**Department of Aerospace Engineering**

**AE602 Mathematics for Aerospace Engineers**

**Assignment No. 5**

**5.1** What matrix has the effect of rotating every vector through 90°and then projecting the result onto the *x*-axis?

**5.2** What matrix represents projection onto the *x*-axis followed by projection onto the -axis?

**5.3** Does the product of 5 reflections and 8 rotations of the plane produce a rotation or a reflection?

**5.4** The matrix yields a *shearing* transformation, which leaves the -axis unchanged. Sketch its effect on the -axis, by indicating what happens to and and and how the whole axis is transformed.

**5.5** What 3 by 3 matrices represent the transformations that

1. Project every vector onto the plane?
2. Reflect every vector through the plane?
3. Rotate the plane through 90°, leaving the *z*-axis alone?
4. Rotate the plane, then the plane, then the plane, all through 90°?
5. Carry out the same three rotations, but through 180°?
   1. Form the cubics to the fourth degree polynomials , what matrix represents multiplication by the columns of the 5 by 4 matrix come from applying the transformation to each basis vector .

**5.7** Find the lengths and the inner product of and .

**5.8** Give an example in of linearly independent vectors that are not mutually orthogonal. Also, give an example of mutually orthogonal vectors that are not independent.

**5.9** Which pairs are orthogonal among the vectors

**5.10** In find all vectors that are orthogonal to and Produce from these vectors a mutually orthogonal system of unit vectors (an orthonormal system) in

**5.11** Find a vector orthogonal to the row space, and a vector orthogonal to the column space, of

**5.12** Find a basis for the nullspace of

And verify that it is orthogonal to the row space. Given split it into a row space component and a nullspace component

**5.13** Show that is orthogonal to if and only if

**5.14** (a) Give any two positive numbers and choose the vector equal to and choose Apply the Schwarz inequality to compare the arithmetic mean with the geometric mean

(b) Suppose we start with a vector from the origin to the point *x*, and then add a vector of length connecting to . The third side of the triangle goes from the origin to . The triangle inequality asserts that this distance cannot be greater than the sum of the first two: . After squaring both sides and expanding , reduce this to the Schwarz inequality.

**5.15** What multiple of is closest to the point Find also the point closest to on the line through

**5.16** Explain why the Schwarz inequality becomes an equality in case and lie on the same line through the origin, and only in that case. What if they lie on opposite sides of the origin?

**5.17** (a) Find the projection matrix onto the through and also the matrix that projects onto the line perpendicular to

(b) Compute and and explain.

**5.18** Prove that the “*trace*” of always equals one. Trace is the sum of diagonal entries­­­­ of a matrix.

**5.19** Show that the length of equal the length of if