**Department of Aerospace Engineering**

**AE602 Mathematics for Aerospace Engineers**

**Assignment No. 8**

**8.1** Find the eigenvalues and eigenvectors of the matrix $A=\left[\begin{matrix}1&-1\\2& 4\end{matrix}\right].$ Verify that the trace equals the sum of the eigenvalues, and the determinant equals their product.

**8.2** With the same matrix $A,$ solve the differential equation ${du}/{dt}=Au, u\_{0}=\left[\begin{matrix}0\\6\end{matrix}\right].$ what are the two pure exponential solutions?

**8.3** Suppose we shift the preceding $A$ by subtracting $7I:$

 $B=A-7I=\left[\begin{matrix}-6&-1\\ 2&-3\end{matrix}\right].$

What are the eigenvalues and eigenvectors of $B,$ and how are they related to those of $A?$

**8.4** Find the eigenvalues and eigenvectors of

$A=\left[\begin{matrix}3&4&2\\0&1&2\\0&0&0\end{matrix}\right]$ and $B=\left[\begin{matrix}0&0&2\\0&2&0\\2&0&0\end{matrix}\right].$

Check that $λ\_{1}+λ\_{2}+λ\_{3}$ equals the trace and $λ\_{1} λ\_{2 }λ\_{3}$ equals the determinant.

**8.5** Suppose that $λ$ is an eigenvalue of $A,$ and $x$ is its eigenvector: $Ax=λx.$

(a) Show that the same $x$ is an eigenvector of $B=A-7I,$ and find the eigenvalue.

(b) Assuming $λ\ne 0,$ show that $x$ is also an eigenvector of $A^{-1}$ and find the eigenvalue.

**8.6** Factor the following matrices into $S⋀S^{-1}:$

$A=\left[\begin{matrix}1&1\\1&1\end{matrix}\right]$ and $A=\left[\begin{matrix}2&1\\0&0\end{matrix}\right].$

**8.7** Find the matrix $A$ whose eigenvalues are 1 and 4, and whose eigenvectors are $\left[\begin{matrix}3\\1\end{matrix}\right]$ and $\left[\begin{matrix}2\\1\end{matrix}\right],$ respectively. $\left(Hint:A=S⋀S^{-1}.\right)$