**Department of Aerospace Engineering**

**AE602 Mathematics for Aerospace Engineers**

**Assignment No. 5**

**5.1** What matrix has the effect of rotating every vector through 90°and then projecting the result onto the *x*-axis?

**5.2** What matrix represents projection onto the *x*-axis followed by projection onto the $y$-axis?

**5.3** Does the product of 5 reflections and 8 rotations of the $x-y $plane produce a rotation or a reflection?

**5.4** The matrix $A=\left[\begin{matrix}1&0\\3&1\end{matrix}\right]$ yields a *shearing* transformation, which leaves the $y$-axis unchanged. Sketch its effect on the $x$-axis, by indicating what happens to $\left(1,0\right)$ and $\left(2,0\right)$ and $\left(-1,0\right)-$ and how the whole axis is transformed.

**5.5** What 3 by 3 matrices represent the transformations that

1. Project every vector onto the $x-y$ plane?
2. Reflect every vector through the $x-y$ plane?
3. Rotate the $x-y$ plane through 90°, leaving the *z*-axis alone?
4. Rotate the $x-y$ plane, then the $x-z$ plane, then the $y-z$ plane, all through 90°?
5. Carry out the same three rotations, but through 180°?
	1. Form the cubics $P\_{3}$ to the fourth degree polynomials $P\_{4}$, what matrix represents multiplication by $2+3t?$ the columns of the 5 by 4 matrix $A$ come from applying the transformation to each basis vector $x\_{1}=1,x\_{2}=t,x\_{3}=t^{2},x\_{4}=t^{3}$.

**5.7** Find the lengths and the inner product of $x=(1,4,0,2)$ and $y=(2,-2,1,3)$.

**5.8** Give an example in $R^{2}$ of linearly independent vectors that are not mutually orthogonal. Also, give an example of mutually orthogonal vectors that are not independent.

**5.9** Which pairs are orthogonal among the vectors

$$v\_{1}=\left[\begin{matrix} 1\\ 2\\\begin{matrix}-2\\ 1\end{matrix}\end{matrix}\right], v\_{2}=\left[\begin{matrix}4\\0\\\begin{matrix}4\\0\end{matrix}\end{matrix}\right], v\_{3}=\left[\begin{matrix} 1\\-1\\\begin{matrix}-1\\-1\end{matrix}\end{matrix}\right]?$$

**5.10** In $R^{3}$find all vectors that are orthogonal to $\left(1, 1, 1\right)$ and $\left(1, -1, 0\right).$ Produce from these vectors a mutually orthogonal system of unit vectors (an orthonormal system) in $R^{3}.$

**5.11** Find a vector $x $orthogonal to the row space, and a vector $y $orthogonal to the column space, of

$$A=\left[\begin{matrix}1&2&1\\2&4&3\\3&6&4\end{matrix}\right].$$

**5.12** Find a basis for the nullspace of

$$A= \left[\begin{matrix}1&0&2\\1&1&4\end{matrix}\right],$$

And verify that it is orthogonal to the row space. Given $x=\left(3,3,3\right),$ split it into a row space component $x\_{r} $and a nullspace component $x\_{n}.$

**5.13** Show that $x-y$ is orthogonal to $x+y$ if and only if $ \left‖x\right‖= \left‖y\right‖.$

**5.14** (a) Give any two positive numbers $x$ and $y,$ choose the vector $b$ equal to $\left(\sqrt{x,}\sqrt{y}\right),$ and choose $a=\left(\sqrt{y,}\sqrt{x}\right).$ Apply the Schwarz inequality to compare the arithmetic mean $\frac{1}{2}\left(x+y\right)$ with the geometric mean $\sqrt{xy.}$

(b) Suppose we start with a vector from the origin to the point *x*, and then add a vector of length $\left‖y\right‖$ connecting $x$ to $x+y$. The third side of the triangle goes from the origin to $x+y$. The triangle inequality asserts that this distance cannot be greater than the sum of the first two: $\left‖x+y\right‖= \left‖x\right‖+\left‖y\right‖$. After squaring both sides and expanding $\left(x+y\right)^{T}\left(x+y\right)$, reduce this to the Schwarz inequality.

**5.15** What multiple of $a=\left(1, 1, 1\right)$ is closest to the point $b=\left(2,4,4\right)?$ Find also the point closest to $a $on the line through $b.$

**5.16** Explain why the Schwarz inequality becomes an equality in case $a$ and $b$ lie on the same line through the origin, and only in that case. What if they lie on opposite sides of the origin?

**5.17** (a) Find the projection matrix $P\_{1}$ onto the through $a=\left[\begin{matrix}1\\3\end{matrix}\right]$ and also the matrix $P\_{2}$ that projects onto the line perpendicular to $a.$

(b) Compute $P\_{1}+P\_{2}$ and $P\_{1}P\_{2}$ and explain.

**5.18** Prove that the “*trace*” of $P={aa^{T}}/{a^{T}a} $ always equals one. Trace is the sum of diagonal entries­­­­ of a matrix.

**5.19** Show that the length of $Ax$ equal the length of $A^{T}x$ if $A A^{T}= A^{T}A.$