## AE 688 Dynamics And Vibration Assignment No. 3

1. The circular disk rotates with a constant angular velocity $\omega=40 \mathrm{rad} / \mathrm{s}$ about its axis, which is inclined in the $y-z$ plane at the angle $\theta=\tan ^{-1}\left(\frac{3}{4}\right)$. Determine the vector expressions for the velocity and acceleration of point $P$, whose position vector at the instant shown is $\mathbf{r}=375 \mathbf{i}+400 \mathbf{j}-300 \mathbf{k} m m$. (Check the magnitudes of your results from the scalar values $v=r \omega$ and $a_{n}=r \omega^{2}$.)

$$
\text { Ans. } \begin{aligned}
\mathbf{v} & =-20 \mathbf{i}+12 \mathbf{j}-9 \mathbf{k} \mathrm{~m} / \mathrm{s} \\
\mathbf{a} & =-600 \mathbf{i}-640 \mathbf{j}+480 \mathbf{k} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Figure 1

2. A V-belt speed-reduction drive is shown where pulley $A$ drives the two integral pulleys $B$ which in turn drive pulley $C$. If $A$ starts from rest at time $t=0$ and is given a constant angular acceleration $\alpha_{1}$, derive expressions for the angular velocity of $C$ and the magnitude of the acceleration of a point $P$ on the belt, both at time $t$.

$$
\text { Ans. } \omega_{c}=\left(\frac{r_{1}}{r_{2}}\right)^{2} \alpha_{1} t, a_{P}=\frac{r_{1}^{2}}{r_{2}} \alpha_{1} \sqrt{1+\left(\frac{r_{1}}{r_{2}}\right)^{4}} \alpha_{1}^{2} t^{4}
$$


3. A device which tests the resistance to wear of two materials $A$ and $B$ is shown. If the link $E O$ has a velocity of $1.2 \mathrm{~m} / \mathrm{s}$ to the right when $\theta=45^{\circ}$, determine the rubbing velocity $v_{A}$.

Ans. $v_{A}=2.76 \mathrm{~m} / \mathrm{s}$

Figure 3

4. Slider $A$ moves in the horizontal slot with a constant speed $v$ for a short interval of motion. Determine the angular velocity $\omega$ of bar $A B$ in terms of the displacement $x_{A}$.

$$
\text { Ans. } \omega=\frac{\sqrt{3 v}}{2 L \sqrt{1-\frac{3}{4} \frac{x_{A}^{2}}{L^{2}}}}
$$

Figure 4

5. The punch is operated by a simple harmonic oscillation of the pivoted sector given by $\theta=\theta_{0} \sin 2 \pi t$ where the amplitude is $\theta_{0}=\pi / 12 \operatorname{rad}\left(15^{\circ}\right)$ and the time for one complete oscillation is 1 second. Determine the acceleration of the punch when $(a) \theta=0$ and $(b) \theta=\pi / 12$.

Ans. (a) $a=0.909 \mathrm{~m} / \mathrm{s}^{2}$ up
(b) $a=0.918 \mathrm{~m} / \mathrm{s}^{2}$ down

Figure 5

6. For the instant represented, crank $O B$ has a clockwise angular velocity $\omega=0.8 \mathrm{rad} / \mathrm{s}$ and is passing the horizontal position. Determine the corresponding velocity of the guide roller $A$ in the $20^{\circ}$ slot and the velocity of point $C$ midway between $A$ and $B$.

Ans. $v_{A}=0.226 \mathrm{~m} / \mathrm{s}, v_{C}=0.1747 \mathrm{~m} / \mathrm{s}$

Figure 6

7. The unit at $A$ consists of a high-torque geared motor which rotates $\operatorname{link} A B$ at the constant rate $\dot{\theta}=0.5 \mathrm{rad} / \mathrm{s}$. Unit $A$ is free to roll along the horizontal surface. Determine the velocity $v_{A}$ of unit $A$ when $\theta$ reaches $60^{\circ}$.

Ans. $v_{A}=305 \mathrm{~mm} / \mathrm{s}$

Figure 7

8. The elements of the mechanism for deployment of a spacecraft magnetometer boom are shown. Determine the angular velocity of the boom when the driving link $O B$ crosses the $y$-axis with an angular velocity $\omega_{O B}=0.5 \mathrm{rad} / \mathrm{s}$ if $\tan \theta=4 / 3$ at this instant.

Ans. $\omega_{C A}=0.429 \mathrm{krad} / \mathrm{s}$

Figure 8

9. The flexible band $F$ is attached at $E$ to the rotating sector and leads over the guide pulley. Determine the angular velocities of $A D$ and $B D$ for the position shown if the band has a velocity of $4 \mathrm{~m} / \mathrm{s}$.

$$
\text { Ans. } \omega_{A D}=12.5 \mathrm{rad} / \mathrm{s}, \omega_{B D}=7.5 \mathrm{rad} / \mathrm{s}
$$

Figure 9

10. The sliding collar moves up and down the shaft, causing an oscillation of crank $O B$. If the velocity of $A$ is not changing as it passes the null position where $A B$ is horizontal and $O B$ is vertical, determine the angular acceleration of $O B$ in that position.

$$
\text { Ans. } \alpha_{O B}=\frac{v_{A}^{2}}{r l} C W
$$

Figure 10

11. The linkage of Prob. $5 / 66$ is shown again here. For the instant when $\theta=\beta=60^{\circ}$, the hydraulic cylinder gives $A$ a velocity $v_{A}=1.2 \mathrm{~m} / \mathrm{s}$ which is increasing by $0.9 \mathrm{~m} / \mathrm{s}$ each second. For this instant determine the angular acceleration of link $B C$.

Ans. $\alpha_{B C}=2.08 \mathrm{rad} / \mathrm{s}^{2} \mathrm{CCW}$

Figure 11

12. For the instant represented, link $C B$ is rotating counterclockwise at a constant rate $\mathrm{N}=4 \mathrm{rad} / \mathrm{s}$, and its pin $A$ causes a clockwise rotation of the slotted member ODE. Determine the angular velocity $\omega$ and angular acceleration $\alpha$ of $O D E$ for this instant.

$$
\text { Ans. } \omega=4 \mathrm{rad} / \mathrm{s} C W, \alpha=64.0 \mathrm{rad} / \mathrm{s}^{2} C C W
$$

Figure 12

13. For the linkage shown, if $v_{A}=500 \mathrm{~mm} / \mathrm{s}$ and is constant when the two links become perpendicular to one another, determine the angular acceleration of $C B$ for this position.

Ans. $\alpha_{C B}=5.76 \mathrm{rad} / \mathrm{s}^{2} \mathrm{CW}$

Figure 13

14. The pin $A$ in the bell crank $A O D$ is guided by the flanges of the collar $B$, which slides with a constant velocity $v_{B}$ of $0.9 \mathrm{~m} / \mathrm{s}$ along the fixed shaft for an interval of motion. For the position $\theta=30^{\circ}$ determine the acceleration of the plunger $C E$, whose upper end is positioned by the radial slot in the bell crank.

Ans. $a_{C}=24.9 \mathrm{~m} / \mathrm{s}^{2}$ up


