**Department of Aerospace Engineering**

**AE602 Mathematics for Aerospace Engineers**

**Assignment No. 4**

**4.1** Decide whether or not the following vectors are linearly independent, by solving

 $c\_{1}v\_{1}+c\_{2}v\_{2}+c\_{3}v\_{3}+c\_{4}v\_{4}=0:$

$$v\_{1}= \left[\begin{matrix}1\\1\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right], v\_{2}= \left[\begin{matrix}1\\0\\\begin{matrix}1\\0\end{matrix}\end{matrix}\right], v\_{3}= \left[\begin{matrix}0\\0\\\begin{matrix}1\\1\end{matrix}\end{matrix}\right], v\_{4}= \left[\begin{matrix}0\\1\\\begin{matrix}0\\1\end{matrix}\end{matrix}\right].$$

Decide also if they span $R^{4},$ by trying to solve $c\_{1}v\_{1}+…+c\_{4}v\_{4}=\left(0, 0, 0, 1\right).$

**4.2** Decide the dependence or independence of

(a) $ \left(1, 1, 2\right), \left(1, 2, 1\right), \left(3, 1,1\right); $

(b) $v\_{1}-v\_{2}, v\_{2-}v\_{3, } v\_{3}-v\_{4}, v\_{4}-v\_{1} $for any vectors $v\_{1},v\_{2, }v\_{3}, v\_{4};$,

(c) $\left(1, 1, 0\right), \left(1, 0, 0\right), \left(0, 1,1\right),\left(x, y, z\right),$ for any numbers $x, y, z$.

**4.3** Prove that if any diagonal element of

$$T=\left[\begin{matrix}a&b&c\\0&d&e\\0&0&f\end{matrix}\right]$$

is zero, the rows are linearly dependent.

**4.4** Is it true that if $v\_{1},v\_{2 ,}v\_{3}$ are linearly dependent, then also the vectors $w\_{1}=v\_{1}+v\_{2,}$ $w\_{2}=v\_{1}+v\_{3,} w\_{3}=v\_{2}+v\_{3,}$ are linearly independent? (Hint: Assume some combination $c\_{1}w\_{1}+c\_{2}w\_{2}+c\_{3}w\_{3}=0,$ and find which $c\_{i }$are possible.)

**4.5** Describe geometrically the subspace of $R^{3}$spanned by

(a) $ \left(0, 0, 0\right), \left(0, 1, 0\right), \left(0, 2,0\right); $

(b) $ \left(0, 0, 1\right), \left(0, 1, 1\right), \left(0, 2,1\right); $

(c) all six of these vectors. Which two form a basis?

(d) all vectors with positive components.

**4.6** To decide whether $b$ is in the subspace spanned by $w\_{1},…, w\_{l}, $let the vectors $w$ be the columns of $A$ and try to solve $Ax=b$ . What is it the result for

(a) $w\_{1}=\left(1, 1, 0\right), w\_{2}=\left(2, 2, 1\right), w\_{3}=\left(0, 0, 2\right), b= \left(3, 4, 5\right);$

(b)$ w\_{1}=\left(1, 2, 0\right), w\_{2}=\left(2, 5, 0\right), w\_{3}=\left(0, 0, 2\right), w\_{4}=\left(0, 0, 0\right), $and any $b$?

**4.7** By locating the pivots, find a basis for the column space of

$$U=\left[\begin{matrix}0\\0\\\begin{matrix}0\\0\end{matrix}\end{matrix} \begin{matrix}1\\0\\\begin{matrix}0\\0\end{matrix}\end{matrix} \begin{matrix}4\\2\\\begin{matrix}0\\0\end{matrix}\end{matrix} \begin{matrix}3\\2\\\begin{matrix}0\\0\end{matrix}\end{matrix} \right]$$

Express each column that is not the basis as a combination of the basic columns. Find also a matrix $A$ with this echelon form $U$, but a different column space.

**4.8** Find the dimension and construct a basis for the four subspaces associated with each of the matrices

$$A=\left[\begin{matrix}0&1&4\\0&2&8\end{matrix} \begin{matrix}0\\0\end{matrix}\right] and U=\left[\begin{matrix}0&1&4\\0&0&0\end{matrix} \begin{matrix}0\\0\end{matrix}\right].$$

**4.9** Find the dimension and a basis for the four fundamental subspaces for both

$ A=\left[\begin{matrix}1&2&0\\0&1&1\\1&2&0\end{matrix} \begin{matrix}1\\0\\1\end{matrix}\right] and $ $U=\left[\begin{matrix}1&2&0\\0&1&1\\0&0&0\end{matrix} \begin{matrix}1\\0\\0\end{matrix}\right]. $

**4.10** Describe the four subspaces in 3-dimensional space associated with

$$A=\left[\begin{matrix}0&1&0\\0&0&1\\0&0&0\end{matrix}\right].$$

**4.11** Find the rank of$ A$ and write the matrix as$ A=uv^{T}$:

$ A=\left[\begin{matrix}1&0&0\\0&0&0\\2&0&0\end{matrix} \begin{matrix}3\\0\\6\end{matrix}\right] and A=\left[\begin{matrix}2&-2\\2&-2\end{matrix}\right].$

**4.12** Find a left-inverse and/or a right –inverse (when they exist) for

 $A=\left[\begin{matrix}1&1&0\\0&1&1\end{matrix}\right] $ and $M=\left[\begin{matrix}1&0\\1&1\\0&1\end{matrix}\right]$ and$ T=\left[\begin{matrix}a&b\\0&a\end{matrix}\right].$