A New Three-Valued Paraconsistent Logic

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Paraconsistent logic and three-valued semantics: The term \textit{Paraconsistent} was first used by the Peruvian philosopher Francisco Miró Quezada in the Third Latin America Conference on Mathematical Logic in 1976. A logic is called paraconsistent if there are formulas \(\phi\) and \(\psi\) such that \(\{\phi, \neg\phi\} \nvdash \psi\). Besides other semantics, the three-valued semantics of (paraconsistent) logics have always received special attentions from logicians like J. Lukasiewicz, S. C. Kleene and others and paraconsistentists like S. Jaśkowski, N.C.A. da Costa, G. Priest, R. Brady, C. Mortensen, D’Ottaviano, W. A. Carnielli, João Marcos etc. Para-inconsistency axioms have been introduced in [5] in a way similar to classical two-valued logic.

Introduction of the three-valued matrix \(\text{PS}_3\): Here we shall introduce a new three-valued matrix, \(\text{PS}_3 := \langle\{1, \frac{1}{2}, 0\}, \wedge, \vee, \Rightarrow, *\rangle\) where \(\{1, \frac{1}{2}, 0\}\) is a distributive lattice and the designated set of \(\text{PS}_3\) has been fixed as, \(\{1, \frac{1}{2}\}\). From the truth tables of \(\text{PS}_3\) it can easily be verified that \(\frac{1}{2} \wedge \frac{1}{2}\) * \(0 = 0\) and hence \(\text{PS}_3\) might be a three-valued semantics of some paraconsistent logic.

Proof theory for \(\text{PS}_3\): The main part of this work is to develop a propositional logic \(\text{LPS}_3\) so that \(\text{PS}_3\) becomes the three-valued semantics of it. We have proved that \(\text{LPS}_3\) is sound and complete with respect to \(\text{PS}_3\). It will then be discussed how does \(\text{LPS}_3\) satisfy Jaśkowski’s criterion (cf. [6]) of being a paraconsistent logic.

Comparison with other existing three-valued paraconsistent logics: A comparison between \(\text{LPS}_3\) and some other paraconsistent logics having three-valued semantics, such as \(\text{LP}\) (Priest’s Logic of Paradox) [8], \(\text{LFI}_1\) (Logic of Formal Inconsistency 1) and \(\text{LFI}_2\) (Logic of Formal Inconsistency
2) [3], J₃ (D’Ottaviano’s logic) [4], RM₃ [2], P₁ (Sette’s three-valued logic) [9], C₀₂ (Mortensen’s paraconsistent logic) [7] will be made. Particularly PS₃ has close connections with the three-valued models of the paraconsistent logics P₁ (or C₀₁) and C₀₂. It is worthwhile to show, how do these logics differ pair wise. It is proved that LPS₃ is maximal relative to the classical propositional logic.

Paraconsistent set theory: The motivation of finding the algebra PS₃ is to build a model of some paraconsistent set theory. The paraconsistent logic LPS₃ can be used in some algebra-valued set theory construction similar to the Boolean-valued construction (cf. [1]) to obtain a model of a (weak) paraconsistent set theory.

References


