## AE 688 Dynamics And Vibration <br> Assignment No. 4 <br> Plane Kinetic of Rigid Bodies

1. The uniform $100-\mathrm{kg}$ log is supported by the two cables and used as a battering ram. If the log is released from rest in the position shown, calculate the initial tension induced in each cable immediately after release and the corresponding angular acceleration $\alpha$ of the cables.

$$
\begin{aligned}
\text { Ans. } T_{A} & =212 \mathrm{~N}, T_{B}=637 \mathrm{~N} \\
\alpha & =2.45 \mathrm{rad} / \mathrm{s}^{2}
\end{aligned}
$$


2. End $A$ of the uniform $5-\mathrm{kg}$ bar is pinned freely to the collar, which has an acceleration $a=4 \mathrm{~m} / \mathrm{s}^{2}$ along the fixed horizontal shaft. If the bar has a clockwise angular velocity $\omega=2 \mathrm{rad} / \mathrm{s}$ as it swings past the vertical, determine the components of the force on the bar at $A$ for this instant.

$$
\text { Ans. } A_{x}=5 \mathrm{~N}, A_{y}=57.1 \mathrm{~N}
$$


3. The uniform slender rod is at rest on a smooth horizontal surface when struck at point A by a bullet travelling normal to the rod. Calculate the distance $x$ from the end of the bar to point $C$, the instantaneous center of zero velocity, about which the rod begins to rotate during the impact.

Ans. $x=100 \mathrm{~mm}$

4. The robotic device consists of the stationary pedestal $O A$, arm $A B$ pivoted at $A$, and arm $B C$ pivoted at $B$. The rotation axes are normal to the plane of the figure. Estimate (a) the moment $M_{A}$ applied to arm $A B$ required to rotate it about joint $A$ at 4 $\mathrm{rad} / \mathrm{sec}^{2}$ counterclockwise from the position shown with joint $B$ locked and (b) the moment $M_{B}$ applied to arm $B C$ required to rotate it about joint $B$ at the same rate with joint $A$ locked. The mass of arm $A B$ is 25 kg and that of $B C$ is 4 kg , with the stationary portion of joint $A$ excluded entirely and the mass of joint $B$ divided equally between the two arms. Assume that the centers of mass $G_{1}$ and $G_{2}$ are in the geometric centers of the arms and model the arms as slender rods.

Ans. (a) $M_{A}=109.8 \mathrm{~N} \cdot \mathrm{~m}$
(b) $M_{B}=5.51 \mathrm{~N} \cdot \mathrm{~m}$

5. The uniform $15-\mathrm{kg}$ semicircular disk is supported in the equilibrium position shown by the two cables which are wrapped around its attached hubs and lead to the identical springs. Each spring has a stiffness $k=2.6 \mathrm{kN} / \mathrm{m}$. If the disk is rotated $90^{\circ}$ so that its mass center is in the lowest possible position and then released from rest, calculate the angular velocity $\omega$ of the disk as it passes the equilibrium position. Neglect the mass of the hubs and shaft.

Ans. $\omega=7.11 \mathrm{rad} / \mathrm{s}$

6. The right-angle plate is formed from a flat plate having a mass $\rho$ per unit area and is welded to the horizontal shaft mounted in the bearing at $O$. If the shaft is free to rotate, determine the initial angular acceleration $\alpha$ of the plate when it is released from rest with the upper surface in the horizontal plane. Also determine the $y$ - and $z$ - components of the resultant force on the shaft at $O$.

$$
\text { Ans. } \alpha=\frac{3 g}{10 b}, O_{y}=\frac{9}{20} \rho b c g, O_{z}=\frac{37}{20} \rho b c g
$$


7. A frozen-juice can rests on the horizontal rack of a freezer door as shown. With what maximum angular velocity $\Omega$ can the door be "slammed" shut against its seal and not dislodge the can? Assume that the can rolls without slipping on the corner of the rack, and neglect the dimension $d$ compared with the $500-\mathrm{mm}$ distance.

Ans. $\Omega=1.135 \mathrm{rad} / \mathrm{s}$

8. A reel of flexible power cable is mounted on the dolly, which is fixed in position. There are 60 m of cable having a mass of 0.65 kg per meter of length wound on the reel at a radius of 375 mm . The empty spool has a mass of 28 kg and has a radius of gyration about its axis of 300 mm . A tension $T$ of 90 N is required to overcome frictional resistance to turning. Calculate the angular acceleration $\alpha$ of the reel if a tension of 180 N is applied to the free end of the cable.

Ans. $\alpha=4.22 \mathrm{rad} / \mathrm{s}^{2}$

9. The solid cylindrical rotor $B$ has a mass of 43 kg and is mounted on its central axis $C-C$. The frame $A$ rotates about the fixed vertical axis $O-O$ under the applied torque $M=30 \mathrm{~N} \cdot \mathrm{~m}$. The rotor may be unlocked from the frame by withdrawing the locking $\operatorname{pin} P$. Calculate the angular acceleration $\alpha$ of the frame $A$ if the locking pin is $(a)$ in place and $(b)$ withdrawn. Neglect all friction and the mass of the frame.

Ans. (a) $\alpha=8.46 \mathrm{rad} / \mathrm{s}^{2}$
(b) $\alpha=11.16 \mathrm{rad} / \mathrm{s}^{2}$

10. The mass center $G$ of the $10-\mathrm{kg}$ wheel is off center by 10 mm . If $G$ is in the position shown as the wheel rolls without slipping through the bottom of the circular path of 2-m radius with an angular velocity $\omega$ of $10 \mathrm{rad} / \mathrm{s}$, compute the force $P$ exerted by the path on the wheel. (Be careful to use the correct mass-center acceleration.)

Ans. $P=100.3 \mathrm{~N}$

11. Determine the magnitude $P$ and direction $\theta$ of the force required to impart a rearward acceleration $a=1.5 \mathrm{~m} / \mathrm{s}^{2}$ to the loaded wheelbarrow with no rotation from the position shown. The combined mass of the wheelbarrow and its load is 190 kg with center of mass at $G$.Compare the normal force at $B$ under acceleration with that for static equilibrium in the position shown. Neglect the friction and mass of the wheel.

$$
\text { Ans. } \begin{aligned}
P & =439 \mathrm{~N}, \theta=49.6^{\circ} \\
B & =1530 \mathrm{~N}, B_{a t}=1553 \mathrm{~N}
\end{aligned}
$$


12. The $10-\mathrm{kg}$ double wheel with radius of gyration of 125 mm about $O$ is connected to the spring of stiffness $k=600 \mathrm{~N} / \mathrm{m}$ by a cord which is wrapped securely around the inner hub. If the wheel is released from rest on the incline with the spring stretched 225 mm , calculate the maximum velocity $v$ of its center $O$ during the ensuing motion. The wheel rolls without slipping.

Ans. $v_{\text {max }}=1.325 \mathrm{~m} / \mathrm{s}$

13. Design tests of the landing sequence for the lunar excursion module are conducted using the pendulum model suspended by the parallel wires $A$ and $B$. If the model has a mass of 10 kg with mass center at $G$, and if $\dot{\theta}=2 \mathrm{rad} / \mathrm{s}$ when $\theta=60^{\circ}$, calculate the tension in each of the wires for this instant.

$$
\text { Ans. } T_{A}=147.9 \mathrm{~N}, T_{B}=21.1 \mathrm{~N}
$$


14. In the rotating assembly shown, arm $O A$ and the attached motor housing $B$ have a combined mass of 4.5 kg and have a radius of gyration about the z -axis of 175 mm . The motor armature and attached $125-\mathrm{mm}$-radius disk have a combined mass of 7 kg and a radius of gyration of 100 mm about their own axis. The entire assembly is free to rotate about the z-axis. If the motor is turned on with $O A$ initially at rest, determine the angular speed N of $O A$ when the motor has reached a speed of $300 \mathrm{rev} / \mathrm{min}$ relative to $\operatorname{arm} O A$.

Ans. $\mathrm{N}=37.4 \mathrm{rev} / \mathrm{min}$


