Recovering cutting tool modal parameters from randomly sampled signals using compressed sensing

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ABSTRACT

A change in the modal parameters of cutting tools could signal tool wear, tool breakage, or other instabilities. The cutting process must be continuously monitored using vibration signals to detect such changes. Since tools vibrate with frequencies of up to a few kHz, continuous monitoring requires sampling at rates of tens of kHz to respect the Nyquist limit. Processing and storing such large data for decision making is cumbersome. To address this issue, this paper discusses the use of a compressed sensing framework that enables non-uniform random sampling at rates below the Nyquist limit. For cutting tools, we show for the first time using synthesized data that it is possible to reconstruct original signals from as few as 1% of the original data. We numerically test the method to characterize the influence of damping, noise, and multiple modes. Recovered modal parameters from the reconstructed signal agree with signals sampled properly.

1. Introduction

A change in the modal parameters, i.e., a change in the natural frequencies, damping ratios, and shapes of vibrations of cutting tools could signal tool wear, tool breakage, or cutting process instabilities (Iglesias et al., 2022). Since wear, breakage, and/or instabilities are detrimental, the cutting process must be continuously monitored using vibration signals to detect such changes. Since tools vibrate with frequencies of up to a few kHz, continuous monitoring requires sampling at rates of tens of kHz to respect the Nyquist limit. Processing and storing such large data for decision making is cumbersome. It is the aim of this paper to discuss solutions targeted at recovery of modal parameters from potentially temporally aliased signals, i.e., signals sampled below the Nyquist limit. Such recovery will make it possible to monitor the process without the problem of processing and storing large data.

Prior work from our research group has shown that it is possible to recover modal parameters from temporally aliased signals (Law et al., 2022; Lambora et al., 2022). However, the approach therein required signals to be sampled at least at two rates that were fractionally uncorrelated. Such an approach would not be easy to leverage for continuous monitoring applications. To address this issue, this paper discusses the use of a compressed sensing (CS) framework that enables non-uniform random sampling at rates below the Nyquist limit (Donoho, 2006; Candes et al., 2006). By exploiting the sparse nature of vibration signals and the incoherence of randomly sampled measurements, CS helps reconstruction of proper signals using fewer samples.

CS has been used successfully in structural monitoring of civil infrastructure (Yang & Nagarajaiah, 2015). Other related work discussed the use of CS techniques with video of vibrating structures (Martinez et al., 2020). They showed recovery of mode shapes even after removing up to 90% of frames from a uniformly sampled video. Related work (Candes & Wankin, 2008) discussed the minimum number of measurements required to successfully reconstruct a signal using CS. Though useful, they did not account for how damping and noise in the signals can potentially influence recovery.

Since CS offers advantages, this paper discusses its use to recover modal parameters of cutting tools from temporally aliased signals. We demonstrate

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the method on systems with one or more modes. We also test the method to characterize the degree of compression being influenced by damping and noise, respectively. Analysis presented herein is new, can be considered our main new technical contribution to the state-of-the-art in condition monitoring of cutting tool systems.

Since the use of CS is still in its infancy, there exist no commercial hardware samplers that can do non-uniform random sampling. Though there exist some reports of prototypes of CS hardware samplers (Yazicigil et al., 2019), those too are not commercially available. As such, like others before us (Yang & Nagarajaiah, 2015; Martinez et al., 2020), we too will demonstrate the use of CS by randomly down-sampling data originally sampled at uniform rates. However, instead of down-sampling real measured signals, we will demonstrate the methods with synthesized signals that are representative of real cutting tool signals.

The remainder of the paper is organized as follows. Section 2 briefly outlines the theoretical framework for CS. We then discuss five different numerical case studies in Section 3 to characterize the degree of compression possible. This is followed by the main conclusions.

2. Overview of the Compressed Sensing Method

This section provides an overview of the compressed sensing (CS) technique to reconstruct proper signals using very few measured samples. Methods presented herein are distilled from (Donoho, 2006; Candes et al., 2006).

Consider a N×1 time series signal vector (f), which is properly sampled according to the Nyquist-Shannon Sampling theorem. A N×N Fourier basis matrix (ψ) decomposes the time series signal into a N×1 frequency domain signal vector (c), also known as loading vector or coefficients of basis:

$$f_{N\times1}= \psi_{N\times N}.c_{N\times1} \qquad \text{..........(1)}$$

The Fourier basis (ψ) in the form of a matrix can be obtained from a Fourier transform. The nature of loadings (c) is sparse in nature, which means that (f) can be characterized by few non-zero coefficients in the vector (c), as is evident from Fig. 1.

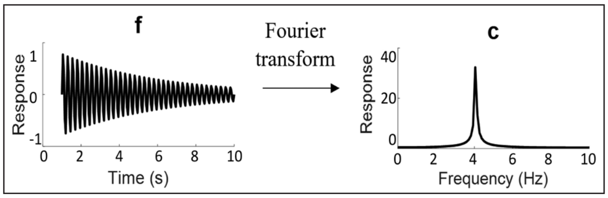

Fig. 1. The time series signal (f) can be decomposed into the sparse signal (c) in the frequency domain using a fourier transform. Most of the information in (c) has near zero values, while a few non-zero coefficients are present, characterised by a peak.

A few 'M' random values are chosen from the vector f, and stored in a measurement vector (b), such that M≪N. The indices of the random measurements chosen in b are stored in the form of a M×N dimension dictionary matrix (ϕ). Therefore, f, b and ϕ can be related as:

$$b_{M\times1}= \phi_{M\times N}.f_{N\times1} \qquad \text{...........(2)}$$

From Eq. (1) and (2), f can be eliminated to give the relation between b and c:

$$b_{M\times1}=[\phi_{M\times N}.\psi_{N\times N}]\ c_{N\times1} \qquad \text{...........(3)}$$

Let $A_{M\times N}= [\phi_{M\times N}\psi_{N\times N}]$, then the above equation can be treated as the following linear system of equations:

$$b_{M\times1}=[A_{M\times N}]\ c_{N\times1} \qquad \text{...........(4)}$$

Our aim is to reconstruct the time series vector f from a known measurement matrix (b), a known Fourier basis (ψ), and a known dictionary matrix (ϕ). Hence, in Eq. (4), b and A are known quantities and c is to be determined. As M≪N, this is an underdetermined linear system of equations. Eq. (4) will have infinitely many solutions for c, until we introduce another constraint for c. To get a unique solution for c, an additional constraint is introduced, to minimize the L^1 norm of c. This is done to get a sparse solution of c, out of the infinite solutions possible by solving only Eq. (4).

Furthermore, there can be finite amount of noise in the real experimental signals, for which Eq. (4) along with the minimum L^1 norm constraint for c, can be posed as the following basis pursuit denoising (BPDN) problem (Gill et al., 2011):

$$\min\|c\|_1 \text{ , subject to } \|b-Ac\|_2^2\leq \delta, \qquad \text{...........(5)}$$

wherein δ controls the level of denoising which can be tuned. δ = 0, would mean no denoising.

We will keep $\delta = 0$ for the entirety of this paper for the sake of generality. Hence, with $\delta=0$, Eq. (5) becomes the basis pursuit problem (Yang & Nagarajaiah, 2015; Chen & Donoho, 1994):

$$\min \|c\|_1 \text{ , subject to } b=Ac \qquad \text{..........(6)}$$

Eq. (6) is a convex optimization problem. We solve it using the CVX module (Grant & Boyd, 2013) in MATLAB. Once c is obtained, f can be restored using Eq. (1).

Having provided an overview of the CS method, we next discuss its use in recovering modal parameters from synthesized data.

3. Numerical Experiments

This section discusses five different numerical experiments to demonstrate recovery of modal parameters from signals randomly sampled below the Nyquist limit. In all cases, we use synthesized data which is representative of real cutting process signals.

At first, we consider recovery for a signal characterized by one frequency component. This may be considered akin to forced undamped vibration response of a cutting tool at its natural frequency. The purpose of this analysis is to characterize the minimum data required to properly recover the natural frequency. This case is discussed in Section 3.1. Following this, in Section 3.2, we introduce varying levels of damping in the same signal (as in Section 3.1) to check the influence of damping on the maximum degree of compression possible for the signal. In the third case, discussed in Section 3.3, we characterize recovery for a cutting tool system with two modes, each with different damping. In the fourth experiment discussed in Section 3.4, we demonstrate recovery and compression in the presence of noisy signals that are likely to occur during condition monitoring of real cutting processes. In the final case, discussed in Section 3.5, we perturb parameters from the system discussed in Section 3.3 to demonstrate that recovery works even when there is a change in system parameters. This case is representative of condition monitoring of cutting processes. Response signals discussed in Sections 3.2 – 3.5 are typical of cutting tool signals when excited by inertial movements of structural members of the machine tool superposed with forced vibration response due to cutting process induced vibrations.

For all cases, we investigate the degree of compression possible, which is defined herein as the ratio of original samples to random samples required for proper modal parameter recovery. To evaluate the degree of compression, a Monte-Carlo approach is used (Metropolis & Ulam, 1949). Since, the sampling is random, the indices of random samples are generated using a random number stream in MATLAB. For every compression ratio of interest, the CS technique is run 200 times to generate 200 different random streams which are used for recovery. For any run within these 200 runs, if the reconstructed signal is not able to recover the modal parameters obtained from the original uniformly sampled signal within a 2% pre-set tolerance, the percentage of random measurements is increased with the least count of increment set to 1%.

To extract modal parameters from all response signals, be it down-sampled data, or original properly sampled data, we use the Eigensystem Realization Algorithm (ERA) (Juang & Pappa, 1985). For additional details about the implementation of ERA, please see (Law et al., 2020; Gupta et al., 2020). All results are benchmarked against parameters extracted from uniformly sampled data that was originally sampled at rates respecting the Nyquist limit.

3.1. Signal with a single mode without damping

Consider a non-decaying sinusoidal function oscillating with natural frequency ω_n = 100 Hz, and that is sampled properly, with the sampling frequency being ω_s = 1000 Hz. This signal is compressed, and the randomly sampled data is overlaid over the original signal in Fig. 2. And as is evident from the spectra in Fig. 2, even with random sampling at the rate of 1% of the original

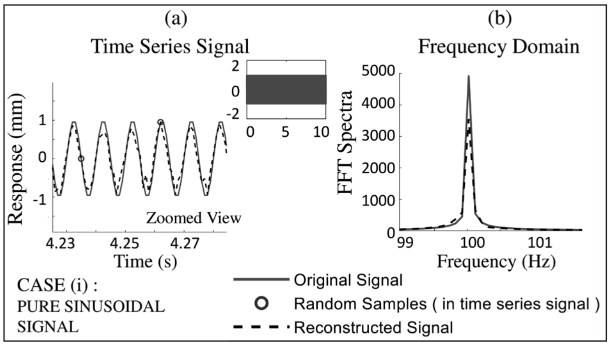

Fig. 2. (a) 1% random samples are taken from original time series signal and reconstructed using CS technique. (b) FFT spectra of the original and reconstructed signal (with only 1% of the original data) are shown.

data, the CS technique can recover the original frequency content.

3.2. Signal with a damped single mode

Consider an underdamped single frequency signal with natural frequency ω_n = 100 Hz and with damping (ζ) that could take on values between 0 and 3%. When properly sampled, ω_s = 1000 Hz. We run the CS algorithm for different values of damping and find the optimal compression ratio for each that results in proper recovery. An example of the original signal with ζ=0.16% is shown in Fig. 3. Also shown in the figure is the randomly sampled data at 6% of original rate. Recovery with this ratio results in a natural frequency estimate of ω_n = 99.91Hz, and a damping ratio of ζ = 0.15%, i.e., a very good match with the original parameters.

Since Fig. 3 only shows recovery for a signal with one damping ratio, we summarize recovery as it changes for different damping ratios in Fig. 4. And as is evident, the degree of compression reduces with increasing damping ratios. For the degree of compression of 5 or less, the number of data points in the randomly sampled data are greater than those required by the Nyquist limit. This suggests that when damping increases to > 1%, CS is not very effective.

3.3. Signal with two damped modes

Consider an underdamped multimodal signal with natural frequencies of ω_{n1} = 100 Hz and ω_{n2} = 250 Hz. Damping ratios for these are ζ_1 = 0.16% and ζ_2 = 0.32% respectively. When originally sampled, ω_s = 1000 Hz. The amplitudes of both modes are assumed equal. For this system, we run the CS technique, and find that recovery is exact with 28% of the original signal. The original signal, the randomly sampled data, and their respective spectra are shown in Fig. 5. The natural frequencies and corresponding damping ratios of the reconstructed signal are ω_{n1} = 99.99 Hz, ζ_1 = 0.14%; ω_{n2} = 249.69 Hz, ζ_2 = 0.25%. We also checked for different degrees of compression, and in general observe that the error in recovering modal parameters increases with an increase in the degree of compression.

3.4. Noisy signal with two damped modes

To investigate the influence of noise in the signal, we add Gaussian noise with different signal to noise (SNR) ratios to the same signal properties as in Section 3.3. Results with a SNR of 5 are shown in Fig. 6. With this noise level, using CS, we can recover modal parameters with 35% of the original data. The degree of compression for a signal with this level of noise is lower than a signal without noise, for which it was 28% - see section 3.3.

Since Fig. 6 shows results for only one level of SNR, a summary for the possible degrees of compression changing with different levels of SNR is shown in Fig. 7. And as is evident, for lower SNR

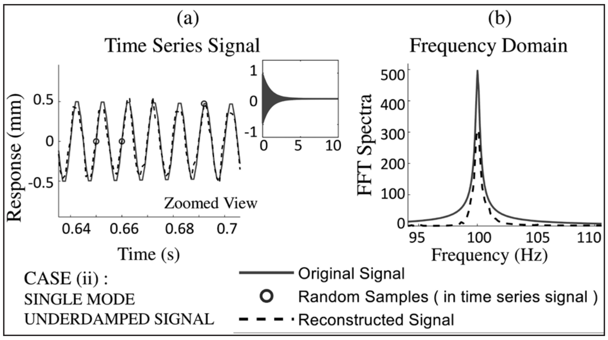

Fig. 3. (a) 6% random samples are taken from original time series signal and reconstructed using CS technique. (b) FFT spectra of the original and reconstructed signal are shown.

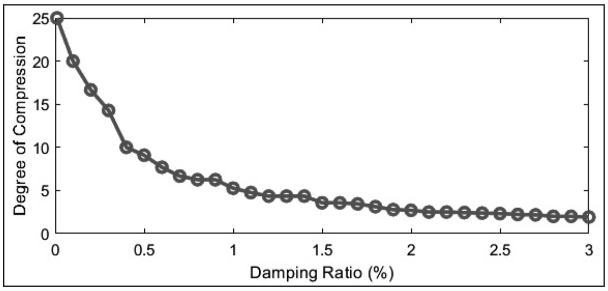

Fig. 4. Parametric study to show the influence of damping ratio on degree of compression of single mode signal. Original signals of 1s duration are considered to generate this plot.

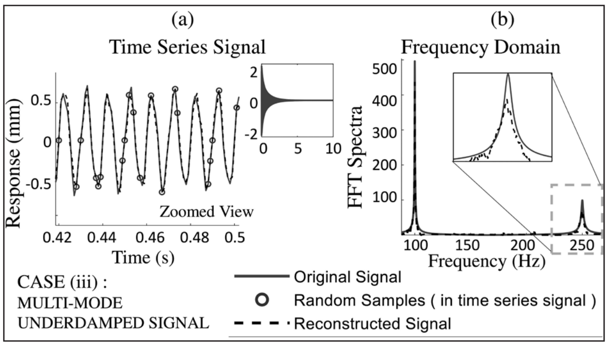

Fig. 5. (a) 28% random samples are taken from original time series signal and reconstructed using CS technique. (b) FFT spectra of the original and reconstructed signal are shown.

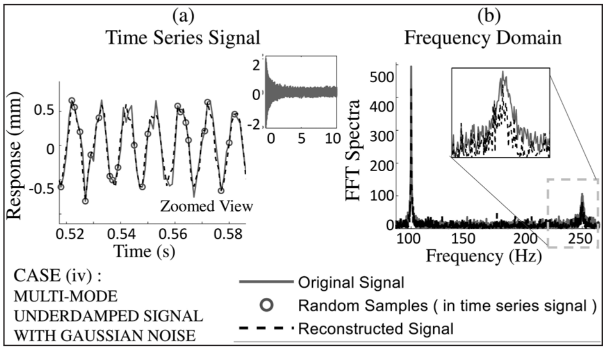

Fig. 6. (a) 35% random samples are taken from original noisy time series signal and reconstructed using CS technique. (b) FFT spectra of the original and reconstructed signal are shown.

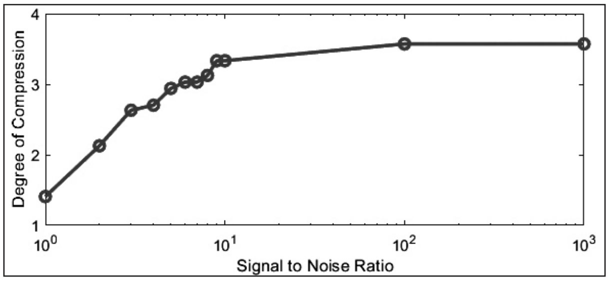

Fig. 7. Parametric study to show the influence of level of noise on degree of compression of signal with 2 damped modes. Original signals of 1s duration are considered to generate this plot.

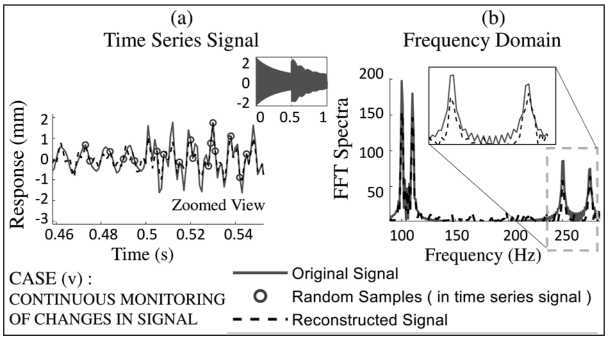

Fig. 8. A sudden change in the signal is also captured while using compressed sensing for continuous health monitoring of machine tools with 28% random samples. (a) Time series signal (b) FFT spectra.

ratios, there is less compression possible than there is for higher SNR. However, beyond a SNR of 10, there appears to be no significant effect of white Gaussian noise on the maximum degree of compression possible for a typical signal.

3.5. Signals with changing parameters

To investigate if CS can detect a change in system parameters during continuous monitoring of cutting tool vibration signals, i.e., a case for which the use of CS is intended, we assume that the system vibrates initially with parameters as discussed in Section 3.3, i.e., the signal has two modes with ω_{n1} = 100 Hz and ω_{n2} = 250 Hz, with their respective damping being ζ_1 = 0.16% and ζ_2 = 0.32%. And during continuous operation, we assume that each of these parameters increase by 10% each, as shown in Fig. 8. We apply CS to this changed signal and find that we are able to recover the changed parameters with the same degree of compression with which we recovered the initial unperturbed system.

4. Conclusion

This paper demonstrated for the first time that it is possible to reconstruct original cutting tool vibration signals from as few as 1% of the original data using a compressed sensing framework. Modal parameters recovered from reconstructed signals were found to agree with those extracted from signals sampled properly. We presented systematic analysis to characterize how the degree of compression changes with damping levels, with multiple modes, and with noisy signals. We observed that for increasing levels of damping, the degree of compression reduces. We also observed that high levels of noise reduce the degree of compression possible. We further showed that recovery works even when there is a change in system parameters.

Since compressed sensing was shown to work effectively, the framework hence extends itself nicely to condition monitoring applications in machine and cutting tool systems which can be done with far lesser data, making data processing, and storing easy. Further work is necessary to evaluate how compressed sensing can be applied to real signals for classifying them.

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Reference

Candes, E. J., Romberg, J., & Tao T. (2006). Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. *IEEE Transactions on Information Theory*, *52*(2), 489-509. https://doi.org/10.1109/TIT.2005.862083

Candes, E., & Wakin, M. B. (2008). An introduction to compressive sampling. *IEEE Signal Processing Magazine*, *25*(2), 21-30. https://doi.org/10.1109/MSP.2007.914731

Chen, S., & Donoho, D. (1994). Basis pursuit. *Proceedings of 1994 28th Asilomar Conference on Signals, Systems and Computers*, *1*, 41-44. https://doi.org/10.1109/ACSSC.1994.471413

Donoho, D. L. (2006). Compressed sensing. *IEEE Transactions on Information Theory*, *52*(4), 1289-1306. https://doi.org/10.1109/TIT.2006.871582

Gill, P. R., Wang, A., & Molnar, A. (2011). The in-crowd algorithm for fast basis pursuit denoising. *IEEE Transactions on Signal Processing*, *59*(10), 4595-4605. https://doi.org/10.1109/TSP.2011.2161292

Grant, M., & Boyd, S. (2013). CVX: MATLAB software for disciplined convex programming, version 2.0 beta. http://cvxr.com/cvx

Gupta, P., Law, M., & Mukhopadhyay, S. (2020). Evaluating tool point dynamics using output-only modal analysis with mass-change methods. *CIRP Journal of Manufacturing Science and Technology*, *31*, 251-264. https://doi.org/10.1016/j.cirpj.2020.06.001

Iglesias, A., Tunç, L. T., Özsahin, O., Franco, O., Munoa J., & Budak E. (2022). Alternative experimental methods for machine tool dynamics identification: A review. *Mechanical Systems and Signal Processing*, *170*, 108837. https://doi.org/10.1016/j.ymssp.2022.108837

Juang, J., & Pappa, R. S. (1985). An eigensystem realization algorithm for modal parameter identification and model reduction. *Journal of Guidance Control and Dynamics*, *8*(5), 620-627. https://doi.org/10.2514/3.20031

Lambora, R., Nuhman, A. P., Law, M., & Mukhopadyay, S. (2022). Recovering cutting tool modal parameters from fractionally uncorrelated and potentially aliased signals. *Annals of the CIRP*, *38*, 414-426. https://doi.org/10.1016/j.cirpj.2022.05.014

Law, M., Gupta, P., & Mukhopadhyay, S. (2020). Modal analysis of machine tools using Visual Vibrometry and output-only methods. *Annals of the CIRP*, *69*, 357-360. https://doi.org/10.1016/j.cirp.2020.04.043

Law, M., Lambora, R., Nuhman, A. P., & Mukhopadhyay, S. (2022). Modal parameter recovery from temporally aliased video recordings of cutting tools. *Annals of the CIRP*, *71*(1), 329-332. https://doi.org/10.1016/j.cirp.2022.03.023

Martinez, B., Green, A., Silva, M. F., Yang, Y., & Mascareñas, D. (2020). Sparse and random sampling techniques for high-resolution, full-field, BSS-based structural dynamics identification from video, *Sensors*. *20*(12), 3526. https://doi.org/10.3390/s20123526

Metropolis, N., & Ulam, S. (1949). The Monte Carlo method. *Journal of the American Statistical Association*. *44*(247), 335-341. https://doi.org/10.2307/2280232

Yang, Y., & Nagarajaiah, S. (2015). Output-only modal identification by compressed sensing: non-uniform low-rate random sampling. *Mechanical Systems and Signal Processing*, 56-57, 15-34. https://doi.org/10.1016/j.ymssp.2014.10.015

Yazicigil, R. T., Haque, T., Kinget, P. R., & Wright, J. (2019). Taking compressive sensing to the hardware level: breaking fundamental radio-frequency hardware performance tradeoffs. *IEEE Signal Processing Magazine*, *36*(2), 81-100. https://doi.org/10.1109/MSP.2018.2880837

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