**Department of Aerospace Engineering**

**AE602 Mathematics for Aerospace Engineers**

**Assignment No. 6**

**6.1** Find the best least squares solution $\overbar{x}$ to $3x=10, 4x=5$. What error $E^{2}$is minimized? Check that the error vector $(10-3\overbar{x, }$ $5-4\overbar{x} )$ is perpendicular to the column $\left(3,4\right).$

**6.2** Suppose the values $b\_{1}=1$ and $b\_{2}=7$ at times $t\_{1}=1$ and $t\_{2}=2 $are fitted by a line $b=Dt $ through the origin. Solve $D=1$ and $2D=7$ by least squares, and sketch the best line.

**6.3** Solve $Ax=b$ by least squares and find $p=A\overbar{x }$ if

$$A=\left[\begin{matrix}1&0\\0&1\\1&1\end{matrix}\right], b=\left[\begin{matrix}1\\1\\0\end{matrix}\right].$$

Verify that the error $b-p$ is perpendicular to the columns of $A$.

**6.4** Write out $E^{2}=\left‖Ax=b\right‖$ and set to zero its derivatives with respect to $u and v$, if

$$A=\left[\begin{matrix}1&0\\0&1\\1&1\end{matrix}\right], x=\left[\begin{matrix}u\\v\end{matrix}\right], b=\left[\begin{matrix}1\\3\\4\end{matrix}\right].$$

Compare the resulting equations with $A^{T}A\overbar{x }=A^{T}b,$ confirming that calculus as well as geometry gives the normal equations. Find the solutions $\overbar{x }$ and the projection $p=A\overbar{x }.$ Why is $p=b?$

**6.5** The following system has no solution:

$$Ax=\left[\begin{matrix}1&-1\\1& 0\\1& 1\end{matrix}\right]\left[\begin{matrix}C\\D\end{matrix}\right]=\left[\begin{matrix}4\\5\\9\end{matrix}\right]=b.$$

Sketch and solve a straight line fit that leads to the minimization of the quadratic $\left(C-D-4\right)^{2}+(C-5)^{2}+(C+D-9)^{2}.$ What is the projection of $b$ onto the column space of $A$?

**6.6** Find the projection of $b$ onto the column space of $A:$

$$A=\left[\begin{matrix} 1& 1\\ 2&-1\\-2& 4\end{matrix}\right], b=\left[\begin{matrix}1\\2\\7\end{matrix}\right]$$

Split $b$ into$ p+q$, with $p$ in the column space and $q $perpendicular to that space. Which of the four subspaces contains$ q$?

**6.7** Find the projection matrix $P$ onto the space spanned by $a\_{1}=(1, 0, 1)$ and $a\_{2}=\left(1, 1,- 1\right).$

**6.8** Find the best straight line fit (least squares) to the measurements

$$b=4 at t=-2, b=3 at t=-1,$$

$$b=1 at t=0, b=0 at t=2$$

Then find the projection of $b=(4, 3, 1, 0) $onto the column space of

A=$\left[\begin{matrix}1&-2\\1&-1\\\begin{matrix}1\\1\end{matrix}&\begin{matrix} 0\\ 2\end{matrix}\end{matrix}\right]$.

**6.9** We want to fit a plane $y=C+Dt+Ez$ to the four points

$$y=3 at t=1, z=1 y=6 at t=0, z=3$$

$$y=5 at t=2, z=1 y=0 at t=0, z=0$$

1. Find 4 equations in 3 unknowns to pass a plane through the points (if there is such a plane).
2. Find 3 equations in 3 unknowns for the best least squares solution.