}\right)^2}}{\left(\frac{1}{p(+_T|u_G)_{N_T, b_i} - P}\right)}$$

$$p(s_G| +_T) = \frac{p(+_T|s_G)p(s_G)}{p(+_T)}$$

$$p(u_G| +_T) = \frac{p(+_T|u_G)p(u_G)}{p(+_T)}$$

$$p(s_T) = p(s_T) + p(s_G)$$

**Fig. 3.** An overview of our implementation of the Bayes' rule to learn stability diagrams.

For a changing depth of cut at the same spindle speed, $N_{\mathrm{T}}$, for example, when $b_i < b_{\mathrm{T}}$, and the data point $b_{\mathrm{T}}$, $N_{\mathrm{T}}$ is stable, the likelihood probability will remain one since for the case of the binary type of classification, every point below $b_{\mathrm{T}}$ will remain stable. However, the probability of a data point remaining stable reduces as $b_i > b_{\mathrm{T}}$, and is assumed to have no influence beyond $b_{\mathrm{T}} + 3\sigma_b$, wherein $\sigma_b$ is the standard deviation along the axial depth of cut, and $b_{\mathrm{T}} + 3\sigma_b$ is the mean for a non-normalized Gaussian probability density. Since the influence of stable results will be higher at lower axial depths of cut, $\sigma_{N_{b_T}}$ is different for the cases of $b_i < b_{\mathrm{T}}$ and for $b_i > b_{\mathrm{T}}$. This is shown in the equations within the flowchart in Fig. 3. Procedures to obtain the likelihood probability of instabilities follows the same logic as for the case of obtaining the likelihood probability of a stable result. The Gaussian distribution however is inverted for the case of the unstable result. For the bistable cases of interest in this paper, since bistable data lies within the stable and unstable regions, the likelihood probability for bistable data is calculated as the average of the likelihoods for the stable and unstable cases.

Using procedures outlined above, a posterior probability is computed using the Bayes' conditional probability theorem using the equations shown in the insets in Fig. 3. To do so, the likelihood and prior probabilities are both used. For a representative test result T, the posterior probability is overlaid on the prior assumption within the inset in Fig. 3, and it clearly shows that probabilities change using the Bayes' rule. This new probability becomes the prior probability for the subsequent experimental data point, and the process is repeated until all grid points on the spindle speed – axial depth of cut map are evaluated.

After probabilities are updated using all test results, a stability lobe prediction from the Bayes' approach can be made at axial depths of cut when the probability of stability is equal to a user-defined threshold. In prior work [5], that threshold was fixed at 0.5. However, since the threshold could influence the accuracy of the decision boundary, as was shown in our concurrent work [8], it is tuned for giving the best decision boundary.

Learning capacity of the Bayesian model is governed not only by the assumed prior distribution of probabilities, and the assumed Gaussian distributions for the likelihood probabilities, but also the choice of $\sigma_N$ and $\sigma_b$ within those distributions. $\sigma_N$ and $\sigma_b$ will both determine the influence of a test point and the posterior probabilities, which in turn, influences the width and amplitude of the stability boundaries after each update. In prior work on the use of the Bayes' approach [5], $\sigma_N$ and $\sigma_b$ and were selected as 3% of the spindle speed range and as 10% of the axial depth of cut range, respectively. Though these values resulted in good prediction accuracies, we have tuned the values of $\sigma_N$ and $\sigma_b$ for this work, based on the quality of prediction. Moreover, the attempt to take $\sigma_N$ as a linearly increasing function within the spindle speed range is made by us in [8], which was found to enhance the accuracy. Hence, following from there, we have chosen $\sigma_N$ as a linearly increasing function within the spindle speed range, rather than a constant value, as was done in [5].

Though the above outlined above procedures can be used to predict/learn a stability boundary, there need to be procedures to quantify the goodness of that prediction. Prior work [5] quantified the learning accuracy by benchmarking against a 'true' stability boundary that was presumably obtained from an analytical model. Instead of relying on a model, we train using the available experimental data, and test the goodness of our predictions against the whole data set that includes the synthesized data and some experimental data using a confusion matrix. From the confusion matrix, we evaluate accuracies and F1 scores to quantify the goodness of fit.

## 4 Learning Stability with Non-linear Characteristics

This section first discusses learning stability for a for a process exhibiting bistable behaviour, followed by a discussion on learning stability for a machining process exhibiting process damping.

### 4.1 Learning Stability for a Process Exhibiting Bistable Behaviour

Processes exhibiting bistable behaviour are characterized by data points that are stable, conditionally stable, and unstable. As such, the model must be trained with representative data from each of these different classes. We hence train using nine stable, four unstable and 27 bistable data points. For testing, we use 242 stable, 106 unstable and 86 bistable data points. Training and testing data is shown in Fig. 4(a). In this case $\sigma_N$ was linearly increased from 3% to 10% of spindle speed range, and $\sigma_b$ was taken to be fixed at 4% of the axial depth of cut range. And, since bistable behaviour is characterized by a globally stable and a globally unstable boundary, two thresholds were used to evaluate these contours. For the globally stable contour, we used a threshold of 0.05, and for the globally unstable contour, we used a threshold of 0.95. These parameters were tuned for this data set. The predicted stability contour is shown in Fig. 4(b). The color map in Fig. 4(b) depicts the probability of stability, with blue representing a very low probability of stability and yellow indicating a high probability of stability. Although the stability boundary is determined by the threshold value, the color map gives an overview of the distribution of the evaluated posterior probabilities. And as is evident, the Bayesian approach can learn stability behaviour even with processes exhibiting bistabilities. The F1-score evaluated for correctly predicting the globally stable and unstable regions is $>90\%$. However, for the bistable data, the F1-score falls to ~75%. This suggests that there is still room for improvement of the algorithm.

Since bistabilities are characterized by the process being stable for small perturbations and unstable for larger ones, learning this behaviour from data can guide selection of cutting parameters to lie outside these zones of conditional instabilities. Moreover, since bistabilities occur due to nonlinearities in cutting force characteristics, which can be difficult to identify and/or model, and since the data learning model used herein is shown to be agnostic to the underlying causes of the observed bistable behaviour, and since it is still able to learn that bistable behaviour, these results are useful.

Furthermore, the computational time required to train and test the model is only ~1.6 s and ~0.0433 s, respectively. These times were for calculations performed on a laptop with a i5-8250U processor running at a base speed 1.8 GHz and with 8 GB or RAM. This makes this method suitable for an in-situ real-time implementation with real data on real machines.

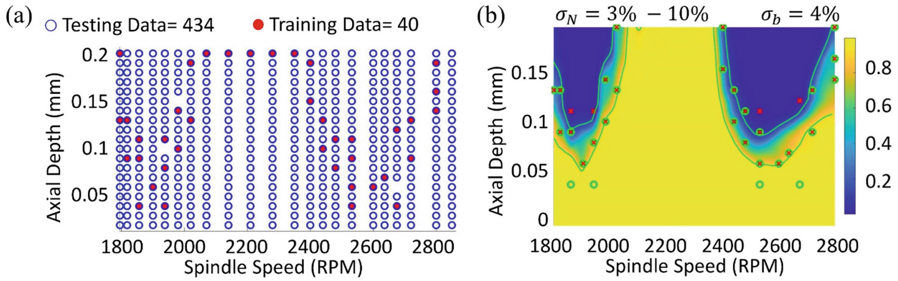



**Fig. 4.** (a) Data for training and testing the process with nonlinear force characteristics, (b) stability charts capturing bistable behaviour with unstable, bistable, and stable data used for training.

## 4.2 Learning Stability with Process Damping

Training and testing data for process damping is shown in Fig. 5(a). The model is trained with 20 unstable data points and is subsequently tested on 972 stable and 255 unstable data points. The model hyperparameters for this data set are again tuned to give maximum F1-score. $\sigma_N$ was linearly increased from 1% to 2% of the spindle speed range. $\sigma_b$ was taken to be fixed at 1% of the axial depth of cut range. The threshold to evaluate stability contours was taken to be 0.05. The predicted stability contour is shown in Fig. 5(b), and as is evident, the process damping phenomena is well-captured with the Bayesian approach that has no knowledge about the underlying process mechanics. The accuracy and F1-score were found to be 96.58% and 97.80%, respectively.

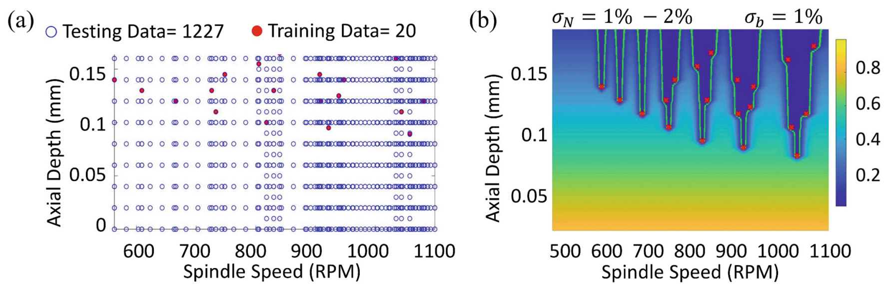



**Fig. 5.** (a) Data for training and testing the process with linear force characteristics exhibiting process damping, (b) stability chart capturing process damping with unstable data used for training.

Since process damping is an interesting phenomenon in which the absolute minimum stability limit improves for lower speeds while remaining unchanged for higher speeds – as is evident from Fig. 5(b), and since improvements are usually observed as the tool wear progresses, and since modelling the tool wear's influence on stability is non-trivial, the hereby demonstration of learning the stability diagram from data will aid selection of cutting parameters to stabilize a process and improve the productivity potential.

The computational cost for the case of learning process damping behavior is lower still than for the case of bistable behavior, taking ~0.48 s for training and ~0.05 s for testing, respectively.

## 5 Conclusion

Analytical model-predicted stability diagrams often fail to guide stable cutting parameter selection in praxis due to the assumptions the models make and due to the vagaries and uncertainties in the inputs to the model. Since machining instabilities should be avoided, this paper demonstrated successfully that the stability diagram can instead be learnt from experimental data using a supervised Bayesian learning model. We successfully quantified the learning accuracy of Bayesian model and tuned the two hyperparameters – standard deviation of Gaussian likelihood distributions, and threshold of probability indicating the stability contour. The tuned model has been shown to be consistent across two different types of datasets gathered from emulated experiments in which the cutting mechanics and dynamics involved non-linear behaviour and were different for both datasets. This suggests that the learning model is agnostic to the underlying process physics. This is the first such report in the literature of a machine learning model being blind to potential nonlinearities in the cutting process. This is also the strength of the learning model.

Since the very nature of the Bayes' rule is that the posterior probability updates with every new data point that is provided to the model, the analysis done in this paper can inform future research to help the community move closer towards self-optimizing and autonomous machining systems in which cutting parameter selection can be adapted autonomously and in real-time based on predictions from a ML model that trains itself directly from data, and captures all possible vagaries of its dynamics.

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