Continuum Damage Mechanics Based Response Prediction in Laminated Composites under Static Damage

P. M. Mohite[1]
[1]Department of Aerospace Engineering, Indian Institute of Technology Kanpur, India-208016
Email: mohite@iitk.ac.in

Abstract: In the present paper continuum damage based meso-model proposed by [1, 2] is used to predict the response of initially damaged laminated composites. The model is based on three foundations of meso-model, internal variable approach and method of local state. The present model is refined by replacing the interface as a very thin layer of resin. Further, this layer uses higher order approximation in transverse direction for displacement field over linear variation used in original model. This model has been implemented in the generalized layerwise finite element model developed by author and his co-researcher [3]. It is seen that the fibre failure mode is dominant comapared to other failure modes in giving significant difference in transverse displacement signature profiling whereas the delamination mode has weak effect. The in-plane strains also show similar behaviour. Here, the results are presented for different shapes and sizes of damage under the transverse load only.

Keywords: Continuum damage meso-model; Static damage; Transverse load; Layerwise finite element model

1. Introduction

Unidirectional fiber-reinforced composites are widely used in aerospace applications. They are used for the critical structural applications. These structures are subjected to complex aerodynamic and service loads and exhibit progressive failure and hence deterioration of elastic moduli. The mechanics by which failure occurs are numerous. These can be broadly categorized as ply level failure mechanisms like brittle fibre failure, fibre matrix debonding, and matrix cracking; and delamination. A exhaustive literature is available for the initiation of damage in the laminates.

The continuum based damage meso-model is capable of predicting both initiation and growth of damage in these laminates. Further, it is also capable of predicting delamination initiation and its growth. The concept was first introduced by Kachanov [4] for creep damage in metal. This concept is taken by Ladevèze and his co-workers and extended to laminated composites. In the following section, the details of the meso-model are presented for the sake of completeness.

2. Meso-Model for Laminated Composites

The concept is based on three foundations as
1) *Meso-scale*: In this approach, the laminate is treated as set of alternate homogeneous ply and a very thin interlaminar interface. The approach of meso-model enables the handling of ply level damage and interfacial damage (delamination). The interface is a two dimensional entity, which transfers displacements and forces from one ply to another ply.
2) *Internal variable Approach*: The effect of damage on mechanical behaviour through degradation of the material's elastic moduli and
3) *The method of local state*: It relates damage to thermodynamic forces associated with strain energy.

2.1 Elementary layer Damage Meso-model

The form of the damage meso-model of the layer involved in this study is based on the assumption that damage within the layer is governed explicitly by the in-plane stress components, and also that layer damage mechanisms solely affect the in-plane stiffness components. It is assumed that all forms of damage provoked by inter-laminar stresses (i.e. delamination) are completely taken into account by the interface. The damaged strain energy density in the layer is thus written as:

$$E_D = \frac{1}{2(1-d_{11})}\left[\frac{\sigma_{11}^2}{E_1^o} - \left(\frac{\nu_{12}^o}{E_1^o} + \frac{\nu^o{}_{21}}{E_2^o}\right)\sigma_{11}\sigma_{22} - \left(\frac{\nu_{31}^o}{E_3^o} + \frac{\nu_{13}^o}{E_1^o}\right)\sigma_{11}\sigma_{33} - \left(\frac{\nu_{32}^o}{E_3^o} + \frac{\nu_{23}^o}{E_2^o}\right)\sigma_{22}\sigma_{33}\right]$$
$$+\frac{1}{2}\left[\frac{<\sigma_{22}>^2_+}{E_2^o(1-d_{22})} + \frac{<\sigma_{22}>^2_-}{E_2^o} + \frac{\sigma_{33}^2}{E_3^o} + \frac{\sigma_{12}^2}{G_{12}^o(1-d_{12})} + \frac{\sigma_{13}^2}{G_{13}^o} + \frac{\sigma_{23}^2}{G_o^{23}}\right] \quad (1)$$

where the subscripts 1, 2, and 3 refer to the principal material coordinate system and, when attached to a damage variable *d*, indicate the orientation of its associated stress component (i.e. damage modes). For example, the variable d_{22} corresponds to the damage process produced by the normal stress component σ_{22}, which, as defined by the meso-model, is attributed to matrix micro-cracking and tensile failure of the fiber/matrix interface. The bracketed notation $<>_{+\text{or}-}$ indicates which term is non-zero depending upon the sign of the normal stress components. It is also important to point out that there is no sign dependence with respect to the contribution of the shear stress σ_{12}. For the damaged strain energy density described in Eq. (1) the thermodynamic force associated with tensile normal stresses σ_{22} is:

$$T_{22} = \frac{\partial << E_D >>}{\partial d_{12}} = \frac{1}{2(1-d_{22})^2}\langle\langle\frac{<\sigma_{22}>^2_+}{E_2^o}\rangle\rangle \quad (2)$$

and the thermodynamic force associated with in-plane shear stresses is:

$$Y_{12} = \frac{\partial << E_D >>}{\partial d_{12}} = \frac{1}{2(1-d_{12})^2}\langle\langle\frac{\sigma_{12}^2}{G_{12}^o}\rangle\rangle \quad (3)$$

where the symbol $\langle\langle\,\rangle\rangle$ indicates that the value is averaged through the thickness of the layer. This represents the key assumption of the meso-model, that the damaged state is uniform through the thickness of each meso-constituent. With respect to fiber damage, tensile fiber rupture is determined via a maximum strain criterion:

$$d_{11} = 0 \quad \text{if} \quad \varepsilon_{11} < \varepsilon_{11}^R \quad \text{else} \quad d_{11} = 1.0 \quad (4)$$

where ε_{11}^R corresponds to the experimentally determined strain at tensile fiber failure.

In order to prevent 'healing' of the material, $\underline{Y}_{22}$ and $\underline{Y}_{12}$ introduce the maximum value of the associated thermodynamics forces in a given load history at time *t* such that:

$$\underline{Y}_{22}(t) = \max_{\tau\le t}(Y_{22}(\tau)), \quad \underline{Y}_{12}(t) = \max_{\tau\le t}(Y_{12}(\tau)) \quad (5)$$

The idea that the damage from tension and shear mechanisms are linearly related is then introduced, as follows, via a single thermodynamic force $\hat{\underline{Y}}$:

$$\hat{\underline{Y}} = (\underline{Y_{12}}(t) + b\underline{Y_{22}}(t)) \quad (6)$$

where the material constant *b* defines the level of coupling between the two effects, a good approximation for which has been shown to be:

$$b = \frac{E_2^o}{G_{12}^o} \tag{7}$$

The values of both internal damage variables of the layer are then defined through:

$$d_{22} = \omega_{22}(\underline{\hat{Y}}) = \frac{\langle\langle \sqrt{\underline{\hat{Y}}} - \sqrt{Y_{22}^o} \rangle\rangle_+}{\sqrt{Y_{22}^c}} \quad \text{if} \quad \omega_{22}(\underline{\hat{Y}}) < 1.0, \quad \text{else} \quad d_{22} = 1.0 \tag{8a}$$

$$d_{12} = \omega_{12}(\underline{\hat{Y}}) = \frac{\langle\langle \sqrt{\underline{\hat{Y}}} - \sqrt{Y_{12}^o} \rangle\rangle_+}{\sqrt{Y_{12}^c}} \quad \text{if} \quad \omega_{12}(\underline{\hat{Y}}) < 1.0, \quad \text{else} \quad d_{12} = 1.0 \tag{8b}$$

where Y_{22}^o and Y_{22}^c are experimentally identified thresholds for the onset and ultimate failure through damage from tensile normal stress σ_{22}. Y_{12}^o and Y_{12}^c are the corresponding values for shear damage.

A further constraint is then applied to the evolution of tensile damage to model the brittle behaviour of tensile failure of the fiber/matrix interface:

$$d_{22} = \omega_{22}(\underline{\hat{Y}}) \quad \text{if} \quad \omega_{22}(\underline{\hat{Y}}) < 1.0 \quad \text{and} \quad \underline{\hat{Y}} < Y_r, \quad \text{else} \quad d_{22} = 1.0 \tag{9}$$

where Y_r is an experimentally-determined threshold which corresponds to rupture of the fiber/matrix interface in tension, and is given as:

$$Y_r = \frac{\langle \sigma_{22} \rangle_+^{2^F}}{2E_2^o(1-d_{22}^F)^2} \tag{10}$$

where the superscript F indicates a value at failure.

Finally, the concept that failure in both modes is obtained when the threshold of one damage mechanism is reached is introduced through the following constraints on the damage evolution laws:

$$d_{22} = \omega_{22}(\underline{\hat{Y}}) \quad \text{if} \quad \omega_{22}(\underline{\hat{Y}}) < 1.0 \quad \text{and} \quad \omega_{12}(\underline{\hat{Y}}) < 1.0, \quad \text{else} \quad d_{22} = 1.0 \tag{11a}$$

$$d_{12} = \omega_{12}(\underline{\hat{Y}}) \quad \text{if} \quad \omega_{12}(\underline{\hat{Y}}) < 1.0; \quad \underline{\hat{Y}} < Y_r \quad \text{and} \quad \omega_{22}(\underline{\hat{Y}}) < 1.0, \quad \text{else} \quad d_{12} = 1.0 \tag{11b}$$

2.2 Damage Meso-model of the Interface

In a form analogous to that of the layer, the damaged strain energy (per unit area) of the interface can be expressed as follows:

$$E_D^I = \frac{1}{2}\left[\frac{\langle \sigma_{33} \rangle_-^2}{E_{33}} + \frac{\langle \sigma_{33} \rangle_+^2}{E_{33}(1-d_I)} + \frac{\sigma_{32}^2}{G_{32}(1-d_{II})} + \frac{\sigma_{31}^2}{G_{31}(1-d_{III})} \right] \tag{12}$$

As with the layer definition, the contribution to the damaged strain energy from normal stress components is separated into tensile and compressive terms in order to distinguish between the mechanical effects of a crack opening versus that of a crack closing. Differentiation of Eq. (12) with respect to each damage variable then yields the following associated thermodynamic forces for each of the three considered delamination modes:

$$Y_I = \frac{1}{2}\frac{\langle \sigma_{33} \rangle_+^2}{E_{33}(1-d_I)^2}, \; Y_{II} = \frac{1}{2}\frac{\sigma_{32}^2}{G_{32}(1-d_{II})^2}, \; Y_{III} = \frac{1}{2}\frac{\sigma_{31}^2}{G_{31}(1-d_{III})^2}$$

(13)

Depending on the assumptions made regarding damage in the interface (i.e. the amount of coupling between the three delamination modes, whether or not a given mode exhibits brittle or ductile failure), a variety of damage evolution laws may be considered. The damage evolution law of the interfacial meso-model involved in this study uses the assumption that delamination growth is governed by a so-called “equivalent damage force” which couples the thermodynamic forces from all three delamination

modes into a single quantity. Approaches based upon this idea are generally referred to as isotropic damage evolution laws. This coupled thermodynamic force has the form:

$$Y_e = [Y_I^{\alpha} + (\lambda_1 Y_{II})^{\alpha} + (\lambda_2 Y_{III})^{\alpha}]^{\frac{1}{\alpha}} \qquad (14)$$
$$\underline{Y}_e(t) = \max_{\tau \le t}(Y_e(\tau))$$

where λ_1 and λ_2 are coupling parameters and α is a material parameter linked to classical fracture mechanics. It is important to note that in Eq. (14) the constraint applied is the same as that used in the damage meso-model of the layer to prevent 'healing' of the material as in Eq. (8). The other key assumption for the isotropic damage evolution laws is that, because of its small thickness, the stiffness of the interface in all three directions is reduced to zero when the threshold of one mode is reached. The three damage variables can then all be linked to a single material function $\omega(\underline{Y_e})$ which relates the coupled thermodynamic force to a single set of damage thresholds:

$$\omega(\underline{Y_e}) = \left[\frac{n}{n+1} \frac{\langle \underline{Y}_e - Y_e^o \rangle_+}{Y_e^c - Y_e^o} \right]^n \qquad (15)$$

$d_I = d_{II} = d_{III} = \omega(\underline{Y}_e)$ if $\omega(\underline{Y}_e) < 1.0$, or $d_I = d_{II} = d_{III} = 1.0$ otherwise

where Y_e^o and Y_e^c are analogous to the experimentally determined parameters used in the meso-model of the layer, and n is a material constant related to the type (brittle, n >1, or ductile , n < 1) of failure of the interface.

Remark: In the above model the interface is treated as a very thin orthotropic layer of the same material as that of the ply, with effective orientation as the bisector of the angle between the adjacent layer fibre directions [1]. In the present study the interface is modeled as a very thin layer of homogeneous matrix material.

Remark: Damage in ply is denoted by three scalars d_{11}, d_{22} and d_{12} as (d_{11}, d_{22},d_{12}).

3. Results and discussion

In this section the effect of static damage on the maximum transverse deflection and in-plane strain components is studied, for different sizes, shapes and intensities of damage, under transverse loading. The material used is high modulus M55J/M18 graphite/epoxy. Here, a [0/90/0] laminate of dimension 64X64 *mm* under transverse uniform load of intensity 0.005N/mm^2 is studied. The thickness of each ply is 0.1 *mm* and that of interface is 0.01 *mm*. The plate is simple supported along all edges. The load chosen is such that it does not initiate damage at other places. Even if damage is initiated, its size and magnitude will be negligible as compared to the initial damage taken.

3.1 Effect of Size of Static Damage on the Response

The plate has centrally located damage of (1) Size 1: 10.7X10.7 *mm* and (2) Size 2: 5.35X5.35 *mm*. The transverse deflection and in-plane strain components along lines passing through the centre of the laminate parallel to *x* axis at the top of laminate are computed. The strain components are non-dimensionalized by the maximum strain value of strain when the laminate is undamaged.

The transverse displacement and in-plane strains components for two cases of existing damage, i.e. damage in bottom ply of intensity (0.1,0.98,0.1) and damage in top ply of intensity (0.98,0.1,0.1), are shown in Fig. 1-2. In this the transverse displacement is also compared with displacement obtained in the absence of damage.

3.2 Effect of Shape of Static Damage on the Response

Here, damage of square size 1 and of circular shape with same area as of square size 1 are considered. The transverse displacement and in-plane strain components are shown in Fig. 3-4.

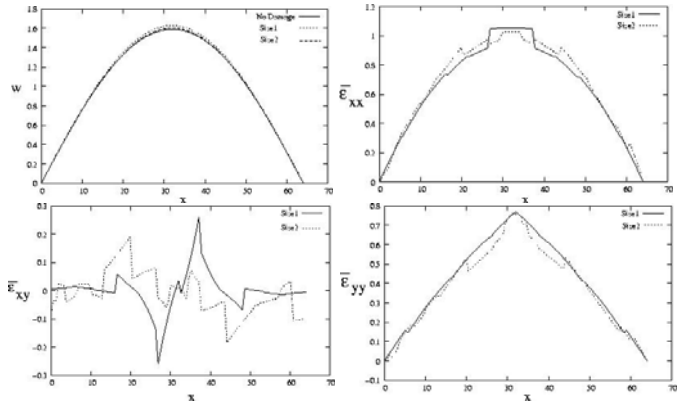

Fig. 1. Effect of size of damage. (Damage in bottom ply of intensity (0.1,0.98,0.1))

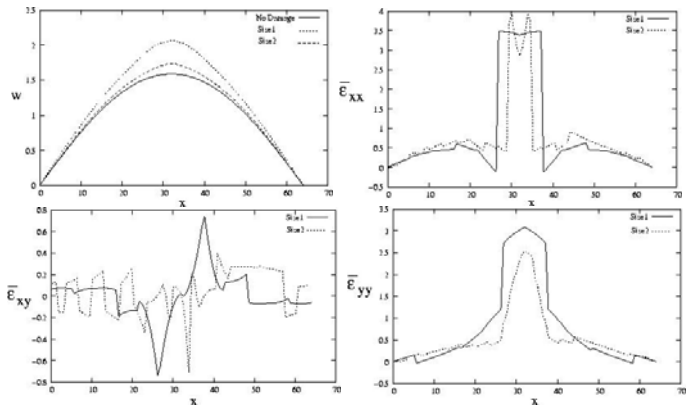

Fig. 2. Effect of size of damage. (Damage in top ply of intensity (0.98,0.1,0.1))

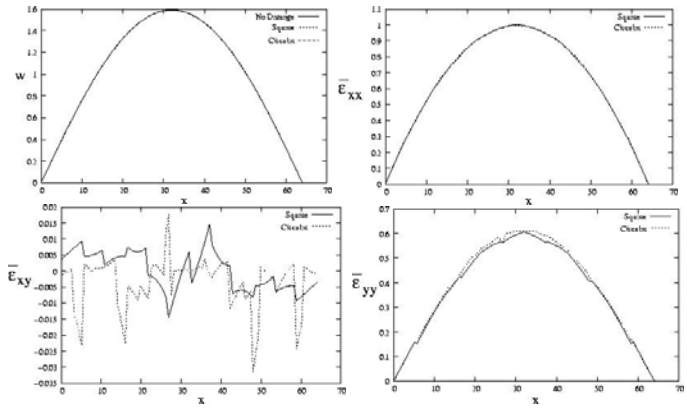

Fig. 3. Effect of shape of damage. (Delamination damage in bottom interface)

From the figures it can be observed that:

1. As the size of damage increases the transverse deflection at the centre of the laminate (where damage is located) increases. This is because the damage reduces the stiffness. Also, the strain components show similar behaviour.
2. The fibre failure mode is more dominant than fibre matrix interface debonding and matrix cracking mode. This can be seen through the change in transverse displacement in Fig. 1 and 2.
3. The change is transverse displacement for square and circular shape is not so significant. The difference for the fibre failure mode is because the circular shape cuts more fibres than square shape of same area thus, reducing more stiffness with circular shape. The strain component becomes more smoothened (see Fig. 4).
4. The delamination damage mode does not have much effect on transverse displacement and strain components (see Fig. 3). This indicates that in the presence of delamination still there is effective load transfer. The shape and size also do not have effect on the response.
5. The new damage sites are initiated in top and bottom lamina. This destroys the symmetry of profiles.

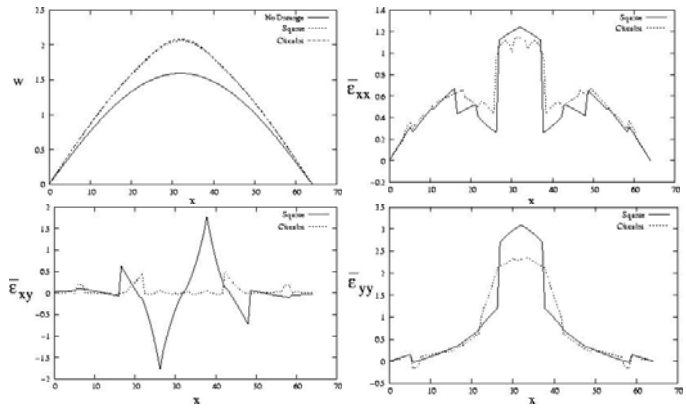

Fig. 4. Effect of size of damage. (Damage in top layer of intensity (0.98,0.1,0.1))

4. Conclusions

1. The size, shape and intensity of the damage in the ply effects the transverse deflection on the top surface of the laminate. As the size and intensity increases the deflection also increases for transverse loading.
2. The size, shape and intensity of damage in interface does not have much effect on transverse deflection on the top surface.
3. The symmetry in the response characteristics in the presence of damage is lost.
4. Due to existing damage, new damage sites will be created at other locations when a static transverse load is applied.

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