Fibre Breaking Damage Model for Unidirectional Fibrous Composite Using Micromechanics

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ABSTRACT

Micromechanics based damage model for fibre breaking for unidirectional fibrous composites in traction and compression is developed and validated in the present study. The present model developed is capable of predicting the initiation and propagation of fibre breaking until final fracture of laminated composites. Two micromechanics based approaches, namely Hill's concentration factors and homogenization are used for the analysis of RVE, which includes a right circular cylindrical fibre in a cubical matrix. The state of micro-stress in fibre given by these two approaches is used in the brittle type fibre breaking damage model. The proposed model is tested for T300/Epoxy composite and the results obtained are in a good agreement with the results reported in literature.

1. INTRODUCTION

With increased use of composites there is a demand for accurate and feasible methods of analysis. The life of the composite structures is such an important issue of the analysis. The initiation and propagation of failure until final fracture of the structure are the main parts in the life prediction. There is a class of laminate failure theories for the prediction of initiation of global failure (see [1]), for example the first ply failure theory. Here, these theories, in general, neither predict the mode of failure initiation nor their propagation until final fracture. It is well understood by now that the failure initiates at the constituent (fibre and matrix) level, which are at micro level. These micro failures are termed as damage. Obviously, laminate failure theories fall short. The other class of models is based on micromechanics. These models provide the fundamental information on the essential structure of governing equations defining the thermodynamic state of the material (compliance) and the kinetics of its change (evolution). A mathematical approach of homogenization is widely applied to damage in composites [2]. A micromechanical approach for laminated composites can be seen in Fish et al [3] and multiscale approach of Ghosh et al [4]. The statistics of fiber distribution is taken into account to represent an actual scenario in [5]. The drawback of this approach is that it is computationally expensive. These analyses, in most of the cases, are limited to single damage mechanism alone. The interaction between the other damage mechanisms is not accounted for. Further, the exhaustive relations between micro analyses to macro level damage indicators usable in field level analysis and design are not available in open literature. Another class of models called "progressive damage models" proposes the damage evolution law in direct relation with the specific damage. Ladevèze et al [6] proposed a meso scale approach where they considered the ply and the interfacial layer as the basic constituents of the composite. They define damage variables as a monotonically increasing function for fiber failure mode and matrix failure mode separately.

In the present study a micromechanics based damage model including both initiation and propagation until final fracture of the structure is developed. The model uses the microstress or microstrain obtained either from Hill's concentration factors approach or homogenization approach. The present model characterizes only fibre failure.

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2. DAMAGE MODEL FOR FIBRE BREAKING

The failures at the micro level are termed as damage and their growth with increasing load or time is termed as damage accumulation. The concept of damage was first introduced by Kachanov in 1958 [7]. The concepts introduced therein were extended for composites by many researchers (for example, Ladevèze et al [6]). This model is thermodynamically consistent and based on three fundamental foundations of (a) Mesoscale: an intermediate scale between micro (fibre; matrix level) and macro (lamina) level, (b) Internal variable approach: which relates the damage to the material elastic constants and (c) Method of local states: which relates the damage to the corresponding thermodynamic force associated with the strain energy of the damaged material. In this model, laminate is seen as alternate sequence of a homogeneous ply and interface as a thin layer of resin material. In the present work, the model similar to one introduced in [6] is formulated. The model in [6] uses the state of stress in the individual constituents (fibre and matrix) based on macroscopic analysis of laminae or laminate, whereas in the proposed model it is done using micromechanics based techniques. In the present study only fibre damage is characterized. The state of (micro) stress/strain in fibre is computed using the Hill's concentration factors and homogenization theory and are explained briefly in the Appendix.

In the present model, a brittle type criterion has been used for fiber breakage, which distinguishes rupture in traction and compression through two thresholds Y_F^T and Y_F^C, respectively. The elastic strain energy e_d^F of the damaged fibers alone is given by the following equation using microstress in the fibre.

$$2e_d^F = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix}^T \begin{bmatrix} \dfrac{1}{E_1^0\left(1-d_F\right)} & -\dfrac{\nu_{12}^0}{E_1^0\left(1-d_F\right)} & -\dfrac{\nu_{12}^0}{E_1^0\left(1-d_F\right)} \\ -\dfrac{\nu_{12}^0}{E_1^0\left(1-d_F\right)} & \dfrac{1}{E_2^0} & -\dfrac{\nu_{23}^0}{E_2^0} \\ -\dfrac{\nu_{12}^0}{E_1^0\left(1-d_F\right)} & -\dfrac{\nu_{23}^0}{E_2^0} & \dfrac{1}{E_2^0} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix}^T + \frac{\sigma_{12}^2}{G_{12}^0} + \frac{\sigma_{13}^2}{G_{13}^0} + \frac{\sigma_{23}^2}{G_{23}^0} \qquad (1)$$

Here, the engineering constants and the stress components belong to the fibre. d_F denotes the fibre breaking damage indicator. Here and in the remaining text, the terms with the superscript *0* denote the undamaged or virgin quantities. The corresponding thermodynamic force for fibre breaking, Y_{d_F} is defined as:

$$Y_{d_F} = -\frac{\partial \left\langle \left\langle e_d^F \right\rangle \right\rangle}{\partial d_F} \qquad (2)$$

The symbol $\left\langle \left\langle \cdot \right\rangle \right\rangle$ denotes the average in fibre. To ensure that the damage increases monotonically, the thermodynamic force at any step assumes the maximum value over previous step. Thus,

$$Y_{d_F} = \sup_{\tau \le t} \left[Y_{d_F|t} \right]$$

The fibre breaking damage indicator assumes the value between 0 and 1, or more precisely 0 and 1 for brittle fracture. The initiation and propagation model for fibre breaking considering brittle fracture is given as follows:

- While $Y_{d_F} < Y_F^T$ and $Y_{d_F} < Y_F^C$, $d_F = 0$
- If $Y_{d_F} > Y_F^T$ and $\sigma_{11} > 0$, then $d_F = 1$ (rupture in traction) (3)
- If $Y_{d_F} > Y_F^C$ and $\sigma_{11} < 0$, then $d_F = 1$ (rupture in compression)

In the present study fibre fracture in traction is considered. The interaction of shear on the rupture is ignored in this model. The fibre is assumed to be transversely isotropic. Further, the computation of strain energy of the damaged fibre uses micro stresses and/or strains as obtained from Hill's concentration factors or homogenization approach.

The macroscopic or lamina level three dimensional constitutive equation for a lamina in the laminate remains the same as detailed in any standard book on the treatment of composite, for example [1]. However, with the introduction of fibre breaking damage, the corresponding Young's modulus in the fibre direction ($E_1^{0(C)}$ where superscript *0* stands for virgin value, *C* stands for composite), is degraded as

$$E_1^{(C)} = E_1^{0(C)}\left(1 - d_F\right) \qquad (4)$$

Similarly, the thermodynamic force on the lamina level can be obtained with the equation same as Eq. (2) with the micro-stress components replaced by the macro or lamina level stress components and all other material constants replaced by the composite/lamina material or effective constants.

3. RESULTS

We have used T300-carbon as the fiber and epoxy as the surrounding matrix. The properties of T300-carbon fibre are as tabulated below in Table 1.

Table 1: Elastic properties of T300 carbon fibre.

Elastic constant	E_1	E_2	G_{12}	ν_{12}	ν_{23}
Value	230 GPa	15 GPa	15 GPa	0.20	0.35

The properties of epoxy are as in Table 2.

Table 2: Elastic properties of expoxy.

Elastic constant	E_1	ν_{12}
Value	3.45 GPa	0.35

T300-carbon is assumed to be transversely isotropic in nature, whereas epoxy is assumed to be isotropic. The sources of the values are the data provided by the organizers of the World Wide Failure Exercise ([8] and references therein). The equivalent properties of composite are computed using homogenization theory. For the above fibre and matrix properties the composite properties are given in Table 3. These properties are used as the effective composite properties during this analysis.

Table 3: Equivalent properties of composite using homogenization theory with no damage.

Elastic constant	E_1^0	$E_2^0 = E_3^0$	$G_{12}^0 = G_{13}^0$	G_{23}^0	$\nu_{12}^0 = \nu_{13}^0$	ν_{23}^0
Value	117.83 GPa	9.13 GPa	4.008 GPa	2.84 GPa	0.27	0.33

The thermodynamic force corresponding to the final fracture, Y_F^T is computed using the ultimate tensile strength of Young's modulus along the fibre direction. The ultimate tensile strength of the this carbon fibre is 3654 *MPa* [1]. The value of Y_F^T is 22.7 *MPa*.

The stress and strain concentration factors are obtained by analyzing the RVE as given in Appendix using Hill's concentration factors and homogenization approach, respectively. For obtaining Hill's stress concentration factors RVE is modeled in Abaqus Standard with 3345, 2249 and 2912 as numbers of elements in fibre, interphase and matrix, respectively and 763, 742 and 909, respectively as the number of nodes in the finite element computations. A finite element code is written for obtaining the strain concentration factors using homogenization. In both cases a linear tetrahedron elements were used (C3D8R in case of Abaqus model). Once the stress or strain concentration factors are obtained, the lamina macro stress or strain state is used to compute the micro stress or strain state. The microstress can be obtained from microstrain using constitutive equation for the fibre.

The proposed damaged model is incorporated in Abaqus Standard using user material subroutine UMAT. The study is carried out for a lamina of 0º orientation under traction. The dimensions of the lamina are given in Figure 1. The thickness of the lamina is 0.1 *mm*. The linear hexahedral elements with reduced integration scheme are used to mesh the lamina. The total number elements and number of nodes are 400 and 902, respectively.

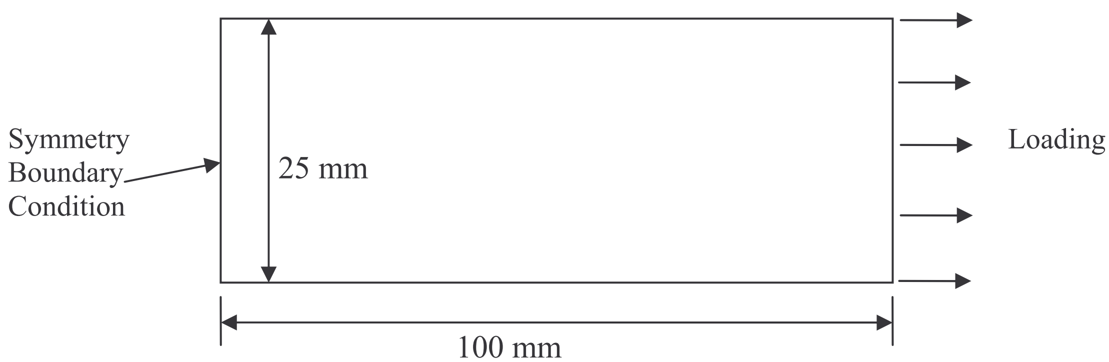

Figure 1: Dimensions and boundary conditions for 0º lamina under traction.

A. Hill's Stress Concentration Factors Approach

The longitudinal stress – longitudinal strain relationship for the lamina obtained is shown in Figure 2. The ultimate stress is predicted as 1704 *MPa* and the corresponding ultimate longitudinal strain is 1.46%. For a typical composite of *T300* fiber and epoxy the value reported in open literature is 1510 *MPa* and 1.5% of strain, respectively (for example see [1]). Further, the Young's modulus along fibre direction $\left(E_1\right)$ from this relationship is computed to be 117.35 *GPa*. This value is very close to the value reported in Table 3. The evolution of fibre breaking damage in lamina with longitudinal strain is shown in Figure 3.

B. Homogenization Approach

In the case of a uniaxial tensile problem the longitudinal stress – longitudinal strain relationship for the lamina is as shown in the Figure 4. This relationship is linear as there is no other damage mechanism other than fibre breaking, which is brittle. The ultimate longitudinal tensile stress was found to be 1655 *MPa* and the ultimate longitudinal strain was found to be 1.41%, a value lower than the one calculated using the Hill's stress concentration factors. However, this value is still higher than the reported value of ultimate stress in open literature as 1510 *MPa*. The evolution of fibre breaking damage in lamina with the longitudinal strain is shown in Figure 5. Further, this relationship gives the Young's modulus along fibre direction $\left(E_1\right)$. The value computed from this relation is 117.38 *GPa*, which is very close to the value reported in Table 3.

It should be noted that here only fibre breaking damage is modeled. The effect of other damage modes on the reduction of modulus along the fibres is not considered. For example, the fibre-matrix debonding also causes the reduction in modulus along fibre direction. The values are bound to decrease once we introduce the interaction of various modes of damage. Further, the longitudinal stress – longitudinal strain relationship will be a nonlinear one.

In both the cases, a delay effect has been incorporated for the damage evolution as given in [6]. However, it should be noted that present study is carried out assuming quasi-static loading. The delay effects give a more realistic damage evolution by ensuring that the damage is not completely brittle as initially assumed and that the rise of damage parameter is not instantaneous.

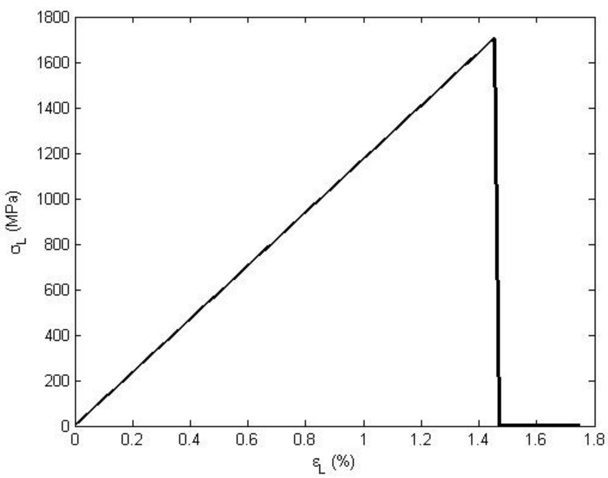

Figure 2: Variation of longitudinal stress with longitudinal strain for 0º lamina under traction along fibre direction (Hill's stress concentration factor approach).

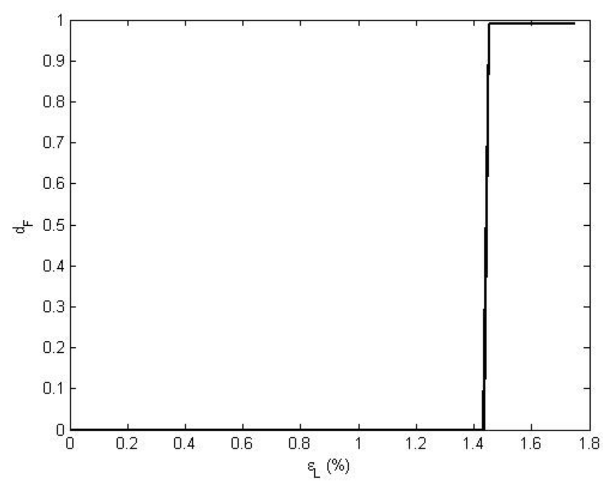

Figure 3: Fibre breaking damage evolution (d_F) with longitudinal strain for 0º lamina under traction along fibre direction (Hill's stress concentration factor approach).

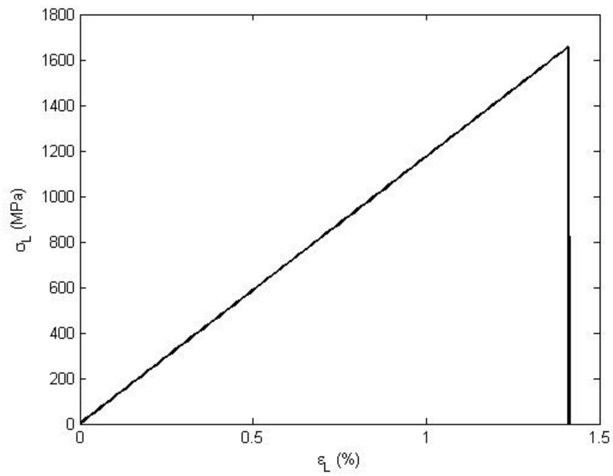

Figure 4: Variation of longitudinal stress with longitudinal strain for 0º lamina under traction along fibre direction (homogenization approach).

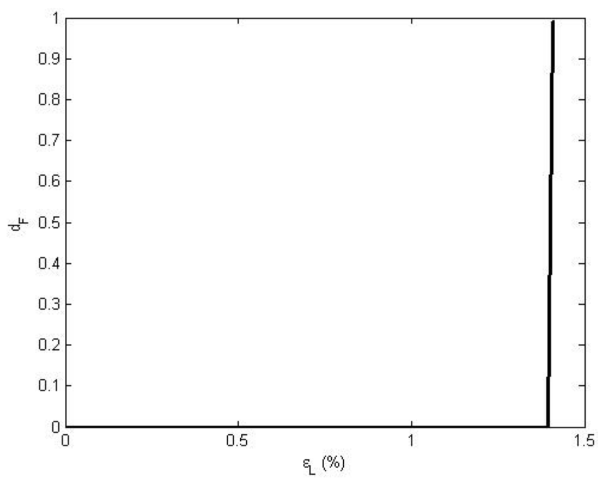

Figure 5: Fibre fracture damage (d_F) evolution with longitudinal strain in 0º lamina under traction along fibre direction (homogenization approach).

4. CONCLUSIONS

A continuum damage model is built for the prediction of fibre failure in unidirectional fibrous composites and is tested for carbon fibre/epoxy matrix under quasi static loading conditions. The model is based on Hill's concentration factors approach and homogenization theory, which include a right circular cylindrical fibre in a cubicle matrix as an RVE. The developed model is incorporated in commercial finite element analysis software – Abaqus/CAE. The preliminary results are presented here.

The results obtained for the ultimate fibre fracture in tension using brittle criterion are in good agreement with the results reported in literature. The Hill's stress concentration factors approach gives the upper bound while homogenization approach gives the lower bound of the tensile strength of lamina.

APPENDIX

To analyze the state of stress and failure in a fibrous composite we divided the composite into three phases as: Fibre, matrix and interphase as a thin layer between fibre and matrix as shown in Figure A1.

The local or micro strains/stresses in these phases are obtained using Hill's concentration factors and homogenization as follows.

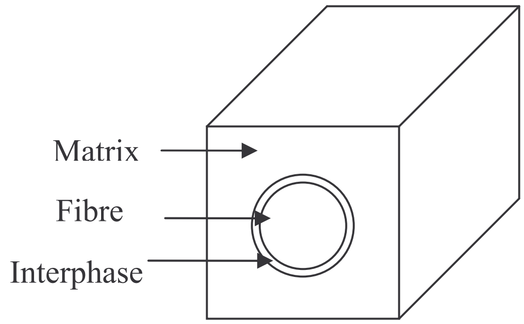

Figure A1: A sample RVE with Fibre, Interphase and Matrix as three separate phases.

A Hill's Concentration Factors

This approach of concentration factors was introduced by Hill [1]. The idea behind this approach is that the local strains at any point ***(x)*** can be written in terms of location dependent strain concentration factors $A_{ijkl}(\mathbf{x})$ and the composite average strains $\overline{\varepsilon}_{ij}$. Similarly, the local stress at any point ***(x)*** can be written in terms of location dependent stress concentration factors $B_{ijkl}(\mathbf{x})$ and the composite average stress $\overline{\sigma}_{ij}$ in the following form.

$$\begin{aligned} &\varepsilon_{ij}^{(F)}(\mathbf{x}) = A_{ijkl}^{(F)}(\mathbf{x})\overline{\varepsilon}_{kl}, \quad \varepsilon_{ij}^{(M)}(\mathbf{x}) = A_{ijkl}^{(M)}(\mathbf{x})\overline{\varepsilon}_{kl}, \quad \varepsilon_{ij}^{(I)}(\mathbf{x}) = A_{ijkl}^{(I)}(\mathbf{x})\overline{\varepsilon}_{kl} \\ &\sigma_{ij}^{(F)}(\mathbf{x}) = B_{ijkl}^{(F)}(\mathbf{x})\overline{\sigma}_{kl}, \quad \sigma_{ij}^{(M)}(\mathbf{x}) = B_{ijkl}^{(M)}(\mathbf{x})\overline{\sigma}_{kl}, \quad \sigma_{ij}^{(I)}(\mathbf{x}) = B_{ijkl}^{(I)}(\mathbf{x})\overline{\sigma}_{kl} \end{aligned} \tag{A1}$$

where, the superscripts *(F)*, *(M)* and *(I)* denote fibre, matrix and interphase, respectively.

The local strains and stresses can be integrated over their respective volumes to get the average quantities in terms of phase-average concentration factors (also called as Hill's concentration factors). Thus,

$$\begin{aligned} &\overline{\varepsilon}_{ij}^{(F)}(\mathbf{x}) = \overline{A}_{ijkl}^{(F)}(\mathbf{x})\overline{\varepsilon}_{kl}, \quad \overline{\varepsilon}_{ij}^{(M)}(\mathbf{x}) = \overline{A}_{ijkl}^{(M)}(\mathbf{x})\overline{\varepsilon}_{kl}, \quad \overline{\varepsilon}_{ij}^{(I)}(\mathbf{x}) = \overline{A}_{ijkl}^{(I)}(\mathbf{x})\overline{\varepsilon}_{kl} \\ &\overline{\sigma}_{ij}^{(F)}(\mathbf{x}) = \overline{B}_{ijkl}^{(F)}(\mathbf{x})\overline{\sigma}_{kl}, \quad \overline{\sigma}_{ij}^{(M)}(\mathbf{x}) = \overline{B}_{ijkl}^{(M)}(\mathbf{x})\overline{\sigma}_{kl}, \quad \overline{\sigma}_{ij}^{(I)}(\mathbf{x}) = \overline{B}_{ijkl}^{(I)}(\mathbf{x})\overline{\sigma}_{kl} \end{aligned} \tag{A2}$$

The composite average strains are multiplied with the strain concentration factors to get the individual state of micro strains. Similarly, composite average stresses are multiplied with the stress concentration factors to get the individual state of micro stress.

To compute the phase averaged stress concentration factors the following procedure is followed.

The stress concentration factors were calculated using the strength of materials approach. For the finite element analysis linear tetrahedral elements with reduced integration scheme are used. The arbitrary macro state of stress $\left(\overline{\sigma}_{ij}\right)$ on RVE is formulated in terms of linear combination of six unit stresses. These six unit stresses are given as:

$$\overline{\sigma}_{ij}^{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{\sigma}_{ij}^{22} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{\sigma}_{ij}^{33} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\overline{\sigma}_{ij}^{12} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{\sigma}_{ij}^{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \overline{\sigma}_{ij}^{23} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{A3}$$

Further, it should be noted that the symmetry of the stresses has been assumed for the remaining stress components. The six fundamental problems with unit stresses are solved in Abaqus to give the pointwise (here, at the centroid of each element) values of stresses. Thus, the states of stress in the three phases are known. Then the volume averaging of these stresses over respective phases yields the phase averaged stress concentration factors.

Remark: Only stress concentration factors for each phase are computed using this method. They are further used to find the state of microstresses in the computation of damage of fibre breaking only.

B Homogenization

The homogenization theory is developed from studies of partial differential equations with rapidly varying coefficients. This theory is based on the two assumptions: the fields vary on multiple scales due to existence of a microstructure and microstructure is spatially periodic. The field variables are approximated by an asymptotic expansion as:

$$u_i^\eta(x_i, y_i) = u_{0i}(x_i, y_i) + \eta u_{1i}(x_i, y_i) + \eta^2 u_{2i}(x_i, y_i) + \cdots \tag{B1}$$

where u_i^η is the exact value of the field variable, u_{0i} is the macroscopic value of the field variable, u_{1i}, u_{2i} etc. are the perturbations in the field variable due to the microstructure, x_i is the global coordinate y_i is the local coordinate. The macro coordinates are related to the micro coordinates by the relation

$$y_i = \frac{x_i}{\eta} \tag{B2}$$

where, η is the ratio between the RVE size and the size of the macroscopic region in which it exists. Now, the small deformation strain tensor can be written as

$$\varepsilon_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i^\eta}{\partial x_j} + \frac{\partial u_j^\eta}{\partial x_i}\right) = \frac{1}{2}\left[\left(\frac{\partial u_{0i}}{\partial x_j} + \frac{\partial u_{0j}}{\partial x_i}\right) + \left(\frac{\partial u_{1i}}{\partial y_j} + \frac{\partial u_{1j}}{\partial y_i}\right) + \eta\left(\frac{\partial u_{1i}}{\partial x_j} + \frac{\partial u_{1j}}{\partial x_i}\right)\right] + h.o.t. \tag{B3}$$

which can be simplified neglecting the terms of $o(\eta)$ and higher order as

$$\varepsilon_{ij} = \overline{\varepsilon}_{ij} + \varepsilon_{ij}^*, \quad \overline{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_{0i}}{\partial x_j} + \frac{\partial u_{0j}}{\partial x_i}\right), \quad \varepsilon_{ij}^* = \frac{1}{2}\left(\frac{\partial u_{1i}}{\partial y_j} + \frac{\partial u_{1j}}{\partial y_i}\right) \tag{B4}$$

where, ε_{ij} is local or microstructural strain tensor, $\overline{\varepsilon}_{ij}$ is average or macroscopic strain tensor and ε_{ij}^* is fluctuating strain tensor. The fluctuating strain tensor is assumed to vary periodically. Now for the virtual displacement or weak form of the equilibrium equations, the virtual displacement $(\mathbf{v})$ and hence the virtual strain $\varepsilon_{ij}(\mathbf{v})$ are also expressed as asymptotic functions of x_i and y_i as

$$\varepsilon_{ij}(\mathbf{v}) = \frac{1}{2}\left(\frac{\partial v_i^\eta}{\partial x_j} + \frac{\partial v_j^\eta}{\partial x_i}\right) = \frac{1}{2}\left[\left(\frac{\partial v_{0i}}{\partial x_j} + \frac{\partial v_{0j}}{\partial x_i}\right) + \left(\frac{\partial v_{1i}}{\partial y_j} + \frac{\partial v_{1j}}{\partial y_i}\right) + \eta\left(\frac{\partial v_{1i}}{\partial x_j} + \frac{\partial v_{1j}}{\partial x_i}\right)\right] + h.o.t. \tag{B5}$$

Further,

$$\varepsilon_{ij}(\mathbf{v}) = \varepsilon^0_{ij}(\mathbf{v}) + \varepsilon^1_{ij}(\mathbf{v}), \quad \varepsilon^0_{ij}(\mathbf{v}) = \frac{1}{2}\left(\frac{\partial u_{0i}}{\partial x_j} + \frac{\partial u_{0j}}{\partial x_i}\right), \quad \varepsilon^1_{ij}(\mathbf{v}) = \frac{1}{2}\left(\frac{\partial u_{1i}}{\partial y_j} + \frac{\partial u_{1j}}{\partial y_i}\right) \tag{B6}$$

The weak form of the equilibrium equations is given by

$$\int_{\Omega^\eta} C_{ijkl}\varepsilon_{ij}(\mathbf{v})\varepsilon_{kl}(\mathbf{u})\, d\Omega^\eta = \int_\Gamma t_i v_i d\Gamma \tag{B7}$$

Here, Ω^η is total macroscopic plus microscopic domain of the composite. The tractions t_i and the boundary displacements, if any, are applied only on the macroscopic boundaries of the composite. The expanded form of the above equation gives

$$\int_{\Omega^\eta} C_{ijkl}\left(\varepsilon^0_{ij}(\mathbf{v}) + \varepsilon^1_{ij}(\mathbf{v})\right)\left(\bar{\varepsilon}_{kl}(\mathbf{u}) + \varepsilon^*_{kl}(\mathbf{u})\right) d\Omega^\eta = \int_\Gamma t_i v_i d\Gamma \tag{B8}$$

It should be noted that the virtual displacement $(\mathbf{v})$ is any arbitrary displacement. It can be chosen to vary on macroscopic or microscopic level. If we choose $(\mathbf{v})$ to vary only on microscopic level and constant on macroscopic level then we get the microscopic equilibrium equation as follows:

$$\int_{\Omega^\eta} C_{ijkl}\varepsilon^1_{ij}(\mathbf{v})\left(\bar{\varepsilon}_{kl}(\mathbf{u}) + \varepsilon^*_{kl}(\mathbf{u})\right) d\Omega^\eta = 0 \tag{B9}$$

as $\varepsilon^0_{ij}(\mathbf{v}) = 0$ for the chosen variation of virtual displacement. However, if we choose $(\mathbf{v})$ to vary only on macroscopic level and constant on microscopic level then we get the macroscopic equilibrium equation as follows:

$$\int_{\Omega^\eta} C_{ijkl}\varepsilon^0_{ij}(\mathbf{v})\left(\bar{\varepsilon}_{kl}(\mathbf{u}) + \varepsilon^*_{kl}(\mathbf{u})\right) d\Omega^\eta = \int_\Gamma t_i v_i d\Gamma \tag{B10}$$

Here, $\varepsilon^1_{ij}(\mathbf{v}) = 0$ for the chosen variation of virtual displacement. Since, ε^*_{kl} varies periodically, Eq. (B9) and Eq. (B10) may be simplified assuming η approaching zero in the limit as

$$\int_\Omega \frac{1}{|V_{RVE}|}\left[\int_{V_{RVE}} C_{ijkl}\varepsilon^1_{ij}(\mathbf{v})\left(\bar{\varepsilon}_{kl}(\mathbf{u}) + \varepsilon^*_{kl}(\mathbf{u})\right) dV_{RVE}\right] d\Omega = 0 \tag{B11}$$

$$\int_\Omega \frac{1}{|V_{RVE}|}\left[\int_{V_{RVE}} C_{ijkl}\varepsilon^0_{ij}(\mathbf{v})\left(\bar{\varepsilon}_{kl}(\mathbf{u}) + \varepsilon^*_{kl}(\mathbf{u})\right) dV_{RVE}\right] d\Omega = \int_\Gamma t_i v_i d\Gamma \tag{B12}$$

The Eq. (B11) to be true, the integration term over the RVE should be zero. This lead to the following condition:

$$\int_{V_{RVE}} C_{ijkl}\varepsilon^1_{ij}(\mathbf{v})\varepsilon^*_{kl}(\mathbf{u})\, dV_{RVE} = -\int_{V_{RVE}} C_{ijkl}\varepsilon^1_{ij}(\mathbf{v})\bar{\varepsilon}_{kl}(\mathbf{u})\, dV_{RVE} \tag{B13}$$

Here, in general, the strain $\bar{\varepsilon}_{ij}$ is not known. However, for a linear problem any arbitrary $\bar{\varepsilon}_{ij}$ can be written as linear combination of unit strains similar to unit stresses given in Eq. (A3) for Hill's concentration factors case. Substituting these unit strains in right hand side of Eq. (B13) we get the stress tensor

$$\sigma^{*kl}_{ij} = C_{ijmn}\bar{\varepsilon}^{kl}_{mn}$$

Now, we solve the resulting auxilliary problem as

$$\int_{V_{RVE}} C_{ijpm}\varepsilon^1_{ij}(\mathbf{v})\varepsilon^{*kl}_{pm}(\mathbf{u})\, dV_{RVE} = -\int_{V_{RVE}} \varepsilon^1_{ij}(\mathbf{v})\sigma^{*kl}_{ij}\, dV_{RVE} \tag{B14}$$

The periodicity of the strain field ε_{ij}^{*kl} is obtained by constraining equal displacements on opposite sides of RVE. Once the ε_{ij}^{*kl} is determined, the solution to Eq. (B13) is recovered by $\varepsilon_{ij}^{*} = -\varepsilon_{ij}^{*kl}\bar{\varepsilon}_{kl}$. Now the relationship between the local RVE strain and the average strain is written as follows:

$$\varepsilon_{ij}^{*} = M_{ijkl}\bar{\varepsilon}_{kl}, \quad M_{ijkl} = \frac{1}{2}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) - \varepsilon_{ij}^{*kl} \tag{B15}$$

where δ_{ij} is Kronecker delta. The relationship between local RVE strain and the average strain is substituted into the macroscopic equilibrium equations to give

$$\int_{\Omega} \frac{1}{|V_{RVE}|}\left[\int_{V_{RVE}} C_{ijkl} M_{klpm} dV_{RVE}\right] \varepsilon_{ij}^{0}(\mathbf{v})\bar{\varepsilon}_{pm}(\mathbf{u})\, d\Omega = \int_{\Gamma} t_i v_i d\Gamma \tag{B16}$$

Here, it should be noted that the terms ε_{ij}^{0} and $\bar{\varepsilon}_{pm}$ are outside the integration over the RVE as they represent the average strain, which is constant, over RVE. The homogenization approach also gives the equivalent properties of the composite laminate. The effective stiffness tensor may be defined as

$$\bar{C}_{ijkl} = \frac{1}{|V_{RVE}|}\int_{V_{RVE}} C_{ijpm} M_{pmkl} dV_{RVE} \tag{B17}$$

Using this effective stiffness tensor, the final form of macroscopic equilibrium equation may be written as

$$\int_{\Omega} \bar{C}_{ijkl}\varepsilon_{ij}^{0}(\mathbf{v})\bar{\varepsilon}_{kl}(\mathbf{u})\, d\Omega = \int_{\Gamma} t_i v_i d\Gamma \tag{B18}$$

It is important to note that the effective stiffness tensor obtained from Eq. (B17) is independent of size of RVE due to periodicity assumption used in obtaining M_{ijkl}.

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