Region-by-region Modeling for Layered Composites

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Summary

In the present paper, a region-by region model for layered composites is presented. The model is compared with equivalent single layer, intermediate and layerwise models. It is shown that the region-by-region model is accurate as that of layerwise model and computationally very economic compared to layerwise model. The efficacy of this model is demonstrated through stress profiles for a domain with multi-material regions.

Introduction

Layered composites are increasingly used in Aerospace industry. Many dimensionally reduced models have been proposed in literature for the study of these structures[1], [2]. The computational cost for these models is independent of number of layers. A major drawback of these models is that the transverse stresses obtained using these models are not accurate. The equilibrium based post-processing approach is used to extract these stress components. This approach is quite effective in most cases. For domains with unsymmetry in layup, existing delamination, ply damage these models are ineffective. Layerwise models are often used to alleviate this issue. But this approach is compute intensive.

Generally, the strong three-dimensional effects are localized[] in the vicinity of boundaries (boundary layer), vertices (vertex singularities), edge (edge singularities), parts of laminae (damage) and local interfaces (delaminations). In these cases beyond local neighborhood of the regions of unsmooth behavior, the solution can be effectively represented using any of the families of dimensionally reduced models. Hence, it will be desired to enrich the approximation only in the regions where the solution is unsmooth. This is achieved in this paper by the proposed region-by-region model. As shown in this paper, this approach leads to tremendous saving in computational cost and gives accurate representation of the state of stress everywhere in the domain.

Plate Model Formulation

The displacement component u^l, for an element in the l^{th} layer, is given as

$$u^l(x,y,z) = \sum_{j=1}^{(p_{xy}+1)(p_{xy}+2)} \sum_{k}^{p_z^u+1} u_{jk} N_j^l(x,y) M_k^l(z) \qquad (1)$$

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where p_{xy} and p_z^u are the in-plane and transverse approximation order (for component u^l) and $N_j(x, y)$ and $M_k(z)$ are in-plane and transverse approximation functions, respectively. Similarly the other components v^l and w^l can be expressed. The layerwise model is denoted by $LMp_{xy}p_z{}^u p_z{}^v p_z{}^w$. In intermediate model some of the layers are lumped together while in the vicinity of the critical layer the layers are left as such. The Intermediate model is denoted by $IMp_{xy}p_z{}^u p_z{}^v p_z{}^w$. For equivalent single layer model all the layers are lumped together to form a single layer. It is to be noted that in all the transverse lumped layers (individual layers in case of layerwise model) the order of transverse approximation for a displacement component is same.

Constrained Approximation

In this section the concept of constrained approximation will be discussed. The ideas are generalization of the concept introduced in [9]. In order to fix ideas let us consider a one-dimensional example. Let us take on interval (0, L) with one element, as shown in Fig. 1 (a). Let us also assume that piecewise linear basis functions (i.e. p=1) are defined over this mesh.

Let

$$v(z) = \sum_{i=1}^{p+1} a_i M_i(z) \tag{2}$$

Be the representation of a function over this domain. Here, M_1 (z) are the linear basis functions defined as shown in Fig. 1(a). Let us now subdivide this element into two equal sub-elements, let the function v (z), given above, be represented in terms of the piecewise linear basis functions (as shown in Fig. 1(b)).

As

$$v(z) = \sum_{i=1}^{2p+1} a_i M_i(z) \tag{3}$$

Where, $\overline{M}_1\ (z)$ are the piecewise linear basis functions defined over the new mesh. Since both equations 5 and 6 represent the same function, the coefficients $\overline{a}_1$ can be expressed in terms of the coefficients a_j. It is obvious that

$$\overline{a}_1 = a_1; \quad \overline{a}_2 = (a_1 + a_2)/2; \quad \overline{a}_3 = a_2 \tag{4}$$

Similarly, the representation of v(z) over any finer mesh can be obtained in terms of the representation over the coarser mesh, with the new fine mesh coefficients

$\overline{a}_j$ constrained by the values of the coefficients a_i for the coarser mesh. This can be easily extended to any p-order approximation defined over the coarser and fine meshes. As shown below, the transverse representation of the finite element solution is defined over a group. However, the basic building block in the analysis is the individual three-dimensional element τ_{3D}. Hence, the approach given above will be employed to represent the element degrees of freedom in terms of the group degrees of freedom.

The region-by-region model is denoted by $RRMp_{xy}p_z^u p_z^v p_z^w$.

This approach is generalization of the planar constrained approximation approach of[4] and the *h-d* approach of[5], given for homogeneous materials.

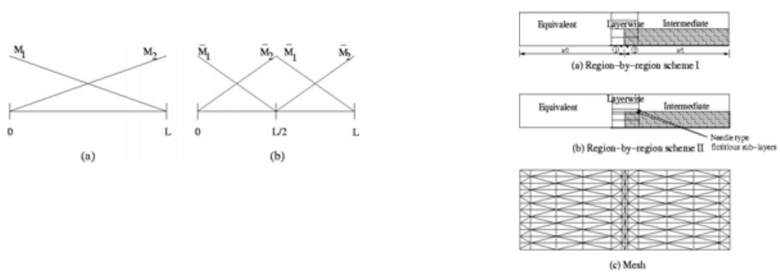

Fig. 1 (a), (b) Linear constrained approximations (c), (d), (e) Beam geometry and meshing

Domain with Multi-material Region

Let us take the rectangular domain of Figure 1(e). The dimension of the plate are *a*=100 mm, *b*=10 mm and *t*=0.508 mm. The plate is clamped along the edge *x*= 0 and is subjected to a uniform transverse load of intensity q_0=1N/mm^2. The plate has four lamina with [165/-165/165-165] in the region $0 \le x \le a/2, 0 \le y \le b.$ the material properties for each lamina are as given in[6]. In the region $a/2 \le x \le a, 0 \le y \le b,$ the plate has a bottom layer of epoxy (with E_{11}=E_{22}=E_{33}=4600MPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.36$ for $-t/2 \le z \le 0$. For $z \ge 0,$ two [165/-165] lamina with material properties given in[5] are present (see Figure 1(d), (e)). The plate essentially mimics a L-shaped domain in two-dimensions for this domain the exact solution will have an edge singularity along the line given by *x=a/2*, *z=0*. To solve this problem we will use *LM3332*, *EQ3332* models and the region-by-region schemes given by:

1. *RR-U* (Figure 2(c)): *EQ3112* in region 1, *LM3332* model in region 2 and *IM3332* model in region 3.

2. *RR-G* (Figure 2(d): *EQ3112* in region 1, *LM3332* model in region 2, with geometrically graded sublaminae near *z*=0 and *IM3332* model in region 3.

For the *IM3332* model, the epoxy layer is taken as one layer and the top two [165/-165] layers are lumped into a second equivalent layer.

Remark: The two-dimensional mesh is as shown in Figure 2(e), with geometrically graded elements in the vicinity of the line *x=a/2*. This along with *RR-G* scheme leads to needle shaped elements in the vicinity of the singular edge. Elsewhere, the reduced models are used.

Displacement Analysis:

The normalized displacement components, along a cutting line given by *y=b/2*, *z=0*, is plotted in Figure 2(a), (b), (c). The normalization of the displacement components is done as $(\bar{u}, \bar{v}, \bar{w}) = E_{22} / \left(tS^3\right)\left(u_1, u_2, u_3\right)$.

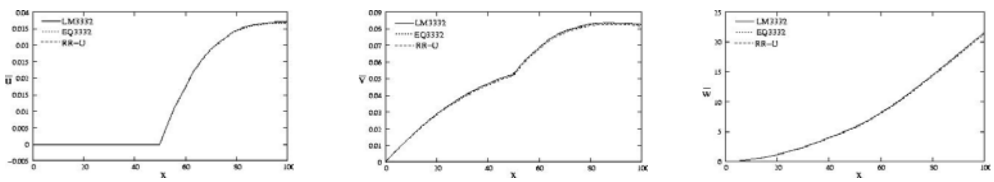

Fig. 2 *u, v* and *w* displacement variation

From the figure we observe that:

1. The *LM3332*, *EQ3332* and *RR-U* strategies give almost identical displacement profiles.

2. The displacements have a sharp change at *x=a/2*. This is because the effective flexural rigidity reduces significantly beyond $x = a/2$. The sharp gradient in the solution in the vicinity of *x=a/2* is apparent.

From the displacement profiles, all the models seen to be very good. However, the state of stress will be domainantly three-dimensional in nature in the vicinity of the singular edge the equivalent (or dimensionally reduced) models will not work. In order to demonstrate this, we study the variation of the stress components next.

Stress Analysis:

The through thickness variation of the stresses components is given for the point: $(50.1, 50, \bar{z})$. The direct transverse stress σ_{zz} obtained using *EQ3332*, *LM3332* and *RR-U* models are shown in Figure 3(a) while the post-processed transverse stresses are shown in Figure 3(b).

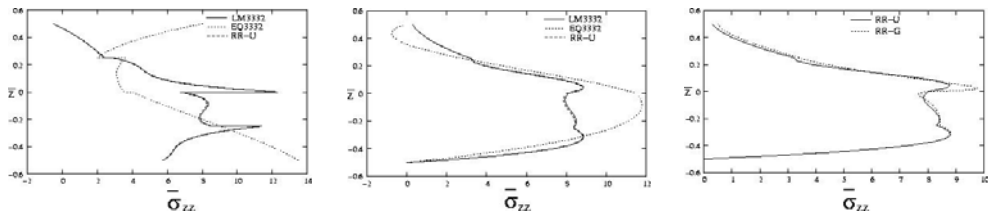

Fig. 3 Variation of σ_{zz} (a) Direct stress (b) equilibrium Stress (c) Equilibrium stresses with and without graded mesh

From the figures we observe that:

1. The *RR-U* strategy gives stress values that are very close to those obtained by the *LM3332* model.
2. The stress component, obtained by the *EQ3332* model is quantitatively and qualitatively different from those obtained by the *LM3332* model
3. The directly computed transverse stress is far from zero (desired value) at the top and bottom of the laminate. With post-processing the violation reduces significantly. However, the *EQ3332* model is still different from the *LM3332* model.

Here, the *LM3332* model was our benchmark since the *EQ3332* model cannot take care of unsmooth ness in the z-direction; it gave significantly different results, as compared to the *LM3332* model. The *RR-U* scheme was very accurate.

The *LM3332* model gives transverse representation layer by layer. In order to resolve the unsmoothness in the solution better, graded sublaminae were used in the *RR-G* model in the vicinity of the singular edge. The *RR-G* strategy is compared with the *RR-U* strategy in Figures 3(c).

From the figure we observe that:

1. Some changes in the stress profiles are observed when the *RR-G* model is used.
2. For *RR-G* strategy, a reduction in the jumps in the direct transverse stresses is observed at interlaminar interfaces.
3. The post-processed transverse stress is obtained by both the *RR-G* and *RR-U* strategies are close.

The number of unknowns for *LM3332*, *EQ3332*, *RR-U* and *RR-G* models are 47915, 15059, 26899 and 33411, respectively. Thus, the layerwise model is compute intensive and region-by-region model is quite cheaper as compared to layerwise model.

The example clearly demonstrates the ineffectiveness of the *EQ* models. It further demonstrates the effectiveness of the *RR-U* and *RR-G* strategies. Here the singular behaviors may be weak and hence the difference between the *RR-G* and *RR-U* strategies is small. However, coupled with suitable a posteriori error estimators (for modeling and discritization error), the *RR-G* strategy will be effective in resolving any of the three-dimensional singular behavior.

Conclusion

1. The layerwise model is very accurate but compute intensive.
2. The concept of region-by-region model is implemented successfully. The concept allows for enriched approximation along with sublayer concept in the *z*-direction.
3. In the case of material dissimilarity, singularity the region-by-region model can effectively capture these three-dimensional effects.
4. The region-by-region model is as accurate as layerwise model and computational cheaper than the layerwise model.

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