AE 670 Aerospace Structural Analysis-I

Assignment No. 5

5.1 The missile fuel tank shown in figure 5.1 is pressurized to 100 psig and is subjected to a torque of 5×10^6 in.-lb during a roll maneuver.

(*a*) Use equations from strength of materials to find the hoop and axial stresses due to the pressure and the shear stress due to torque.

(*b*) Find the principal and maximum shear stresses and the angle to each.

5.2 Show that in polar coordinates the two-dimensional equations of equilibrium in the radial and circumferential directions are

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}+R=0$$

$$\frac{\partial\sigma_{\theta r}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+2\frac{\sigma_{r\theta}}{r}+\Theta=0$$

where R and Θ are body forces in the r and θ directions. Refer figure 5.2.

5.3 From the results of example 5.2 show that the equation of equilibrium of a circular disk rotating about its axis at a constant angular velocity Ω is

$$r\frac{d\sigma_{rr}}{dr}+\sigma_{rr}-\sigma_{\theta\theta}+\frac{r^2\omega\Omega^2}{g}=0$$

where ω is the specific weight of the disk. **Hint**: Due to axisymmetry, the stress $\sigma_{r\theta}$ can be taken to be zero.

5.4 Show that the two dimensional equilibrium equations in polar coordinates given in example 5.2 are identically satisfied by an Airy stress function $\varphi(r,\theta)$ defined by

$$\sigma_{rr}=\frac{1}{r}\frac{\partial\varphi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}+V \qquad \sigma_{\theta\theta}=\frac{\partial^2\varphi}{\partial r^2}+V$$

$$\sigma_{r\theta}=\frac{1}{r^2}\frac{\partial\varphi}{\partial\theta}-\frac{1}{r}\frac{\partial^2\varphi}{\partial r\partial\theta}$$

Where R and Θ are conservative body forces derivable from a potential function $V(r,\theta)$ by

$$R=-\frac{\partial V}{\partial r},\quad \Theta=-\frac{\partial V}{r\partial\theta}$$

5.5 The elementary theory of beams from strength of materials gives the stresses

$$\sigma_{xx} = -\frac{My}{I} \qquad \sigma_{yy} = 0 \qquad \sigma_{xy} = -\frac{VQ}{Ib}$$

where

$$Q = \frac{b}{2}\left[\left(\frac{h}{2}\right)^2 - y^2\right]$$

for the rectangular beam shown in figure 5.3. Do these satisfy the equations of equilibrium and the boundary conditions for a cantilever support at $x = L$ for (*a*) a shear force P at $x = 0$, (*b*) a uniform pressure on the upper surface, (c) the beam under the action of its own weight?

5.6 Using the equations of the compatibility in polar coordinate system show that the compatibility equation for a circular disk rotating at a constant angular velocity Ω about its axis and subjected to a temperature change $T(r)$ is

$$\frac{d^4\varphi}{dr^4} + \frac{2}{r}\frac{d^3\varphi}{dr^3} - \frac{1}{r^2}\frac{d^2\varphi}{dr^2} + \frac{1}{r^3}\frac{d\varphi}{dr} = -E\left(\frac{d^2\alpha T}{dr^2} + \frac{1}{r}\frac{d\alpha T}{dr}\right) + 2(1-\nu)\frac{\omega\Omega^2}{\mathbf{g}}$$

where ω is the specific weight of the disk and φ is the stress function defined in example 5.4.

5.7 The state of stress at a point is $[\sigma] = \begin{bmatrix} 7 & 1.4 & 0 \\ 1.4 & -42 & -2.8 \\ 0 & -2.8 & 0 \end{bmatrix} \times 10^6$ Pa

Find the components of the surface traction vector for an interface whose normal vector is **n** = 0.11 **i** + 0.35 **j** + 0.93 **k**.

5.8 What will be state of stress at the same point as in example 5.7 for new set of axes (x_1', x_2', x_3') obtained by rotating the coordinate system (x_1, x_2, x_3) about x_3 axis by angle 60°?

5.9 Determine the principal stresses for the stress tensor with rectangular Cartesian components given in the matrix below (unit are kN). Determine also the direction cosines for each principal axis. $[\sigma] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

5.10 Evaluate the three invariants *I*, *II*, *III* for the stress tensor in example 5.9 by using the given components and also by using the values obtained in that problem for the principal stresses and check that the same results are obtained by the two procedures.

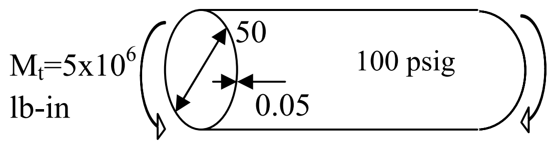

Figure 5.1

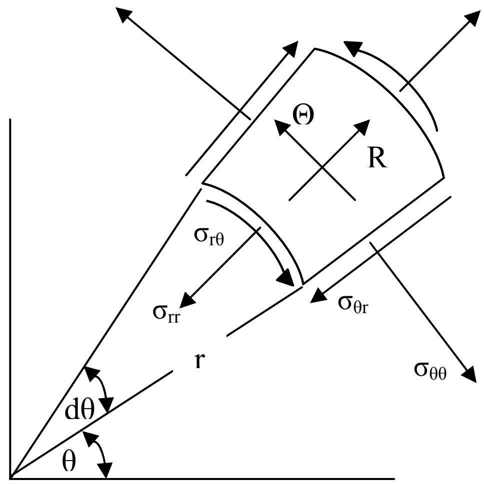

Figure 5.2

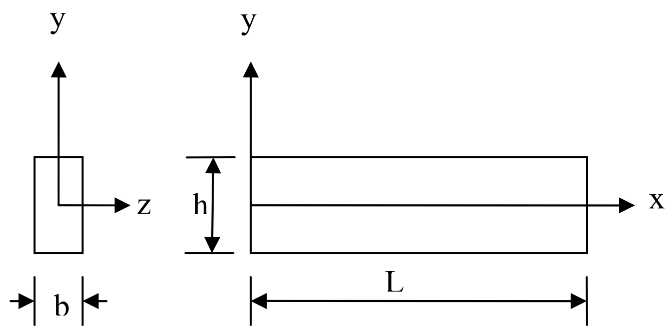

Figure 5.3