**Department of Aerospace Engineering**

**AE602 Mathematics for Aerospace Engineers**

**Assignment No. 7**

**7.1** (a) Write down the four equations for fitting $y=C+Dt$ to the data

$$y=-4 at t=-2, y=-3 at t=-1.$$

$$ y=-1 at t=1, y=0 at t=2.$$

Show that the columns are orthogonal.

(b) Find the optimal straight line, draw a graph, and write down the error $E^{2}$.

(c) Interpret the fact that the error is zero in terms of the original system of four equations in two unknowns: Where is the right side $b$ with relation to the column space, and what is its projection $p?$

**7.2** If $Q\_{1}$ and $Q\_{2}$ are orthogonal matrices, and therefore satisfy $Q^{T}Q=I$, show that $Q\_{1}Q\_{2}$is also orthogonal. If $Q\_{1}$is rotation through θ, and $Q\_{2}$ is rotation through ф, what is $Q\_{1}Q\_{2}?$ Can you find the trigonometric identities for $\sin(\left(θ+∅\right))$ and $\cos(\left(θ+∅\right))$ in the matrix multiplication $Q\_{1}Q\_{2}?$

**7.3** If $u$ is a unit vector, show that $Q=I-2uu^{T}$ is an orthogonal matrix. (It is a reflection, also known as a Householder transformation.) Compute $Q$ when $u^{T}=\left[\frac{1}{2} \begin{matrix}\frac{1}{2}&-\frac{1}{2}&-\frac{1}{2}\end{matrix}\right]$.

**7.4** Find a third column so that the matrix

$$Q= \left[\begin{matrix}{1}/{√3}&{1}/{√14}&\\{1}/{√3}&{2}/{√14}&\\{1}/{√3}&{-3}/{√14}&\end{matrix}\right] $$

**7.5** Apply the Gram-Schmidt process to

$$a=\left[\begin{matrix}0\\0\\1\end{matrix}\right], b=\left[\begin{matrix}0\\1\\1\end{matrix}\right], c=\left[\begin{matrix}1\\1\\1\end{matrix}\right] $$

 and write the result in the form $A=QR.$

**7.6** Suppose the given vectors are

$$a=\left[\begin{matrix}1\\1\\0\end{matrix}\right], b=\left[\begin{matrix}1\\0\\1\end{matrix}\right], c=\left[\begin{matrix}0\\1\\1\end{matrix}\right] $$

Find the orthonormal vectors $q\_{1},q\_{2},q\_{3}$.

**7.7** Find an orthonormal set $q\_{1},q\_{2},q\_{3}$ for which $q\_{1},q\_{2}$ span the column space of

$$A=\left[\begin{matrix} 1& 1\\ 2&-1\\-2& 4\end{matrix}\right].$$

Which fundamental subspace contains $q\_{3}?$ What is the least squares solutions solution of $Ax=b$ if $b=\left[\begin{matrix}1&2&7\end{matrix}\right]^{T}?$

**7.8** Express the Gram-Schmidt orthogonalization of

$a\_{1}=\left[\begin{matrix}1\\2\\2\end{matrix}\right], $ $a\_{3}=\left[\begin{matrix}1\\3\\1\end{matrix}\right] $

as $A=QR.$ Given $n$ vectors $a\_{i},$ each with $m$ components, what are the shapes of $A, Q, and R ?$