A Novel Subdomainwise Modeling Approach for Analysis of Layered Composite Structures

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Nomenclature

$\mathbf{u}$	Generalized displacement field
p_{xy}	Inplane approximation order
p_z^i	Transverse approximation order for i^{th} displacement component
$\{\sigma\}$	Stress vector
$\{\epsilon\}$	Strain vector

I. Introduction

Laminated structures are vastly used in automobile, aerospace and other applications. The analysis of these structures requires a study of layered medium. In the literature, several dimensionally reduced models have been proposed for the study of these structures. The models based on displacement formulation, like shear deformable[1] and zig-zag models,[2] are very popular as they are independent of the number of layers in the structure.

However, a major drawback of these theories is that the transverse state of stress represented by them is not accurate. To alleviate this problem the equilibrium based post-processing approach must be used to get better representation[3]-.[4] This approach is accurate with these models for most of the cases. However, for domains with cut-outs and unsymmetry in lay-up sequence, these models become ineffective. This problem is overcome by using a more refined model. In layerwise models, the standard models are used layerwise. In these models, the continuity of displacements (and transverse stresses shear stresses in some cases) is imposed. Intermediate model is obtained by lumping one or more laminae below or above the lamina of interest. The layerwise model proposed here is adapted from Ahmed and Basu.[5] It is modified according to Schwab[8] for bending and membrane dominated problems. In the present paper only bending case is studied.

The strong three dimensional effects are localized in the vicinity of boundaries (boundary-layer), vertices (vertex singularity), edges (edge and vertex edge singularity), damaged lamina and delaminations (see Fig. 1). In these regions it is desired to enrich the approximation. This is achieved by proposed region-by-region model. This approach leads to tremendous savings in computational cost and gives accurate state of stress

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and displacements. This approach is a generalization of the planar constrained approximation approach of Demkowicz et al[6] and the $h-d$ approach of Stein et al,[7] given for homogeneous materials.

In the present paper, the comparison of layerwise, intermediate, equivalent and region-by-region models on the basis displacements and state of stress is carried out. Further, their effect on predicting first-ply failure load is also studied. In the detailed paper some more numerical examples will be studied.

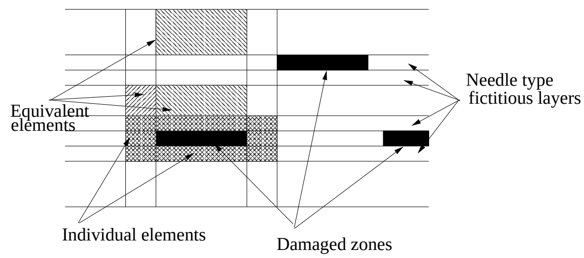


Figure 1. General scenario in lminated composites

II. Plate Models

Analysis of thin laminated structures is based on using predefined director functions in the z-direction, with the displacement field given as a series in terms of products of the director functions and planar functions. Various families of plate models can be defined based on the specific definitions of the director functions. The plate models employed in this study belong to the families of plate models given below.

A. Equivalent Models (EQ)

These are the conventional popular plate models, with CLPT and HSDT models as special cases. The displacement fields corresponding to these models can be defined as:

$$\{u\} = \left\{ \begin{array}{c} u_1\,(x,y,z) \\ u_2\,(x,y,z) \\ u_3\,(x,y,z) \end{array} \right\} = \left\{ \begin{array}{c} \sum_{i=1}^{p_z^1+1} u_{1i}\,(x,y)\,\phi_i\,(z) \\ \sum_{i=1}^{p_z^2+1} u_{2i}\,(x,y)\,\psi_i\,(z) \\ \sum_{i=1}^{p_z^3+1} u_{3i}\,(x,y)\,\zeta_i\,(z) \end{array} \right\} \tag{1}$$

Here, p_z^i, $i=1,2,3$ is the order of the polynomial director functions in the z-direction for u_1, u_2 and u_3 , respectively. Generally $p_z^1 = p_z^2$ is chosen. Here, z is chosen with respect to the middle plane of the plate. Note that for isotropic plates and also for symmetric laminates subjected to transverse loads, $u_1(x,y,z)$ and $u_2(x,y,z)$ are antisymmetric, while $u_3(x,y,z)$ is symmetric with respect to z. Hence, following Schwab[8] and Reddy,[12] $p_z^1 = p_z^2 = 1$ and $p_z^3 = 0$ is chosen as the first order shear deformable theory. This is represented as the $(1,1,0)$ model. The $HSDT$ model is generally taken as $(3,3,0)$. The director functions are polynomials defined over the full thickness. Following Schwab,[8] a natural hierarchy of such models is given by (1,1,0), (1,1,2), (3,3,2), (3,3,4),$\cdots\cdots$. These models generally correspond to bending effect (i.e. plate under transverse loading). In the current study, both the conventional definition and the sequence of models due to Schwab[8] are used. Conventionally, the functions $\phi_i\,(z) = \psi_i\,(z) = \zeta_i\,(z)$ are taken as the monomials z^{i-1}. Here we have taken $\phi_i\,(z) = \psi_i\,(z) = \zeta_i\,(z) = M\,(\hat{z})$, where $M(\hat{z})$ is the Legendre polynomial based hierarchic shape function (see Figure 2(a)) defined in terms of $\hat{z} = \frac{2}{t}z$, where t is the total thickness of the plate.

B. Intermediate models (IM)

Generally, the critical local quantities of interest are desired in particular lamina or at the interface of two laminae. For example, analysis of initially damaged laminates with embedded lamina level damage or delaminations requires accurate representation of the local displacement, strain and stress state. In this case, the equivalent models cannot be used. The intermediate models are based on defining the director functions for a group of laminae and not the full laminate. From Figure 2(b), it can be seen that the director functions are defined as the one dimensional hierarchic basis functions $\bar{M}_i(z)$, with the displacement field defined as:

$$\{u\} = \left\{ \begin{array}{c} u_1(x,y,z) \\ u_2(x,y,z) \\ u_3(x,y,z) \end{array} \right\} = \left\{ \begin{array}{c} \sum_{i=1}^{n_1} u_{1i}(x,y)\,\bar{M}_i(z) \\ \sum_{i=1}^{n_2} u_{2i}(x,y)\,\bar{M}_i(z) \\ \sum_{i=1}^{n_3} u_{3i}(x,y)\,\bar{M}_i(z) \end{array} \right\} \tag{2}$$

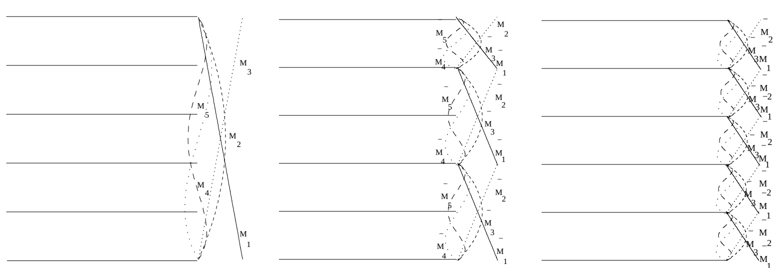
Generally, $n_1 = n_2$. Note that now n_1 and n_3 depend on the number of lamina groups in the thickness direction, with the order of the director functions taken to be the same in each group of laminae, given by p_z^1, p_z^2 and p_z^3. This leads to an increase in the number of director functions $\bar{M}_i(z)$ and hence the number of unknown functions $u_{ij}(x,y)$. Similar to the convention employed above, these models will be denoted by $IMp_{xy}p_z^1p_z^2p_z^3$.

C. Layerwise Models (LM)

This is the most general three-dimensional representation of the displacement field. Each lamina is taken as a separate group and the director functions are defined as the one dimensional basis functions defined over the lamina. From Figure 2(c), it can be seen that the representation of the displacement field is given by:

$$\{u\} = \left\{ \begin{array}{c} u_1(x,y,z) \\ u_2(x,y,z) \\ u_3(x,y,z) \end{array} \right\} = \left\{ \begin{array}{c} \sum_{i=1}^{n_1} u_{1i}(x,y)\,\bar{M}_i(z) \\ \sum_{i=1}^{n_2} u_{2i}(x,y)\,\bar{M}_i(z) \\ \sum_{i=1}^{n_3} u_{3i}(x,y)\,\bar{M}_i(z) \end{array} \right\} \tag{3}$$

where $n_1 = n_2$ and n_3 depend on the order of approximation $p_z^1 = p_z^2$, p_z^3 and the number of laminae (or layers) nl in the laminate. Hence, here the number of unknowns grows with the number of laminae. Members of this family of models will be represented by $LMp_{xy}p_z^1p_z^2p_z^3$. The present model is adapted from Ahmed and Basu.[5]



(a) Equivalent (b) Intermediate (c) Layerwise

Figure 2. Director approximations over layerwise, intermediate and equivalent models

III. Region-by-region Model

In a structural component, the "hot-spot" are generally localized in the vicinity of structural details, boundaries of the domain (faces and edges), re-entrant corners, cut-outs, existing delaminations and ply-failure zones. The solution is unsmooth in the vicinity of these details, while it is very smooth in the remaining part of the domain (see Figures 1 and 3). In order to get an accurate representation of the solution everywhere, it is desirable to use an enriched approximation model (LM or IM with sublaminae if desired) only in the vicinity of the "hot-spot", while in the rest of the domain, a lower order model will suffice. In this study, p_{xy} is uniform over the whole domain, while the approximation enrichment is done by using either a higher value of p_z^i and/or a more refined model, e.g. IM or LM. Thus it is important to build the capability to put any desired model in a specified region, rather than doing an overkill by using a higher model everywhere in the domain (which will be computationally very expensive). This concept has been introduced through the region-by-region modeling approach.

In the detailed paper the concept of regions, concept of groups and imposition of constraints and finite element implementation will be described.

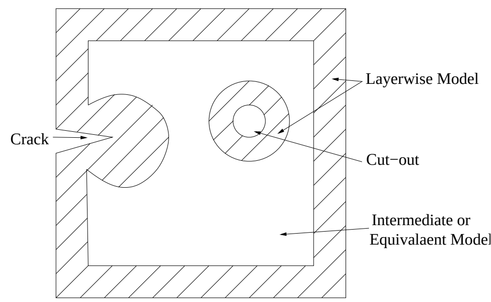


Figure 3. A typical plate domain with cut-outs

IV. Numerical Results

The major goal of this paper is to present an approach through which the local three-dimensional state of stress can be obtained in a laminated plate structure, in a designated region of interest, with optimal computational effort. Hence, the local accuracy of the EQ, IM and LM families of plate models will be analyzed first. This will be followed by a study of the efficacy of region-by-region implementation, for various problems.

A. Effect of model on accuracy of point-wise data

In this section the EQ, IM and LM models are compared for the transverse deflection and state of stress.

1. Comparison of transverse deflection

These models are compared with the exact three-dimensional elasticity solution for 7 layer [0/90/0/90/0/90/0] cross-ply, square laminate and 9 layer [0/90/0/90/0/90/0/90/0] cross-ply, square laminate. Total thickness of 0° layers is equal to total thickness of 90° laminae and laminae of same orientation are of equal thickness. The material properties are as given in Pagano and Hatfield.[9] The plate has dimension $a = b = St$ along x-axis and y-axis, respectively. Here, $S = \frac{a}{t}$ is aspect ratio with $t = 6\,mm$ for 7 layers and $t = 10\,mm$ for 9 layers. The plate is subjected to transverse sinusoidal loading of the form

$$T_3(x, y) = q_0(x, y)\,\sin\left(\frac{\pi\,x}{a}\right)\,\sin\left(\frac{\pi\,y}{b}\right)$$

All edges of the laminate are soft simple supported. The transverse deflection ($w = u_3$) is nondimensionalized as $\bar{w} = \frac{\pi^4\,Q\,w\left(\frac{a}{2},\frac{b}{2},0\right)}{12\,q_0\,S^4\,t}$, where

$$Q = 4\,G_{12} + [E_{11} + E_{22}\,(1 + 2\,\nu_{23})]\,/\,(1 - \nu_{12}\nu_{21})$$

The nondimensional thickness is defined as $\bar{z} = \frac{z}{t}$. The values of $\bar{w}$ are tabulated in Table 1 and 2. The models used are $LM3332$, $IM3332$ and $EQ3332$. The number in parenthesis are the percentage error in the computed transverse displacements. Note that for the intermediate model, the bottom four layers of the laminate are lumped together to form an equivalent layer whereas the top three layers are individual layers. For 9 layered laminate, bottom six layers are lumped together to form an equivalent layer.

Table 1. Comparison of transverse displacement ($\bar{w}$) for cross-ply 7 layer square laminate under sinusoidal loading

S	Exact	LM3332	IM3332	EQ3332
2	12.342	12.341 (0.008)	11.866 (3.86)	13.790 (-11.73)
4	4.153	4.153 (0.00)	3.846 (7.39)	3.712 (10.62)
10	1.529	1.529 (0.00)	1.473 (3.66)	1.417 (7.32)
20	1.133	1.133 (0.00)	1.119 (1.25)	1.104 (2.55)
50	1.021	1.021 (0.00)	1.019 (0.19)	1.017 (0.39)
100	1.005	1.005 (0.00)	1.005 (0.00)	1.004 (0.09)

Table 2. Comparison of transverse displacement ($\bar{w}$) for cross-ply 9 layer square laminate under sinusoidal loading

S	Pagano[9]	LM3332	IM3332	EQ3332
2	12.288	12.306 (-0.16)	12.121 (1.35)	13.579 (-10.51)
4	4.079	4.079 (0.00)	3.713 (8.97)	3.653 (10.44)
10	1.512	1.512 (0.00)	1.438 (4.89)	1.406 (7.01)
20	1.129	1.129 (0.00)	1.109 (1.77)	1.101 (2.48)
50	1.021	1.021 (0.00)	1.019 (0.19)	1.016 (0.48)
100	1.005	1.005 (0.00)	1.005 (0.00)	1.004 (0.09)

From the table we observe that

1 The $LM3332$ model accurately predicts the transverse deflection accurately for all the aspect ratios. The error in the values ranges from $0 - 0.16\%$.

2 The $IM3332$ and $EQ3332$ models are far from the exact one for the aspect ratios upto $S = 10$, i.e. for thick plates. The error for these aspect ratios ranges from $2 - 12\%$.

3 For the $IM3332$ and $EQ3332$ models with aspect ratios $S > 10$ the displacement is close to exact. The error is $0 - 2.5\%$.

4 The $IM3332$ model is more accurate, as compared to the $EQ3332$ model.

2. Comparison of state of stress

Here we consider various types of problems to study the accuracy of the region-by-region modeling approach. This modeling approach is compared with EQ , LM and IM models.

Here, various stress components for symmetric and antisymmetric laminates, under cylindrical bending, are compared with the exact values given in Pagano.[10] The cylindrical bending load is of the form $T_3(x, y) = q_0 \sin\left(\frac{\pi x}{a}\right)$. The $LM3332$ and $EQ3332$ models are used for computing the stresses.

Here, $[150/-150/150]$ laminate is considered such that the thickness of each layer is $[1, 2, 1]$ and $t = 4mm$. The normalised in-plane stress $\bar{\sigma}_{xx}$ is plotted at $\left(\frac{a}{2}, \frac{b}{2}, \bar{z}\right)$ while the transverse shear stresses are plotted at $\left(0, \frac{b}{2}, \bar{z}\right)$. Since the transverse stresses are shown for a point on the boundary, the $RR-I$ scheme is used for these results. The in-plane stresses are given for an interior point, hence, the $RR-II$ scheme shown in Figure 4(b) is used. Here the darker region has $LM3332$, grey region has $LM3112$ and remaining region has $EQ3112$. The transverse shear stresses are also obtained by another scheme ($RR-III$) shown in Figure 4(c) where the darker region has $EQ3332$ and remaining region has $EQ3112$ model. Note that the result are also obtained by using $EQ3332$, $EQ3112$ and $LM3332$ models in the full domain for comparison with the, $RR-II$ and $RR-III$ schemes. The in-plane stresses and transverse shear stresses for $S = 4$ are shown in Figure 6 and Figure 7, respectively.

Remark: In Figure 4 truncated mesh (in the y-direction) is shown.

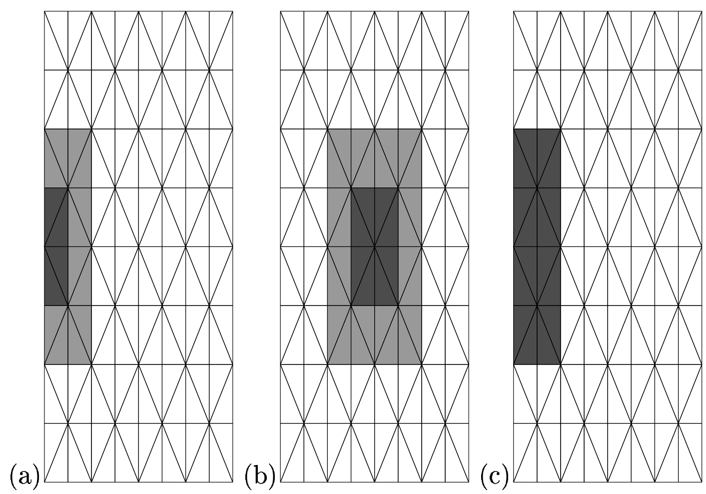


Figure 4. Region-by-region schemes

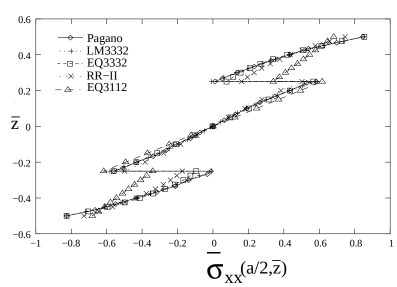


Figure 5. Comparison of in-plane stress for $[150/-150/150/-150]$ laminate

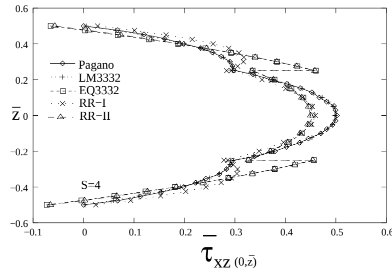


Figure 6. Comparison of transverse stress for $[150/-150/150/-150]$ **laminate (direct stresses)**

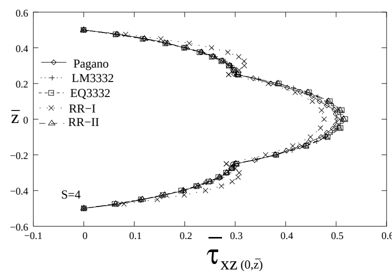


Figure 7. Comparison of transverse stress for $[150/-150/150/-150]$ **laminate (equilibrium based post-processing)**

From the results it is observed that:

1 The direct values of the transverse stresses obtained using $RR-I$ are very accurate, while the $EQ3332$ model gives inaccurate values of the local transverse stresses.

2 For $[150/-150/150]$ laminate the stresses obtained by the $RR-III$ are very close to those obtained by the $EQ3332$ model.

3 The values of $\bar{\sigma}_{xx}$ obtained by the $EQ3112$ model are not close to the exact one. However, the $EQ3332$ model gives accurate values of $\bar{\sigma}_{xx}$.

4 The transverse stress components obtained by equilibrium approach of postprocessing for $RR-III$ are in good agreement with the exact ones.

5 The in-plane stresses obtained by the $RR-II$ are much closer to the exact one as compared to that obtained using the $EQ3112$ model everywhere. Here, the $EQ3332$ model is more accurate than the $EQ3112$ model.

In Table 3 the number of unknowns for all the models used in these cases are compared.
From this table it is observed that:

1 The $LM3332$ approach is very compute intensive.

2 The $RR-I$, $RR-II$ and $RR-III$ schemes leads to tremendous saving in computational cost, as compared to $LM3332$ (or $EQ3332$ model for $RR-III$), and also give local accuracies comparable to the $LM3332$ model (or $EQ3332$ model for $RR-III$).

3 The saving in cost and gain in local accuracy becomes more significant as the number of laminae, in the laminate increase.

Table 3. Number of unknowns for models and numerical examples

Model	Quantity	Unknowns
Throughout $LM3332$	$\bar{\sigma}_{xx}, \bar{\tau}_{xz}, \bar{\tau}_{yz}$	16875
Throughout $EQ3332$	$\bar{\sigma}_{xx}, \bar{\tau}_{xz}, \bar{\tau}_{yz}$	6875
Region-by-region (Scheme I)	$\bar{\tau}_{xz}, \bar{\tau}_{yz}$	5239
Region-by-region (Scheme II)	$\bar{\sigma}_{xx}$	5147
Region-by-region (Scheme III)	$\bar{\tau}_{xz}, \bar{\tau}_{yz}$	4715

B. First-ply failure load prediction

The first-ply failure load is obtained by using Tsai-Wu[11] criterion is given by

$$FI_{TW} = F_i\,\sigma_i + F_{ij}\,\sigma_i\,\sigma_j \geq 1 \tag{4}$$

where F_i, F_{ij}, and F_{ijk} are the strength tensor terms and σ_i are the stress components. As an example the $[-45/45/-45/45]$ laminate is taken. The plate is either clamped on all edges or hard simple supported. The top face of the plate is subjected to uniform transverse load $T_3(x, y) = q_0$. The plate dimensions the material properties and lamina thickness used in these computations are given in Reddy and Reddy.[12]

The results obtained from the present analysis are compared with those reported in Reddy and Reddy.[12] The loads are computed using $LM2332$, $EQ2332$ and $RR2332$. For the region-by-region model the failure load is first computed by $EQ2332$ model. The elements with failure index above 0.8 are found. One element neighborhood of these elements shaded dark grey the $LM2332$ model is used, while in the remaining region the $EQ2332$ model is used (for example see Figure 8). The first-ply failure loads are given in Tables 4-7.

Table 4. First-ply failure loads; all edges clamped, $[-45/45/-45/45]$ laminate under uniform transverse loading, (direct stresses); $p_{xy} = 2$

Model	FLD	Coordinates		Layer	Location	Unknowns	Max.σ
		x	y				
Reddy[12]	39354.8	≈ 115.00	≈ 125.00	1	bottom	-	-
$EQ2332$	39612.5	121.38	126.43	1	bottom	6875	σ_{22}
$RR2332$	39604.2	107.51	0.56	1	bottom	9323	σ_{22}
$LM2332$	39581.4	107.51	0.56	1	bottom	21875	σ_{22}

Table 5. First-ply failure loads; all edges hard simple supported, $[-45/45/-45/45]$ laminate under uniform transverse loading, (direct stresses); $p_{xy} = 2$

Model	FLD	Coordinates		Layer	Location	Unknowns	Max.σ
		x	y				
Reddy[12]	32513.5	≈ 113	≈ 65	4	top	-	-
$EQ2332$	32464.1	227.69	19.83	4	top	6875	σ_{22}
$RR2332$	32745.5	1.20	107.16	4	top	15587	σ_{22}
$LM2332$	32742.6	1.20	107.16	4	top	21875	σ_{22}

Table 6. First-ply failure loads; all edges clamped, $[-45/45/-45/45]$ laminate under uniform transverse loading, (equilibrium based post-processed stresses); $p_{xy} = 2$

Model	FLD	Coordinates		Layer	Location	Unknowns	Max.σ
		x	y				
Reddy[12]	39354.8	≈ 115.00	≈ 125.00	1	bottom	-	-
$EQ2332$	32382.6	121.38	126.43	1	bottom	6875	σ_{22}
$RR2332$	32386.1	107.51	0.56	1	bottom	8987	σ_{22}
$LM2332$	32549.2	107.51	0.56	1	bottom	21875	σ_{22}

Table 7. First-ply failure loads; all edges hard simple supported, $[-45/45/-45/45]$ laminate under uniform transverse loading, (equilibrium based post-processed stresses); $p_{xy} = 2$

Model	FLD	Coordinates		Layer	Location	Unknowns	Max.σ
		x	y				
Reddy[12]	32513.5	≈ 113	≈ 65	4	top	-	-
$EQ2332$	30821.2	227.69	19.83	4	top	6875	σ_{22}
$RR2332$	31155.0	1.20	107.16	4	top	15227	σ_{22}
$LM2332$	31078.2	1.20	107.16	4	top	21875	σ_{22}

In the present study, the transverse stress components obtained from the equilibrium based post-processing approach, have been used in computing the failure load.

We observe that:

1 The first-ply failure loads obtained by $RR2332$ and $LM2332$ models are close.

2 The locations predicted by the $RR2332$ and $LM2332$ models are close.

3 The number of unknowns for $LM2332$ model are significantly higher as compared to the $RR2332$ model.

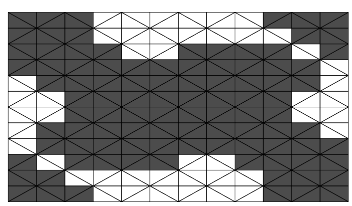


Figure 8. Layerwise and equivalent single layer regions for [-45/45/-45/45] laminate with all edges hard simple supported (equilibrium based post-processing)

V. Conclusion

In this paper a family of plate models is compared for their accuracy in predicting the pointwise quantities like displacement and state of stress at a point. Further, a new region-by-region approach is introduced. Following are the major conclusions of this study.

1 The layerwise model accurately captures the local state of stress for all laminated composite plates, for different plate thicknesses.

2 The equivalent and intermediate models are more reliable for thin plates while for thicker plates these models can lead to erroneous results.

3 For thicker laminates the layerwise shear effects can be very pronounced and hence the equivalent models cannot capture this effect.

4 The in-plane stress components computed by all the models are accurate.

5 The transverse stress components computed by direct use of finite element data for layerwise model are in good agreement with exact one.

7 The equilibrium approach for computing transverse stresses is accurate for all the models.

8 The failure loads obtained by region-by-region model are very close to that obtained using layerwise model.

9 The failure loads obtained using equilibrium based post-processing approach are lower than those obtained using direct stresses.

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